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Impurity transport in turbulent plasmas

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Impurity control is very important for the development of fusion devices. This is a complex problem related to confinement and transport in plasma and to plasma-wall interaction. In this frame, we analyze here the particular problem of impurity transport in turbulent plasmas. Particle motion in a stochastic potential was extensively studied in the guiding center approximation. It is well known that for slowly varying or large amplitude turbulence the $\mathbf{E} \times \mathbf{B}$ drift determines a process of dynamical trapping of the trajectories. It consists of trajectory winding around the extrema of the stochastic potential and strongly influences the transport. Important progresses in the study of this nonlinear process were recently obtained [1], [2].

The guiding center approximation is not adequate for the impurity ions which can have Larmor radii comparable or larger than the correlation length of the turbulence. In these conditions the trajectories have to be determined from the Lorentz force. The aim of this paper is to determine the effect of finite Larmor radius on particle transport in a turbulent magnetized plasmas. We thus compare the transport induced by the Lorentz force (Lorentz transport) with the guiding center approximation (drift transport). The direct application of these results concerns the transport regimes of impurities in turbulent plasmas.

We consider a constant confining magnetic field directed along z axis, $\mathbf{B} = B\mathbf{e}_z$ (slab geometry) and an electrostatic turbulence represented by an electrostatic potential $\phi'(\mathbf{x}, t)$, where $\mathbf{x} \equiv (x_1, x_2)$ are the Cartesian coordinates in the plane perpendicular to $\mathbf{B}$. The motion of an ion with charge q and mass m is determined by the Lorentz force:

$$m\frac{d^2\mathbf{x}(t)}{dt^2} = q\left\{-\boldsymbol{\nabla}\phi'(\mathbf{x}, t) + \mathbf{u} \times \mathbf{B}\right\} \tag{1}$$

where $\mathbf{x}(t)$ is the ion trajectory, $\mathbf{u}(t) = d\mathbf{x}(t)/dt$ is its velocity and $\boldsymbol{\nabla}$ is the gradient in the (x_1, x_2). The initial conditions are $\mathbf{x}(0) = \mathbf{0}$, $\mathbf{u}(0) = \mathbf{u}_0$. This equation is transformed into an equivalent system of first order equations by introducing the instantaneous Larmor radius $\rho_i(t) \equiv -\varepsilon_{ij}u_j(t)/\Omega$ and the guiding center position $\boldsymbol{\xi}(t) \equiv \mathbf{x}(t) - \boldsymbol{\rho}(t)$

$$\frac{d\xi_i}{dt} = -\varepsilon_{ij}\frac{\partial\phi(\boldsymbol{\xi} + \boldsymbol{\rho}, t)}{\partial\xi_j} \tag{2}$$

$$\frac{d\rho_i}{dt} = \varepsilon_{ij}\left[\frac{\partial\phi(\boldsymbol{\xi} + \boldsymbol{\rho}.t)}{\partial\xi_j} + \Omega\rho_j\right]. \tag{3}$$

where $\Omega = qB/m$ is the cyclotron frequency, ε_{in} is the antisymmetric tensor ($\varepsilon_{12} = -\varepsilon_{21} = 1$, $\varepsilon_{11} = \varepsilon_{22} = 0$) and $\phi(\mathbf{x}, t) \equiv \phi'(\mathbf{x}, t)/B$.

The electrostatic potential $\phi(\mathbf{x}, t)$ is a stochastic field and thus Eqs. (2-3) are Langevin equations. We determine here the mean square displacement and the time dependent diffusion coefficient for the guiding center trajectories $\boldsymbol{\xi}(t)$. The potential is considered to be a stationary and homogeneous Gaussian stochastic field, with zero average and given two-point Eulerian correlation, $E(\mathbf{x}, t)$.

Three dimensionless parameters describe the Lorentz transport:

$$K = \frac{V\tau_c}{\lambda_c} = \frac{\tau_c}{\tau_{fl}}, \quad \overline{\rho} = \frac{|\boldsymbol{\rho}(0)|}{\lambda_c}, \quad \overline{\Omega} = \Omega\tau_{fl}. \tag{4}$$

The first, the Kubo number, does not appear in the equations but characterizes the stochastic potential. It is the ratio of the correlation time τ_c to the average time of flight with the stochastic drift velocity V over the correlation length λ_c, $\tau_{fl} = \lambda_c/V$. The other two parameters are specific to the Lorentz transport. $\overline{\Omega}$ appears in Eq. (3) and determines the relative importance of the cyclotron and drift motion in the evolution of the Larmor radius and $\overline{\rho}$, the ratio of the initial Larmor radius to the correlation length, appears in the initial condition and essentially describes the effect of the initial kinetic energy of the ions.

