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Magnetic Reconnection and Intrinsically Associated Thermal Energy Transport *

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1. Introduction

The so-called drift tearing mode that was described first in Ref. [1] couples the effects of magnetic reconnection, driven primarily by the plasma current density gradient, with those of the gradient of the longitudinal electron pressure in a strongly magnetized plasma. This mode has found a renewed appreciation recently in view of its relevance to current experiments. We have pointed out originally that for modes involving singular perturbations the effects of nonlinearities become important at very small amplitudes. In order to analyze these effects we have started by reformulating the linearized theory of the drift-tearing mode, in the limit where the electron thermal conductivity along the field is significant. Since the plane geometry can simulate cylindrical and toroidal modes for $m^0>1$ modes, we refer to this geometry for simplicity. Therefore the equilibrium field is taken as $\mathbf{B}=B_z(x)\mathbf{e}_z+B_y(x)\mathbf{e}_y$ with $B_y^2<<B_z^2\approx B^2$; and its perturbation $\hat{\mathbf{B}}=\tilde{\mathbf{B}}(x)\exp(-i\omega t+ik_y y+ik_z z)$, with $k_y^2>>k_z^2$ and $k^2\approx k_y^2$. The scale lengths of the density and the magnetic field gradients along the axis y are assumed to be at the same order. The longitudinal electron momentum balance equation that we adopt includes all the components that are relevant to a rather weakly collisional regime, that is

$$0\approx -en\mathbf{E}_{\|}-\nabla_{\|}p_e-\alpha_T n\nabla_{\|}T_e+en\eta_{\|}\mathbf{J}_{\| } \tag{1}$$

Here α_T is the finite numerical coefficient associated with the thermal force and the other terms are easily identifiable. We anticipate that the longitudinal electron pressure gradient terms are represented by the frequencies: $\omega_{*e}\equiv -k_y cT_{e\|}(dn/dx)/(eBn)$, and $\omega_e^T=-k_y c(dT_e/dx)/(eB)$. In addition we take $\hat{\mathbf{E}}_{\perp}+\hat{\mathbf{v}}_E\times\mathbf{B}/c\approx 0$ to describe the transverse electron guiding center motion. Equation (1) then becomes

$$(\omega-\omega_{e\|}^T)\tilde{B}_x\approx ck_y\eta_{\|}\tilde{J}_{\|}-\frac{c}{eB}(\mathbf{k}\cdot\mathbf{B})\left[ik_y\tilde{T}_e(1+\alpha_T)+ik_yT_e\frac{\tilde{n}_e}{n}\right]-(\mathbf{k}\cdot\mathbf{B})(i\omega\tilde{\xi}_{Ex}) \tag{2}$$

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where $\hat{J}_{\parallel} \approx (c/4\pi)\partial\hat{B}_y/\partial x \approx (c/4\pi)/(-ik_y)\partial^2\hat{B}_x/\partial x^2$, $\omega_{e\parallel}^T \equiv (1+\alpha_T)\omega_e^T + \omega_{*e}$, and $\hat{\xi}_{Ex} \equiv \hat{v}_{Ex}/(-i\omega)$. The equations that we consider at first are for the linearized "reconnection" layer δ_L<<a , around x=x_0, where a is the plasma radius, $k_\parallel B \equiv \mathbf{k}\cdot\mathbf{B}(x=x_0) = 0$, $0<x_0<a$ and k_y^2 can be neglected relative to the operator $\partial^2/\partial x^2$ when applied to the perturbed quantities. Within the δ_L-layer the effects of finite resistivity $\eta_\parallel$, electron thermal conductivity $\kappa_{e\parallel}$ and ion gyroradius represented by the frequency $\omega_{di} = k_y c(dp_{i\perp}/dx)/(eBn)$ are to be taken into account.

2. Importance of Finite Electron Thermal Conductivity and Electron Compressibility

The relevant theory, that includes the effects of electron thermal conductivity and electron compressibility, is considerably more complex than that the original one given in Ref.[1] but is necessary for its application to high temperature regimes. In particular the complete form of the electron thermal energy balance has to be considered. Thus we analyze the case $\gamma<|\omega_R|\sim\nu_{e\parallel}$, for $\omega\equiv\omega_R+i\gamma$, and note that in this case reconnection depends on the effective collision frequency

$$\nu_{eff} \approx \frac{2}{3}(1+\alpha_T)\frac{\nu_{e\parallel}^2}{\omega_R^2+\nu_{e\parallel}^2}\left(\frac{k_\parallel^2 T_e}{m_e \nu_{ee}\nu_{e\parallel}}\right)\nu_{ee} \sim \nu_{ei}^\parallel, \text{ for } \omega_R^2 \sim \nu_{e\parallel}^2, \tag{3}$$

