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Runaway electron generation in a cooling plasma

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The classic calculation of Dreicer generation of runaway electrons assumes that the plasma is in a steady state. In a tokamak disruption, this is not necessarily true since the plasma cools down quickly and the collision time for electrons at the runaway threshold energy can be comparable to the cooling time. The electron distribution function then acquires a high-energy tail which can easily be converted to runaways by the rising electric field. Far more runaways can be produced in this way than by the usual mechanism.

Expansion of the kinetic equation

In a disruption, the plasma cooling is caused by excitation and ionisation of impurities. Mostly thermal electrons lose energy this way, and the suprathermal electrons lose energy primarily by collisions with thermal electrons. The pitch-angle averaged kinetic equation for fast electrons is

$$\frac{\partial f}{\partial t} = \frac{\hat{\nu}_{ee} v_T^3}{v^2} \frac{\partial}{\partial v} \left(f + \frac{T}{mv} \frac{\partial f}{\partial v} \right),$$

where $\hat{\nu}_{ee} = ne^4 \ln \Lambda / 4\pi\epsilon_0^2 m v_T^3$ is the collision frequency at the thermal speed $v_T(t) = [2T(t)/m]^{1/2}$. Normalising $ds = \hat{\nu}_{ee} dt$, $x = v/v_T$, and $\tilde{f} = f v_T^3 \pi^{3/2}/n$ gives

$$\frac{\partial \tilde{f}}{\partial s} + \delta \left(3\tilde{f} + x \frac{\partial \tilde{f}}{\partial x} \right) = \frac{1}{x^2} \frac{\partial}{\partial x} \left(\tilde{f} + \frac{1}{2x} \frac{\partial \tilde{f}}{\partial x} \right), \tag{1}$$

where

$$\delta = -\frac{1}{2\hat{\nu}_{ee}} \frac{d \ln T}{dt}.$$

In the early stages of the thermal quench the energy loss rate from the plasma is expected to be proportional to $T^{-1/2}$, so that $\delta = 1/(3\hat{\nu}_{ee}(0)t_0)$ is independent of time and

$$T(t) = T_0(1 - t/t_0)^{2/3} \quad \text{or} \quad T(s) = \exp(-2s\delta). \tag{2}$$

Eq. (1) has been solved for an initially Maxwellian plasma by matching asymptotic expansions in the small parameter δ in five regions of velocity space.

Region I: $x \sim 1, \quad \tilde{f} = f_0 + \delta f_1 + \cdots$

A straightforward regular perturbation expansion gives $f_0 = e^{-x^2}$ and $f_1 \rightarrow 2x^5 e^{-x^2}/5$ for

$t \to t_0$ ($s \to \infty$). The expansion breaks down when $x \sim \delta^{-1/5}$, so a different procedure is required.

Region II: $x \sim \delta^{-1/5}, \quad F = \delta^{-2/5}\left(F_0 + \delta^{2/5}F_1 + \cdots\right)$
Let $u = x\delta^{-1/5}$, $s' = \delta^{3/5}s$ and expand $F = \ln\tilde{f}$ as above. One finds that $F_0(u,s') = -u^2$ as expected and

$$F_1(u,s) = \frac{2u^5}{5} - \frac{2}{5}\left(u^3 - 3s'\right)^{5/3}\theta(u^3 - 3s'),$$

The expansion breaks down when $x \sim \delta^{-1/3}$.

Region III: $x \leq \delta^{-1/3}, \quad F = \delta^{-2/3}\left(F_0 + \delta^{2/3}F_1 + \cdots\right)$
Letting $y = x\delta^{-1/3}$ the final state as $s \to \infty$ becomes to the lowest order

$$F_0(y,\infty) = \begin{cases} -y^2 + 2y^5/5, & y < 1 \\ -3/5, & y > 1 \end{cases}.$$

The constant value $-3/5$ is approached for $y > 1$ approximately at $x_{\text{tail}} \simeq \sqrt{C}\exp(s\delta) = (CT_0/T)^{1/2}$, with $C \simeq 2\ln(x_{\text{tail}})$. In the next order, one finds that

$$F_1(y,\infty) = \begin{cases} 0, & y < 1 \\ -\ln\left(y^3 - 1\right), & y > 1 \end{cases}.$$

The perturbation expansion breaks down in a narrow region around $y = 1$ when $s \to \infty$. This has to be treated separately.

