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Predator-Prey Phenomena in Interchange Mode Turbulence

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I. It is now widely accepted that the low-frequency drift-type wave turbulence itself is capable to generate the large scale wing of the wave spectrum, i.e. the so-called *zonal flows* and *streamers*. In the present paper we investigate the properties of these large-scale structures generated, via Reynolds stress, by small-scale interchange (flute) mode turbulence. Flute modes are defined as low-frequency, $\omega << \Omega_{ci}$ (the ion cyclotron frequency), drift-type fluctuations driven by the plasma pressure gradient perpendicular to the ambient magnetic field B. These gradient-specific modes are known to be two-dimensional vortex motions. It is well known that the related mode turbulence exhibits dual turbulent cascades. Namely, as the wave number spectrum broadens through nonlinear dynamics, the energy is transferred primarily to larger scales. Furthermore, as long as the large-scale structures are excited, they form an environment for the parent flute modes and therefore these two disparate components are deeply influencing each other. We focus the discussion on the dynamics that the complex system composed of small-scale turbulence (prey) and large-scale structures (predators) is believed to follow. To this end, the evolution equations for the mean flow generation are derived by averaging the model equations over the fast small scales and a numerical analysis of the nonlinear evolution of the interchange instability is performed within the so-called zero-dimensional approach.

II. To describe the flute modes in a weakly inhomogeneous magnetized plasma we use the two-fluid macroscopic equations. Then in the electrostatic limit, $\mathrm{E} = -\nabla\phi$, these basic equations are reduced to a pair of coupled nonlinear equations for the dimensionless density $n(\mathrm{r},t) = \delta n / n_0$ and the potential $\Phi(\mathrm{r},t) = e\phi / T$,

$$\frac{\partial n}{\partial t} + \mathrm{v}^* \frac{\partial \Phi}{\partial y} + D\nabla^2(\Phi - n) = -[\Phi, n] \,, \tag{1a}$$

$$\left(\frac{\partial}{\partial t}\nabla^2\Phi - \mathrm{v}^* \frac{\partial}{\partial y}\nabla^2\Phi\right) + \mathrm{v}_0 \frac{\partial n}{\partial y} - \mu\nabla^4(\Phi + n) = -\left([\Phi, \nabla^2\Phi] + \nabla\cdot[n, \nabla\Phi]\right). \tag{1b}$$

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Here $\mathrm{v}^* = \kappa c T / eB_0 \mathrm{v}_{th}$ and $\mathrm{v}_0 = g / \Omega_{ci} \mathrm{v}_{th}$ are the ion diamagnetic and gravitational drift velocities normalized to the ion thermal velocity, v_{th}, $\kappa = -(d \ln n_0 / dx) > 0$, $\kappa \propto L_n^{-1}$ and $g = T / m_i R$, R is the characteristic scale length of the magnetic field inhomogeneity, D and μ are collision parameters. Moreover, we introduce the dimensionless space $\mathrm{r} \to \mathrm{r} / \rho_i$ and time $t \to t / \Omega_{ci}$ variables, where $\rho_i = \mathrm{v}_{th} / \Omega_{ci}$ is the ion Larmor radius. The Jacobian, or Poisson bracket, $[a, b]$, is defined as $(\nabla a \times \nabla b) \cdot \mathrm{z}$. Linear analysis for small perturbations, $(n, \Phi) \sim \exp(-i\omega t + i\mathrm{k} \cdot \mathrm{r})$, yields that flute modes belong to the class of reactive systems, i.e. they contain the mechanism to produce instability without external means. It is clear from this analysis that streamers $(k_y \neq 0)$, which have short extent in the direction of translation symmetry, can be generated even by the linear instability mechanism, whereas zonal flows $(k_y = 0)$ can be generated only through some nonlinear interaction mechanism.

