Modelling of Toroidal Alfvén Eigenmodes and Fishbones in the START Spherical Tokamak

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1. Introduction
Spherical tokamaks (STs) have densities comparable to those of conventional tokamaks but much lower magnetic fields, and hence lower particle energy thresholds for the excitation of toroidal Alfvén eigenmodes (TAEs). Such modes were observed for the first time in ST geometry during neutral beam injection (NBI) experiments on the UKAEA Small Tight Aspect Ratio Tokamak (START) [1]. Hydrogen (H) atoms were injected with energy \( E \approx 30 \text{ keV} \) and hence speeds \( v \approx 2.4 \times 10^6 \text{ m s}^{-1} \): a typical value of the Alfvén speed \( c_A \) was \( 10^6 \text{ m s}^{-1} \). Fusion \( \alpha \)-particles in ST plasmas would also be born with \( v > c_A \) [1], and so NBI on START has provided an opportunity to simulate \( \alpha \)-particle effects in an ST power plant: \( \alpha \)-driven TAEs could have the effect of making ignition more difficult to achieve. Energetic ions can also drive unstable internal kink modes, giving rise to “fishbone” oscillations, which can degrade energetic ion confinement. As in the case of TAEs, the use of NBI on START made possible the first experimental investigation of fishbones in ST geometry.

2. Observations
NBI-heated START plasmas had major radius \( R_0 \approx 0.3-0.37\text{ m} \) and minor radius \( a \approx 0.23-0.3\text{ m} \). H was co-injected tangentially to the magnetic axis into deuterium target plasmas. Modes with fixed frequencies \( \nu \approx 50 - 350 \text{ kHz} \), lasting for 1–5 ms and with poloidal mode numbers \( m \sim 1-4 \), were detected using Mirnov coils in discharges with beam power \( P_{\text{NBI}} \lesssim 500 \text{ kW} \) or during the early phase of discharges with \( P_{\text{NBI}} \approx 800 \text{ kW} \). In the latter case electron density \( n_e \) and current had not reached flat-top, so the absorbed beam power was relatively low. The toroidal beta \( \beta_T \) was also low when these modes were observed (\( \beta_T \lesssim 3-5 \% \)). The frequencies were comparable to the TAE gap frequency \( \nu_A \equiv c_A/4\pi qR_0 \approx 200 \text{ kHz} \) (\( q \) being the safety factor). Because these modes appeared infrequently, and for periods shorter than the rise time of \( n_e \), it was not possible to determine a scaling of \( \nu \) with \( n_e \). The power spectrum of a fixed-frequency mode observed during the early phase of a discharge with \( P_{\text{NBI}} \approx 800 \text{ kW} \) is shown in Fig. 1. The dominant toroidal mode number \( n \) was odd (probably \( n = 1 \)); the dominant poloidal mode numbers were \( m = 1, 2 \). Several distinct peaks can be seen, the strongest at \( \approx 220-250 \text{ kHz} \).
In shots with higher $\beta_T$ (≥ 3–5%) strong fishbone-like bursts were observed at $\nu < 50$ kHz, often sweeping down in frequency and coinciding with sawtooth crashes (as in Fig. 2). Fixed-frequency fishbones, and frequency-sweeping fishbones occurring in the absence of sawteeth, were also observed. Although there is no strong evidence of fishbones affecting heating or confinement, such modes were weak or absent in the highest performance shots ($\beta_T ≥ 30\%$).

3. Modelling of Toroidal Alfven Eigenmodes

We have used the CSCAS [2] and NOVA [3] codes to compute continuous magnetohydrodynamic spectra in START, with input data consisting of an equilibrium (reconstructed using the EFIT code [4]) corresponding to the power spectrum in Fig. 1. In the TAE frequency range both codes indicate the existence of a very wide toroidicity-induced gap in the Alfven continuum, centred on $\nu \sim \nu_A$ [1]. Similar predictions have been made for the National Spherical Torus Experiment [3]. The plasma displacement corresponding to a TAE with $n = 1$, computed using NOVA, is shown in Fig. 3. The frequency eigenvalue is 306 kHz; an almost identical mode with $\nu = 310$ kHz was found using the code MISHKA-1 [1,5]. As observed in shot 35305, poloidal harmonics $m = 1, 2$ are dominant.

MISHKA-1 and NOVA computations using the same equilibrium parameters have revealed the existence of several other TAFs with $n = 1$ and dominant $m = 1, 2$: frequency eigenvalues obtained using NOVA include 144 kHz, 169 kHz, 204 kHz and 272 kHz. The predicted existence of several modes in the Alfven frequency range, consistent with the multiple spike structure of Fig. 1, is due in part to low magnetic shear $s \equiv d\ln q/d\ln r$ [6], a characteristic feature of ST plasma cores. The eigenvalues depend on $q(r)$ and $n_x(r)$: neither profile was determined experimentally at the time of the spectral measurements in Fig. 1 (although a subsequent absence of sawtooth activity for ~5 ms suggests...
q(0) > 1, and interferometry data giving the line-integrated n_e are available). Uncertainties in q and n_e, combined with plasma rotation, can account for differences between computed frequency eigenvalues and the peak frequencies in Fig. 1. The fine structure in this spectrum may also be due in part to n = 2 modes or additional low shear modes (if the true q-profile were significantly flatter than that used in the modelling).