Region IV: $x \simeq \delta^{-1/3}$
Writing $x = \delta^{-1/3} + w$ gives in the lowest order

$$\tilde{f}(w,\infty) = \left[c_1 - c_2\mathrm{erf}\left(w\sqrt{3}\right)\right]e^{3w^2}.$$

Region V: $x > \delta^{-1/3}$
Here, a regular perturbation expansion holds and the final state becomes $c_3/(y^3-1)$. Combining regions I – V the solution at $s \to \infty$ becomes (shown in Figure 1)

$$\lim_{t\to t_0} f(y,t) = n\left(\frac{m}{2\pi T}\right)^{3/2} \times$$
$$\times \begin{cases} \exp\left[-\frac{1}{\delta^{2/3}}\left(y^2 - \frac{2y^5}{5}\right)\right], & y < 1 \\ \frac{1}{2}\exp\left[\frac{3}{\delta^{2/3}}\left((y-1)^2 - \frac{1}{5}\right)\right]\mathrm{erfc}\left(\frac{\sqrt{3}(y-1)}{\delta^{1/3}}\right), & y - 1 \sim \delta^{1/3} \\ \frac{\delta^{1/3}}{2}\left(\frac{3}{\pi}\right)^{1/2}\exp\left(-\frac{3}{5\delta^{2/3}}\right)\frac{1}{y^3-1}, & y > 1. \end{cases}$$

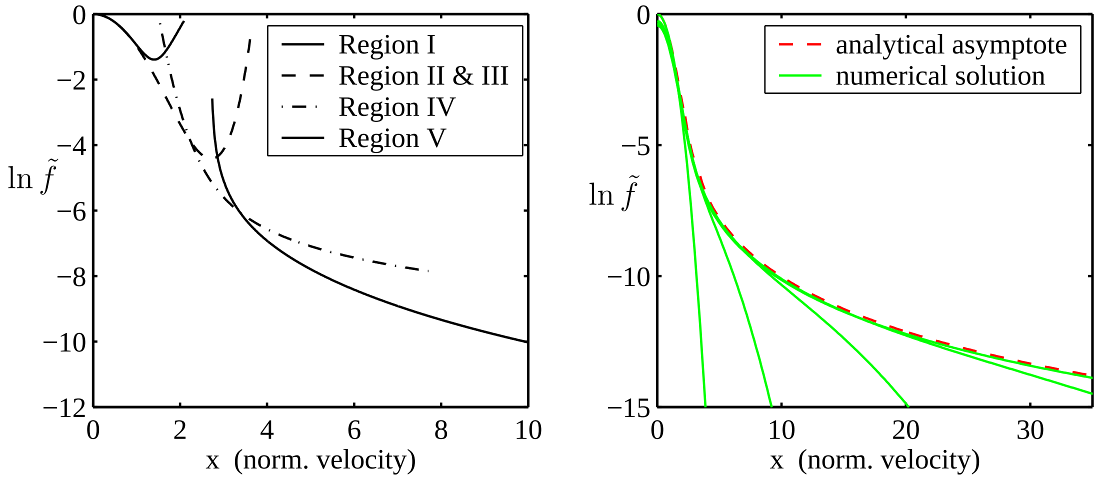

Figure 1: Left: The analytical expansion of the asymptotic distribution function $\tilde{f}$ as $t \to t_0$ ($s \to \infty$) for $\delta = 0.05$, which is typical of a JET disruption. **Right:** Comparison with a numerical solution given at the times $s = 0$, 20, 40, 60 and 80 (from left to right).