III. To describe the evolution of the coupled system (wave turbulence + large scale plasma flows) we represent the perturbed potential Φ as a sum of a large scale flow $\overline{\Phi}$ quantity and a small scale turbulent part $\widetilde{\Phi}$. A similar representation has been chosen for the density *n*. The large scale plasma flow varies on a longer time scale compared to the small scale turbulent fluctuations, so we may employ a multiple scale expansion, thus assuming that in the Fourier representation there is a sufficient spectral gap separating large scale (*q*) and small scale (*k*) motions, $|\mathrm{q}| << |\mathrm{k}|$. Averaging (1) over the fast small scales, we obtain the evolution equations for the *q*th Fourier component of the large-scale flow:

$$\frac{\partial n_q}{\partial t} + iq_y \mathrm{v}^* \Phi_q - q^2 D(\Phi_q - n_q) = -\sum_k \hat{\mathrm{z}} \cdot (\mathrm{k} \times \mathrm{q})(\Phi_k n_{q-k} - \Phi_{q-k} n_k), \tag{2a}$$

$$q^2 \left(\frac{\partial \Phi_q}{\partial t} - iq_y \mathrm{v}^* \Phi_q \right) - iq_y \mathrm{v}_o n_q + \mu q^4 (\Phi_q + n_q) =$$
$$-\sum_k \hat{\mathrm{z}} \cdot (\mathrm{k} \times \mathrm{q}) \left[k^2 - (\mathrm{k} - \mathrm{q})^2 \right] \Phi_k \Phi_{q-k} - \sum_k \hat{\mathrm{z}} \cdot (\mathrm{k} \times \mathrm{q}) \left[\mathrm{k} \cdot \mathrm{q} \Phi_k n_{q-k} - \mathrm{q} \cdot (\mathrm{q} - \mathrm{k}) \Phi_{q-k} n_k \right]. \tag{2b}$$

It is seen that a small-scale turbulence can indeed drive large-scale flows. To proceed with the analysis we have to estimate the correlators $\langle \Phi_k \Phi_{k1} \rangle$, $\langle \Phi_k n_{k1} \rangle$ and $\langle \Phi_{k1} n_k \rangle$ for short scale perturbations, where we have defined $\mathrm{k}_1 = \mathrm{q} - \mathrm{k}$. Following the standard renormalization technique we can find the required relations for n_{q-k} and Φ_{q-k}:

$$n_{q-k} = \tau(\mathrm{k}\times\mathrm{q})\cdot\hat{\mathrm{z}}(\Phi_q n_{-k} - \Phi_{-k} n_q)\,, \tag{3a}$$

$$\Phi_{q-k} = \tau(\mathrm{k}\times\mathrm{q})\cdot\hat{\mathrm{z}}(\Phi_q + n_q)\Phi_{-k}\,. \tag{3b}$$

Here, τ is the nonlinear decorrelation time for the turbulent fields.

Coupling of the small-scale fluctuations to the mean flow can be described by the wave kinetic equation for the wave packets and corresponds to the conservation of an action-like invariant of the wave turbulence, $N_k(\mathrm{r},t)$, with slowly varying parameters due to the large-scale flow:

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_{k,r}}{\partial \mathrm{k}}\frac{\partial N_k}{\partial \mathrm{r}} - \frac{\partial \omega_{k,r}}{\partial \mathrm{r}}\frac{\partial N_k}{\partial \mathrm{k}} = St\{N_k\}\,, \tag{4}$$

where the adiabatic action invariant is given by $N_k = 4k^2\frac{R}{L_n}(1-k^2\frac{R}{4L_n})|\Phi_k|^2$. The source term $St\{N_k\}$ describes the wave growth and damping due to linear and non-linear damping mechanisms. We assume that the small-scale turbulence is close to the stationary state, so that $St\{N_k\}\rightarrow 0$. The linear frequency of flute modes is now modified by the flows:

$$\omega_{k,r} = \omega_k + \mathrm{kV}_0 + \mathrm{kV}_1\,,\ \mathrm{V}_0 = -[\nabla\overline{\Phi}\times\mathrm{z}],\quad \mathrm{V}_1 = -[\nabla\overline{n}\times\mathrm{z}]. \tag{5}$$

