Global, Two-fluid, Electromagnetic, Nonlinear Simulations of Tokamak Turbulence and Transport Scaling Laws

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1. Introduction: The CUTIE global, nonlinear, electromagnetic turbulence simulation code\(^1\) has been developed further and used to study scalings of global particle and energy confinement in systematic scans of \(\rho_s, \beta, \nu_s\), and isotope mass at constant \(q\) and geometry. The code results show gyro-Bohm dependence of energy and particle confinement times on \(\rho_s\) for the larger values of this parameter. In accordance with ITER, JET and DIII-D data bases, the dependence of the turbulence on \(\nu_s\) is found to be weak. However, in agreement with previous CUTIE results, there is a significant degradation of confinement with \(\beta\). Thus the simulations imply \(\tau_E \propto \tau_{Bohm} \rho_s^{-1} \beta^{-1.5}\). While the degradation with \(\beta\) is consistent with L-mode scaling, the ITER98H scaling gives a weaker \(\beta\) dependence \((\propto \beta^{-0.5})\). Simulations with flows show substantial improvement in confinement depending on whether the \(\mathbf{E} \times \mathbf{B}\) flow shear is increased or decreased by the imposed toroidal flow. Simulations under realistic high \(\beta\) COMPASS-D conditions\(^2\) with monotonic \(q\) profiles indicate stabilization of turbulence in the core where an internal transport barrier in electron and ion temperatures and particle density is predicted to occur. Verification or otherwise of these predictions will be addressed in future experimental campaigns. The simulations suggest generally that turbulent transport in systems where the sources are prescribed (as opposed to gradients) has an intermittent or ‘bursty’ character, and is strongly influenced by transient ‘corrugations’ of the plasma density, pressure and current profiles generated by the nonlinear modal interactions and the self-consistent radial electric fields and bootstrap currents implied by them. The CUTIE code solves the well-known\(^1\) two-fluid, quasi-neutral plasma equations of motion for the variables: \(n_e, T_e, T_i, V_i\), the potentials, \(\phi, \psi\), and the vorticity, \(\Theta \equiv \nabla \cdot (n \nabla \phi)\). Periodic cylinder equilibrium including mode-coupling and destabilizing curvature effects. The equilibrium profiles and the turbulent fluxes are self-consistently co-evolved. Thus, the fields are evolved by:

\[
\frac{\partial \Theta}{\partial t} + i \mathbf{k} \cdot \mathbf{v}_0 \Theta + i k \| \mathbf{V}_A \rho_s^2 \nabla^2 \psi = \Sigma_\Theta
\]

\[
\frac{\partial \psi}{\partial t} + i \mathbf{k} \cdot \mathbf{v}_\phi \psi + i k \| \mathbf{V}_A \phi = \Sigma_\psi
\]

\[
\Sigma_\Theta = \mathbf{V}_A \rho_s \left[ \frac{1}{r} \frac{\partial \psi^*}{\partial \theta} \frac{4\pi \rho_s d_j}{cB} + \mathbf{V}_0 \rho_s \frac{1}{r} \frac{\partial (\psi^*, \rho_s^2 \nabla^2 \psi^*)}{\partial (r, \theta)} \right.
\]

\[
+ \mathbf{V}_0 \rho_s \left[ \frac{1}{r} \frac{\partial (\Theta^*, \phi^*)}{\partial (r, \theta)} + \left( \frac{N^* T_{i0}}{n_0(r) T^*} \right) \frac{1}{r} \frac{\partial (\Psi^*, n^*)}{\partial (r, \theta)} \right]
\]

\[
- \frac{2}{R_0} \left[ \frac{\cos \theta}{r} \frac{\partial p^*}{\partial \theta} + \sin \theta \frac{\partial p^*}{\partial r} \right] + \nabla \cdot (D_\Theta \nabla \Theta)
\]
\[ \Sigma_\psi = V_{th} \rho_s \left[ \frac{1}{r} \frac{\partial (\psi^*, \phi^*)}{\partial (r, \theta)} - \left( \frac{N^* T_\phi}{n_0(r) T^*} \right) \frac{1}{r} \frac{\partial (\psi^*, n^*)}{\partial (r, \theta)} \right] + V_A \left( \frac{N^* T_\phi}{n_0(r) T^*} \right) \nabla \cdot \nabla + \nabla (D_\psi \nabla \psi) \]

In addition, the code solves the electron continuity, ion parallel momentum and the two energy equations. Detailed definitions of various symbols and descriptions of the solution procedure can be found in Ref. 1. The fluctuating potentials, \( \delta \phi, \delta \psi \) are in Gaussian units with non-dimensional forms: \( \phi^* = \frac{\delta \phi}{T}, \psi^* = \frac{\delta \psi}{B_{Boh} \beta^{1/2}} \). Aliasing at high \( k \) is avoided by a turbulent viscosity, \( \nu_{turb} \approx V_{ih}\tau(R(\delta)^2 + \delta \Theta^2) \).