as well as on $\nu_{ei}^\parallel$. Here $\nu_{e\parallel} \equiv k_\parallel^2 D_{e\parallel}$, $D_{e\parallel} \equiv 2\kappa_{e\parallel}/(3n)$ and $\nu_{ei}^\parallel \equiv \eta_\parallel n e^2/m_e$. We define δ_{LT} as $\omega_{e\parallel}^T \equiv (k_\parallel' \delta_{LT})^2 D_{e\parallel}$ where $k_\parallel' = (\mathbf{k}\cdot\mathbf{B}')/B$. Then for $|x-x_0| > \delta_{LT}$ the electrons can be treated as isothermal, for $|x-x_0| < \delta_{LT}$ as adiabatic [1], and for $|x-x_0| \sim \delta_{LT}$ the complete expression for $\tilde{T}_e/T_e$ should be taken into account. Following a similar procedure to that indicated in Ref.[1] we are led to consider the following equation

$$-\left(1-\frac{\omega_{di}}{\omega_{e\parallel}^T}\right)A\frac{d^2W}{d\bar{x}^2} = -i\left[1-\frac{\omega_{*e}}{\omega_{e\parallel}^T}-\frac{\omega_{Te}}{\omega_{e\parallel}^T}F(\bar{x})\right]\frac{\bar{x}^2 W}{1+D_{mT}(\bar{x})/D_m} - \left(\frac{\gamma - i\delta\omega_R}{\omega_{e\parallel}^T}\right)\frac{\bar{x}}{1+D_{mT}(\bar{x})/D_m} \tag{4}$$

where $\bar{x} \equiv (x-x_0)/\delta_{LT}$, $\omega \approx \omega_{e\parallel}^T + \delta\omega_R + i\gamma$, with $\gamma\sim\delta\omega_R$, $\varepsilon_\delta^T \equiv \delta_{LT} k$,

$$\frac{D_{mT}}{D_m} \approx \left(\frac{T_e}{m_e \nu_{ei}^\parallel D_{e\parallel}}\right)\left\{\frac{2}{3}(1+\alpha_T)\left(\frac{\bar{x}^4}{\bar{\omega}_R^2+\bar{x}^4}\right) + i\bar{x}^2\left[\frac{1}{\bar{\omega}_R} + \frac{2(1+\alpha_T)\bar{\omega}_R/3}{\bar{\omega}_R^2+\bar{x}^4}\right]\right\},$$

$\bar{\omega}_R \equiv \omega_R/\omega_{e\parallel}^T \approx 1$, $W \equiv i(\mathbf{k}\cdot\mathbf{B}')\delta_{LT}\tilde{\xi}_{Ex}/\tilde{B}_{x0}$, $F(\bar{x}) \approx (1+\alpha_T)\left(|\bar{\omega}|^2 - i\bar{x}^2\bar{\omega}\right)/(\bar{\omega}_R^2+\bar{x}^4)$ with $\bar{\omega} \equiv \omega/\omega_{e\parallel}^T$. $A \equiv \omega_{e\parallel}^T D_m k^2 (\varepsilon_\delta^T)^{-4}\omega_A^{-2}$ and D_{mT} involves ν_{eff} as is evident from its expression. Here $\omega_A^2 \equiv (\mathbf{k}\cdot\mathbf{B}')^2/(4\pi\rho k^2)$. We find that the width of the transition layer is of the order of

$$\delta_L \approx \left(\frac{k}{r_p}\frac{\nu_{ei}^{\parallel}}{\Omega_{ce}}\right)^{1/4}\left(\rho_s\frac{B}{B_p}\right)^{1/2}\frac{1}{k} > \rho_i\,, \tag{5}$$

where $\rho_s = v_s/\Omega_{ci}$, $v_s^2 \equiv T_e/m_i$, $\rho_i = (2T_i/T_e)^{1/2}\rho_s$, and the corresponding growth rate is $\gamma \sim (D_m k^2)^{3/4}\omega_A^{1/2}{}^2/|\,\omega_{e\parallel}^T|^{1/4}$. We observe that δ_L is rather insensitive to the temperature and therefore the validity of the analysis presented here should persist over a significant range of temperatures.

3. Nonlinear Model

The nonlinear effects which are included in the simple model equation that we have analyzed are related to i) the (quasilinear) decrease of $dp_{e\parallel}/dx$ and ii) the fact that $\hat{B}_x\,\partial\,\hat{p}_{e\parallel}/\partial\,x$ becomes important relative to $\mathbf{B}\cdot\nabla\hat{p}_{e\parallel} = i(\mathbf{k}\cdot\mathbf{B})\hat{p}_{e\parallel}$ as $\mathbf{k}\cdot\mathbf{B}$ tends to vanish within δ_L while $|\partial\,\hat{p}_{e\parallel}/\partial\,x|/|\hat{p}_{e\parallel}|\sim 1/\delta_L$ tends to become singular. Consequently, the width of the layer where these two effects are comparable is $\delta_{NL}\sim|\tilde{B}_{x0}/(\mathbf{k}\cdot\mathbf{B}')|^{1/2}$ and this width can exceed easily that of the resistive reconnection layer [1] for quite small amplitudes of the reconnected field $\tilde{B}_{x0}$ at $x=x_0$.