IV. A coupled system of Eqs. (2-4) self-consistently describes the non-linear dynamics of flute modes and can be treated as a “predator-prey” system consisting of the two disparate components of wave turbulence: the population of waves (prey), growing via linear instability, generates large scale structures (predators) through the Reynolds stress. Concomitantly, the large scale structures (predators) regulate the wave population (prey). To demonstrate the “predator-prey” phenomena in the flute mode turbulence, we perform a numerical analysis of Eqs. (2-4) in the so-called zero-dimensional approach, according to which the predator and prey populations are functions of time only. In these studies we construct a minimal dynamical model with only three principal components, one for the small-scale wave turbulence (prey, x), one for zonal flows (predator 1, y), and one for streamers (predator 2, z). Then, Eqs.(2-4) can be phenomenologically written as follows:

$$\frac{dx}{dt} = rx - \frac{r}{K}x^2 - axy - bxz\,,\quad \frac{dy}{dt} = aexy - dy\,,\quad \frac{dz}{dt} = cz - bfxz - hyz\,, \tag{6}$$

where all the 9 constants are assumed to be positive. The evolution equation for the prey x contains the linear growth given by r and a damping due to zonal as well as streamer flow generation expressed by the mixed terms ($-axy$) and ($-bxz$). Moreover, a nonlinear growth term proportional to x^2 is present with the so-called carrying capacity *K*, i.e. the prey value in the absence of predators. The evolution equation for zonal flows (*y*) consists of terms responsible for the generation due to nonlinear interaction of short-scale fluctuations (*aexy*) and for the linear damping (-*dy*). The streamer evolution is described by the linear growth (*cz*) together with generation due to interaction of small-scale fluctuations (-*bfxz*) and a nonlinear interaction term (-*hyz*) standing for the damping due to zonal shear flows. The latter term is not present in Eq. (2) and was added to account for such interaction.

V. The results of the numerical analysis of Eqs. (6) indicate that the dissipation parameters play an important role in guiding the system toward a particular symmetry state and can be summarized as follows. The presence of low dissipation (small values of *d* and *c*) preferentially gives rise to zonal structures, while a strong dissipation results in streamer formation. On the other hand, for intermediate values of dissipation our analysis has shown that the system can either evolve toward a state where zonal flows and streamers coexist (Fig. 1) or to a state characterized by the extinction of both these components. Moreover, we have encountered explosive instability dynamics, known from the interaction between positive and negative energy modes; it was shown that they can be stabilized by introducing the interaction between zonal flows and streamers (-*hyz*). Finally, examples of intermittency evolutions may also be observed (Fig. 2).

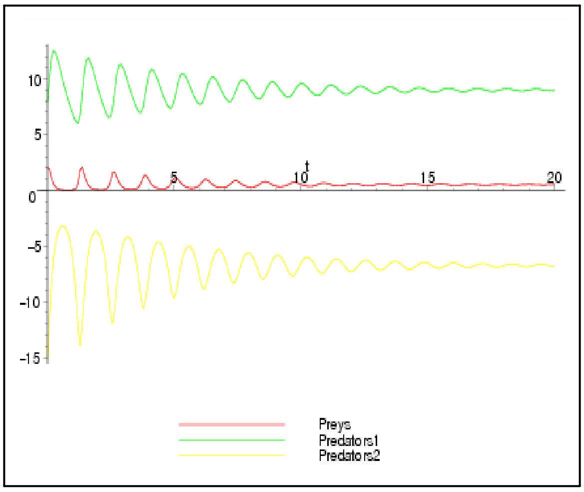

Fig. 1: Evolution of the system “zonal flows + streamers + turbulence”. The damping oscillatory evolution of all the three components is observed. It follows the predator-prey dynamics, in the form of periodical solutions.

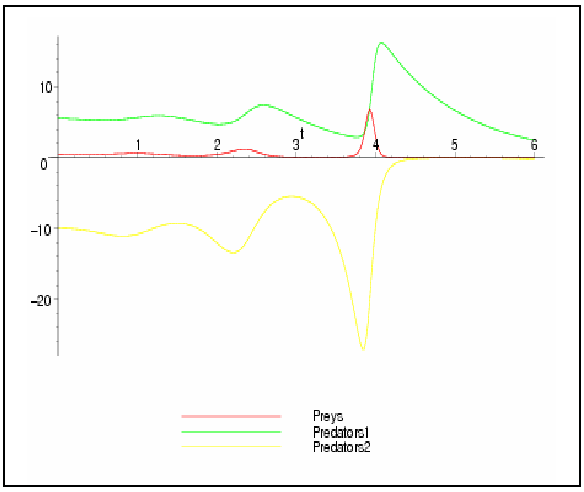

Fig. 2: Another evolution of the system “zonal flows + streamers + turbulence”. It may be regarded as an example of intermittency behaviour.