2. Global confinement scaling studies: The code has been applied to study global confinement \( (\tau_E/\tau_{Bohm}) \) as a function of \( \rho_s, \beta, \nu_s, A \) and \( Ma \), where \( A = m_i/m_e \) and \( Ma \) is the flow Mach number of any imposed toroidal plasma flows. It includes electromagnetic effects measured by the ‘drift Alfvén’ parameter, \( \Delta_A = \frac{\rho_s}{\omega_A} = \frac{\nu_s}{\omega_A k_1^{3/2}} \propto \beta^{3/2} \left( \frac{\rho_{Boh}}{\rho_s} \right) \). Fixing \( Z_{eff}, R/a, q_{a0} \), the parameters, \( \rho_s, \nu_s, \beta, A \) have been separately varied. Conditions studied: \( \beta \approx 0.8\%, \beta_N \approx 1.55, \nu_s \approx 0.14 - 0.28, \rho_s \approx 0.05 - 0.015, q_{a0} = 3.5, R/a = 3, Ma = 0 \). The following results were obtained: \( \tau_E, \tau_{Bohm} \) have gyroBohm dependence on \( \rho_s \), except for the smallest value \( (=0.015) \) tried. There is little or very weak \( \nu_s \) dependence (consistent with the ITER scaling, \( \nu_s^{0.11} \)). A scan over \( \beta \) shows degradation with \( \beta \) in both particle and energy confinement: \( \frac{\tau_E}{\tau_{Bohm}} \approx \beta^{-1} \) when \( \beta_N \) is varied from 1.55 to 2. For these conditions, \( \frac{\tau_E}{\tau_{Bohm}} \approx \beta^{-3/2} \). This is like L-Mode but different from JET, DIII-D databases in H-Mode. A favourable isotope effect is found when \( \rho_s, \beta, \nu_s, q_{a0}, Ma \) are fixed and \( A \) is varied. In many cases, at high \( \beta_N \), the \( (2,1) \) mode is prominent (cf. Ref. 2) and grows algebraically with time.

3. Corrugated profiles and meso-scale dynamics of transport barriers: The turbulent fluxes are quadratic in the amplitudes which implies, in addition to the time and magnetic-surface averaged flux components which are usually described in terms of an ‘effective transport matrix’, the existence of ‘corrugated’ components. These drive relatively rapid (‘mesoscale’) spatio-temporal variations in the density, temperature and current density profiles. This transient flux effect of turbulence on the \( m = n = 0 \) plasma properties such as the radial electric field and plasma current density has important feed-back effects on the growth, saturation and spectral distributions of the turbulence. Recent calculations with CUTIE have simulated internal transport barriers (ITBs) which have been seen on several machines. In a simulation of COMPASS-D in ECH heated conditions\(^2\), transport barriers form ‘spontaneously’ near the \( q = 4/3 \) surface (\( q_0 \approx 1.1 \)).
Both density and temperatures appear to develop sharp gradients close to the resonant point and this implies an increase in the local bootstrap current as well as the radial electric field shear. In turn, these quantities react on the turbulence. The $q$ profiles tend to flatten locally and the turbulence is suppressed in a region close to the rational surface. Some simulation results are illustrated in Figs. 1a-d for centrally heated (ECH) plasmas in typical COMPASS-D conditions: $R_0 = 0.55m$, $a = 0.2m$, $\n_i > \approx 10^{19}m^{-3}$, $B_\phi = 1.1T$, $I_p \approx 112kA$ , $q_0 = 1.14$ , $q_{55} = 3.54$, $P_{ECH} = 1MW$ radially distributed like $\exp(\n - (r/a)^2$, $\rho_s = 0.05$, $\beta = 1.9\%$, $\beta_N \approx 3$. Both $T_e$, $T_i$ also show strong barriers near $q = 1.33$, with $T_e$ gradients of order 1 keV/m at the ITB. Movies have been made of the transient dynamics of the process. They suggest that close to rational values of $q$, reduced magnetic shear and increased electric drift shear have a synergistic effect on the turbulence and introduce profile corrugations which in turn support the barriers. These findings also apply to TEXT-U$^3$ and RTP$^4$ conditions. Simulations for the TEXT-U conditions (ECH heated discharge with a $q = 1$ surface) are illustrated in Figs. 2a, 2b. The electron temperature and density (not shown) develop ‘ears’ near the $q = 1$ barrier(Fig.2a). There is a slowly growing $m = 1$ ‘snake’ (cf. Ref.3) which actually occurs just within the barrier(Fig. 2b).

4. Conclusions: The results presented suggest that CUTIE simulations are in qualitative agreement with some of the remarkable results obtained by the RTP group$^1$, and the observations in TEXT-U$^3$. They indicate interesting structural features in the turbulence: there appears to be a strong inverse cascade to relatively long wavelength, low $n$ balloon type modes in addition to gross modes like ‘snakes’, which form near mode rational radii. The mode dynamics (rotation, oscillations) is often complex and involves locking, periodic relaxation oscillations and bursts of intermittent, high frequency turbulence. The frequency and wave number spectra are computed and have been analysed. They reveal a wealth of fascinating detail, many of which resemble experimental observations in JET and other machines. It is therefore reasonable to conclude that the dynamical system modelled by CUTIE is sufficiently rich to capture several interesting features of actual tokamaks. In particular, it seems important to recognize that features like the ‘ears’ in the profiles observed in RTP and TEXT-U are indicative of the complex nonlinear, nonlocal interactions between rapidly varying components of the turbulent fluxes and the plasma profiles. Oscillatory, highly sheared radial electric fields and bootstrap currents appear to play key roles in the evolution of transport barriers as simulated by CUTIE and possibly also in real tokamaks.

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References


Fig.1a $n_{e0}(r,t)$, for COMPASS-D

Fig.1b $eE_z(r,t)/B$ for COMPASS-D

Fig.1c $\delta B_z(r,t)/B$ (rms)

Fig.1d Enstrophy spectrum (3.35 ms)

Fig.2a $T_e$, for TEXT-U (Note ‘ears’)

Fig.2b $\psi^*$, for TEXT-U ($m=1$ ‘snake’)