Runaway electron generation

The runaway electron density n_{run} can be estimated by the number of tail particles left above the runaway threshold v_c when the temperature has dropped to some temperature T_1. When v_c is inside Region V, the estimate is

$$\begin{aligned} n_{\text{run}}/n &= \int_{v_c}^{v_{\text{tail}}} f 4\pi v^2 dv = \\ &= \frac{2}{\sqrt{3}\pi}\delta^{-2/3}\exp(-\frac{3}{5}\delta^{-2/3})\ln\left(\frac{\delta x_{\text{tail}}^3 - 1}{\delta x_c^3 - 1}\right), \end{aligned} \tag{3}$$

where $x_c = v_c/v_T = ((T_1/T_0)^{1/2}E_{D0}/E_{\|0})^{1/2}$, $x_{\text{tail}} = v_{\text{tail}}/v_T \simeq (2\ln(x_{\text{tail}})T_0/T_1)^{1/2}$, E_{D0} is the initial Dreicer field and $E_{\|0}$ the initial electric field. The fact that $f(v)$ is assumed to be isotropic makes the estimate uncertain by a factor of order unity.

Eq. (1) has also been solved numerically using a finite difference scheme. Then it is also possible to consider a temperature evolving as

$$T(t) = T_\infty + (T_0 - T_\infty)\exp(-t/t_0). \tag{4}$$

The resulting number of particles that end up with $v > v_c$ should be compared to the classical Dreicer generation [1]

$$\frac{dn_{\text{run}}}{dt} = kn\hat{\nu}_{ee}\left(\frac{E_D}{E_\|}\right)^{3/8}\exp\left(-\frac{E_D}{4E_\|} - \sqrt{\frac{2E_D}{E_\|}}\right), \tag{5}$$

where k is a factor of order unity. The results are presented in Figure 2.

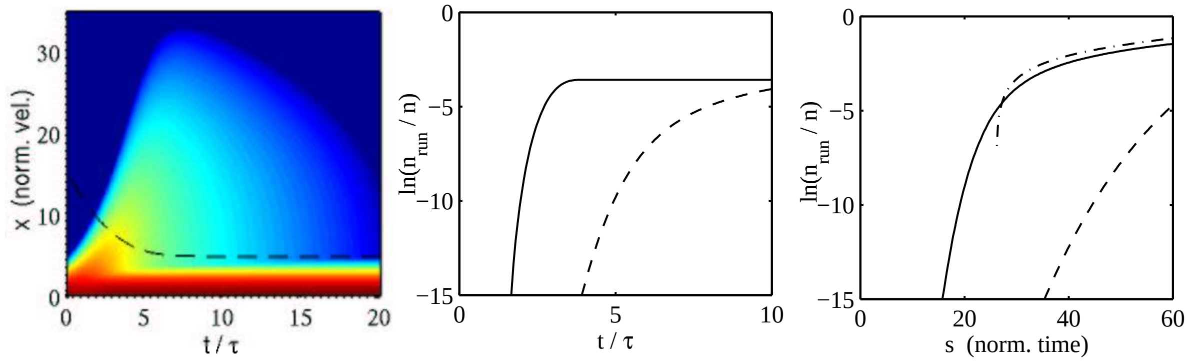

Figure 2: Left: $\tilde{f}(x,t)$ is plotted in colour against normalised velocity and time for a case where $T(t)$ is given by Eq. (4) with $T_\infty/T_0 = 0.01$, $E_{D0}/E_{\|0} = 250$ and $t_0 = \tau$, where $\tau = 1/\hat{\nu}_{ee}(0)$. The dashed line shows the runaway threshold. **Middle:** The number of particles in the runaway region obtained from numerical solution (solid) compared with usual Dreicer production Eq. (5) (dashed) in the same case. **Right:** Comparison of Eq. (3) (dash-dotted) with numerical solution (solid) and usual Dreicer production (dashed). Here $T(t)$ is from Eq. (2), $E_{D0}/E_{\|0} = 200$ and $\delta = 0.05$.

Conclusions

- Far more Dreicer runaway electrons can be produced both in JET and ITER due to the short cooling time effect than by ordinary Dreicer generation.
- The runaway electrons are produced earlier than ordinary Dreicer generation predicts.
- The exact number of runaways produced by this effect is sensitive to the time evolution of the electron temperature.

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References

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