We argue that the transition into a fully non-linear regime can be made more probable in the presence of previously excited modes that can provide a stronger nonlinear drive. In this case the simplest of model equations we have considered is

$$\gamma(1-\alpha_{NL} f_{NL})\tilde{B}_x \approx i\,\gamma(\mathbf{k}\cdot\mathbf{B}')(x-x_0)\tilde{\xi}_{Ex} + D_m\frac{d^2\tilde{B}_x}{dx^2}\,, \tag{6}$$

and the total momentum conservation equation corresponding to $\nabla\cdot\hat{\mathbf{v}}_{pi} \approx \nabla_{\parallel}\hat{u}_{e\parallel}$ is assumed to remain unchanged through the non-linear stage. Thus

$$\gamma^2\frac{d^2\tilde{\xi}_{Ex}}{dx^2} \approx \frac{i(\mathbf{k}\cdot\mathbf{B}')}{4\pi\rho}(x-x_0)\frac{d^2\tilde{B}_x}{dx^2}\,. \tag{7}$$

Here $\tilde{B}_{x0}\approx\tilde{B}_x(x=x_0)$, α_{NL} is a constant parameter, and we have taken $f_{NL}=|d\tilde{\xi}_{Ex}/dx|^2$. We note that within the δ_{NL}-layer $\tilde{B}_x \approx \tilde{B}_{x0}[1+\varepsilon_\delta\,\varphi(\bar{x})]$, where $\varepsilon_\delta = k\delta_{NL}$, $\bar{x}\equiv(x-x_0)/\delta_{NL}$ and $d^2\varphi/d\bar{x}^2 = O(1)$. The solution for $\tilde{\xi}_{Ex}(\bar{x})$ of these equations develops a singularity in the curvature of $\tilde{\xi}_{Ex}$ ($d^2\tilde{\xi}_{Ex}(\bar{x})/d\bar{x}^2$) that is removed by the effects of finite resistivity (D_m). The growth rate of the mode evaluated by the considered model equation is enhanced relative to that found by the linearized resistive theory and can be of the order of $\omega_A \varepsilon_\delta^{3/2}$. In this case, $\tilde{\xi}_{Ex}$ in the outer region remains finite and given by the ideal MHD approximation $\tilde{B}_x = i(\mathbf{k}\cdot\mathbf{B})\tilde{\xi}_{Ex}$. The nonlinear Eq.(6) can be reduced to the following form by making suitable variable transformation: $-\varepsilon\tilde{\xi}''_{Ex}(\bar{x})/\bar{x} + \left(\tilde{\xi}'_{Ex}(\bar{x})\right)^2 + \bar{x}\,\tilde{\xi}_{Ex}(\bar{x}) = 1$, where ε is a small number.

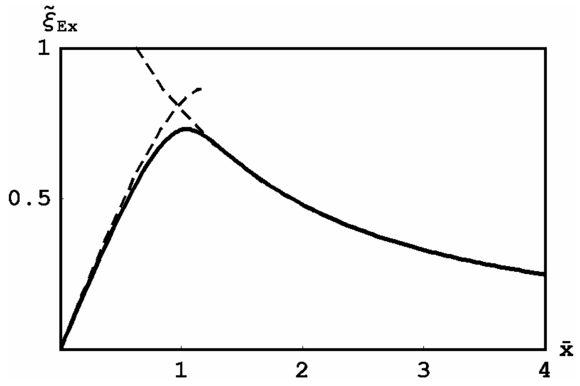

Fig.1. Solution of the nonlinear model equation. The dashed curves are the solutions of the unperturbed equation. Here, $\tilde{\xi}_{Ex}(0) = 0$, $\tilde{\xi}'_{Ex}(0) = 0.947$ for ε=0.1 .

We note that tridimensional stability analyses concerning both cylindrical [2] (limited to m^0=1 modes) and toroidal [3] configurations have indicated that when the effects of the parallel pressure gradient are included, the rate of reconnection produced by the excited modes increases considerably, and the width of the reconnection region is definitely broader. In fact we consider that nonlinear drift-tearing modes can provide the explanation for modes involving magnetic reconnection that have been observed experimentally [4], and do not appear to correspond to neoclassical tearing modes.

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