![Graph](image)

**Fig. 3.** TAE plasma displacement eigenfunction ξ(r) obtained using equilibrium parameters corresponding to Fig. 1. The square root of normalized poloidal flux is denoted by r/a, curves labelled C, D, E, ... denote m = 1, 2, 3, ..., and the q-profile is indicated by Q.

NOVA–K, a modified version of NOVA which includes leading order kinetic effects [3], was used to evaluate various contributions to the drive and damping γ/ω of the mode shown in Fig. 1. Temperatures and densities required as input to NOVA–K were measured or inferred from similar shots; the beam ion distribution f_b was computed using the LOCUST code [1], and approximated by an analytical function of energy, pitch angle and toroidal momentum. The results are shown in Table 1: the drive was computed taking into account finite orbit width (FOW) and finite Larmor radius (FLR) effects (with Larmor radii estimated locally at each point on the guiding centre orbit). Despite strong ion Landau damping, and FLR stabilization, the net drive is positive: this is due mainly to strong spatial gradients in f_b. An independent estimate of the ion Landau damping rate obtained using the CASTOR–K code [7] was somewhat lower than the NOVA–K value. Neither code includes radiative damping, which may affect the net drive.

<table>
<thead>
<tr>
<th>Damping/Drive Process</th>
<th>γ/ω (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>trapped electron collisional</td>
<td>-0.031</td>
</tr>
<tr>
<td>electron Landau</td>
<td>-0.005</td>
</tr>
<tr>
<td>plasma ion Landau</td>
<td>-1.401</td>
</tr>
<tr>
<td>beam drive (FOW effects only)</td>
<td>+3.600</td>
</tr>
<tr>
<td>beam drive (FOW + FLR effects)</td>
<td>+1.470</td>
</tr>
</tbody>
</table>

**Table 1.** Contributions to damping and drive of mode shown in Fig. 1.

### 4. Modelling of Fishbones

Because of charge exchange effects and the use of tangential co-injection, few beam ions in START were trapped, and the precessional drift fishbone mode discussed in [8] could not be driven [1]. Betti and Freidberg [9] obtained a dispersion relation for the m = 1 internal kink mode in the presence of passing energetic ions undergoing finite radial drift orbit excursions Δ_k. McClements et al. [1] noted that when the ideal kink mode is close to marginal stability this dispersion relation has real solution.  

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\[ \hat{\omega} \approx -\frac{2}{3} \frac{r_1}{R_0} \frac{\Delta_b \beta_{b0}}{r_{b0} s^3}, \]

where \( \hat{\omega} = \omega R_0/(c_A s) \), \( c_A \) and \( s \) being evaluated at the \( q = 1 \) radius \( r_1 \), \( \beta_{b0} \) is the beam ion poloidal beta, \( r_{b0} \) is the beam ion pressure scale length, and \( \Delta_b \) is evaluated at the maximum beam ion energy. Spatial redistribution of beam ions resulting from the sawtooth crashes in shot 34839 may have caused \( \beta_{b0} \), and hence \( \hat{\omega} \), to fall on the observed sub-millisecond timescale (Fig. 2). Evolution of the beam ion velocity distribution resulting from nonlinear interactions with the fishbone mode would also affect both \( \beta_{b0} \) and \( \Delta_b \), thereby accelerating the rate of frequency–sweeping. As in the case of TAEs, uncertainties in beam and plasma parameters (notably \( 1/s^3 \)) make it difficult to compare predicted mode frequencies with observations. However, using the best available estimates of the relevant parameters, we find that Eq. (1) gives \( \nu \sim 50 \text{kHz} \), as observed. To obtain Eq. (1) the real part of the potential energy \( \delta W \) associated with kink perturbations was assumed to be strictly zero: when \( \delta W \) has a small positive real part, \( \omega \) is still given approximately by Eq. (1), but there is also a finite growth rate. When \( \nu \sim 50 \text{kHz} \), a growth rate \( \gamma/\omega \geq 1\% \) is sufficient for the mode to be amplified on a timescale comparable to the observed fishbone rise time. We conclude that such bursts could have resulted from kink mode excitation by passing beam ions. Frequency–sweeping fishbones excited via this mechanism have not previously been observed.

5. Summary

Benign instabilities observed during NBI in START include modes identified as TAEs and frequency–sweeping fishbones. Continuous shear Alfvén spectra corresponding to START equilibria contain wide gaps, within which multiple TAEs could be driven unstable by beam ions. Frequency eigenvalues computed using two different codes are consistent with frequencies of modes excited in START when \( \beta_T \) was relatively low ( \( \lesssim 3-5\% \)). Damping of these modes was dominated by ion Landau damping, instability being driven mainly by beam pressure gradients. In shots with \( \beta_T \gtrsim 3-5\% \) fishbones were observed at frequencies below \( 50 \text{kHz} \); these can be attributed to \( m = 1 \) kink mode excitation arising from resonant interactions with passing beam ions.

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References