The guiding center approximation obtained by taking $\boldsymbol{\rho} = \mathbf{0}$ in Eq.(2) was recently studied by developing a semi-analytical approach, the decorrelation trajectory method, [1], [2]. Using this approach an important progress was obtained in the understanding of the intrinsic trapping process specific to the $\mathbf{E} \times \mathbf{B}$ drift. We have developed and adapted this method to the Lorentz transport described by Eqs. (2-3). The correlation of the Lagrangian drift velocity $L(t)$ corresponding to given Eulerian correlation $E(\mathbf{x}, t)$ of the stochastic potential is determined in terms of a set of smooth, simple trajectories which represent average trajectories in subensembles of realizations of the stochastic potential. Explicit results for $L(t)$ and for the time dependent diffusion coefficient $D(t)$ are obtained by effectively calculating these trajectories. The results presented in the figures are obtained for $E(\mathbf{x}, t) = 1/(1 + \mathbf{x}^2/2)\exp(-t/K)$.

We consider first a static potential $(K, \tau_c \to \infty)$ and compare the results with those obtained for the drift transport [1], [2]. The aim is to identify the effects of the Larmor radius. The time dependent diffusion coefficient is presented in Fig. 1. The time dependence of the Lorentz diffusion coefficient is rather complex and a strong influence of the Larmor radius can be observed. At small time the diffusion coefficient increases nonuniformly, in steps. Averaging these steps a linear time dependence can be observed, similar with that obtained in the drift transport. This behavior extends to times much longer than the flight time. At later times $D(t)$ decays as a consequence of trapping, with a time dependence that is approximately the same as in the drift case, but larger with a factor of about 2. Thus the transport in the static case is subdiffusive.

The correlation of the Lagrangian drift velocity is presented in Fig. 2. It decays very fast (in a time much smaller than τ_{fl}) and then it presents a series of peaks with decreasing amplitude and eventually has a negative tail. The peaks appear around multiples of the cyclotron gyration period $T = 2\pi/\overline{\Omega}$.

A clear story of the physical process can be deduced from the time evolution of $D(t)$ and $L(t)$. Starting from $t = 0$, at very small time $(t \ll T)$ $D(t)$ is equal with the drift diffusion coefficient. Then, the cyclotron motion with a large Larmor radius ($\overline{\rho} = 1$ in Figs. 1, 2) averages the stochastic field along the trajectory. Consequently the guiding center has a very small displacement and $D(t)$ is much reduced compared to the drift case. After a period the trajectories come back near the initial position (all in phase because T is a constant) and a coherent motion of the guiding centers appears during the passage of the particles, which produces a step in $D(t)$. Thus the evolution of the guiding centers is determined mainly by short coherent kicks appearing with period T. Their displacement is thus slower (with the factor $1/2\pi\overline{\rho}$ that represents the fraction of the cyclotron period spend around the initial point). Consequently, the trapping appears later and the linear increase of $D(t)$ extends to longer times leading to values of $D(t)$ that are higher than for the drift diffusion. The effective trapping time can be estimated as $\tau_{eff} = \tau_{fl} 2\pi\overline{\rho}$. When the displacements of the guiding centers increase the coherence of the periodic kicks is progressively lost and the peaks of $L(t)$ becomes smaller and thicker.

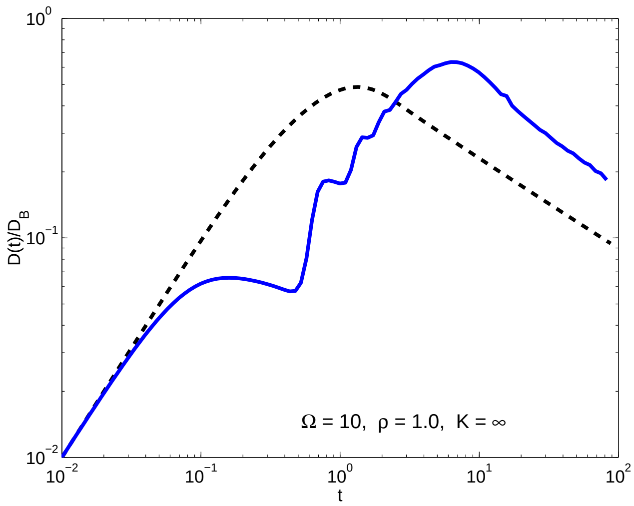

Figure 1: The time dependent diffusion coefficient for the Lorentz (blue line) and drift transport (black dashed line).

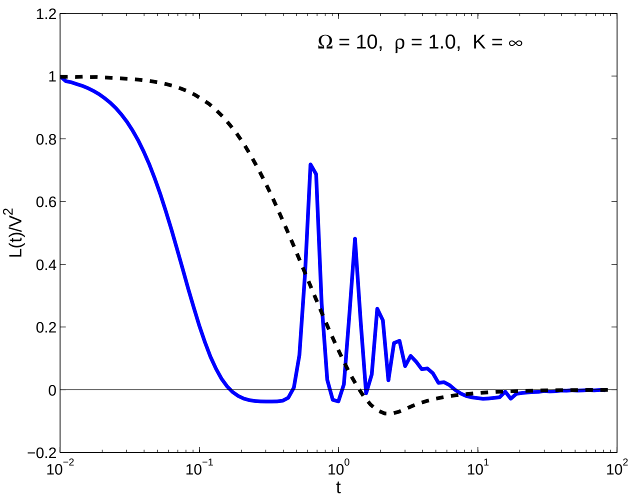

Figure 2: The correlation of the Lagrangian drift velocity for the Lorentz (blue line) and drift transport(black dashed line).

The asymptotic diffusion coefficient in a time dependent stochastic potential with Kubo number K is obtained from the time dependent diffusion coefficient for the static potential as

$$D(\infty|K, \overline{\rho}, \overline{\Omega}) \cong D(K|\infty, \overline{\rho}, \overline{\Omega}), \tag{5}$$

where the parameters are explicitly written. The dependence of the asymptotic diffusion coefficient on the Kubo number for several values of $\overline{\rho}$ is presented in Fig. 6. One can see that the modification of the diffusion coefficient due to Larmor radius is complex and it may consist of a strong decrease as well as of a strong increase, depending on the conditions.

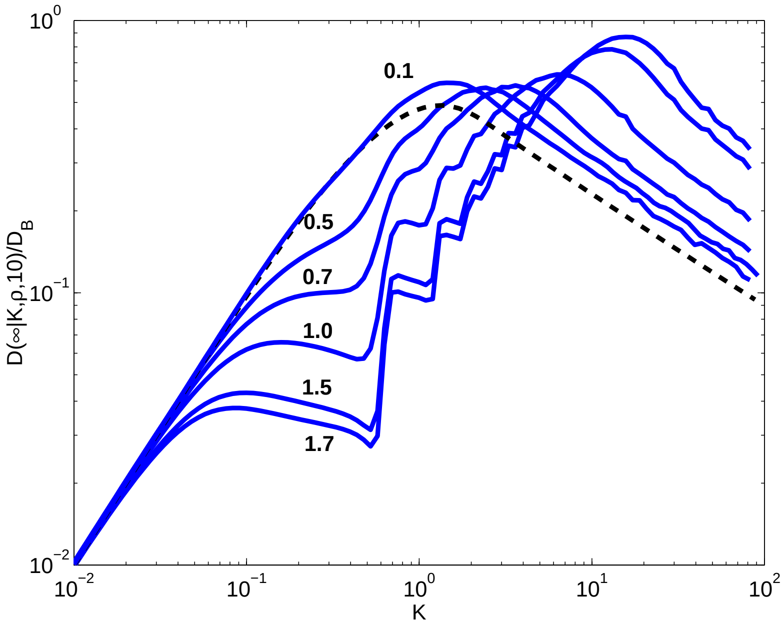

Figure 3: The asymptotic diffusion coefficient for the Lorentz transport as a function of K for the values of $\overline{\rho}$ that label the curves and for $\overline{\Omega} = 10$. The dashed line is the zero Larmor limit.

At small Kubo numbers the diffusion coefficient is much smaller than in the drift approximation. It increases in steps that appear, for all values of $\overline{\rho}$, at values of K which are multiples of the cyclotron period $T = 2\pi/\overline{\Omega}$. Apart from these steps that are attenuated at larger K, there is a global increase with K as $D(\infty|K, \overline{\rho}, \overline{\Omega}) \sim D_B K = (\lambda_c^2/\tau_c)K^2$ in this regime. This is similar with the quasilinear regime of the drift transport and corresponds to initial ballistic motion. This regime extends up to $K \cong 2\pi\overline{\rho}$. The diffusion coefficient at this value of K has a value much larger than for the drift transport. At larger values of K ($K > 2\pi\overline{\rho}$), the trapping becomes effective and the diffusion coefficient has approximately the same K dependence as in the drift transport. The effect of the Larmor radius consists in an amplification factor in the diffusion coefficient that is independent of K in this regime. This factor increases with the increase of $\overline{\rho}$.

In conclusion, the Larmor radius has a strong effect on impurity ion transport in turbulent plasmas. The generally accepted idea that the effective diffusion is reduced due to the cyclotron motion which averages the stochastic potential, is not always true. At given Larmor radius, the transport can be reduced or increased, depending essentially on the value of the Kubo number.

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[2] 5. M. Vlad, F. Spineanu, J. H. Misguich, J.-D. Reusse, R. Balescu, K. Itoh, S. -I. Itoh, Plasma Physics and Controlled Fusion 46, 1051 (2004).