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Landau resonance between charged particles and plasma waves

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Abstract

Possible interactions between plasma waves and charged particles are considered, where the particles move with slow to relativistic speeds relative to a thermalised plasma. The electromagnetic perturbation is formulated as an elliptically polarised wave, and the collisionless plasma is described by a Maxwellian distribution in phase space, which is realised in cylindrical coordinates. The linearised Vlasov equation is solved to obtain the perturbed distribution function in the rest frame of the observer. The perturbed currents supported by the ionized medium are then calculated, so that an expression can be written for the total amount of energy available for transfer through the Landau mechanism.

In the literature, Landau resonances are described analytically by explicitly using the dielectric tensor, which invoke rather complicated calculations [1,2]. In a previous work [3], it was shown that the calculations can be simplified by employing an identity derived in [4]. It thus transpires that there are only certain discrete channels allowed for the transfer of energy through the Landau mechanism. The total energy available for transfer can be written as [3]

$$\begin{aligned} W &= -\frac{\pi^2 q^2 \omega}{2c^2}\left(\frac{\omega}{k_3} - v_R\right)\int_0^\infty dp_\perp p_\perp^3 \\ &\quad \cdot \Big[\Big\{ (a_1+a_2)^2 f_0'\left[p_\perp^2,(p_\parallel^+)^2\right] + (a_1-a_2)^2 f_0'\left[p_\perp^2,(p_\parallel^-)^2\right] \Big\} J_0^2(\chi) \\ &\quad + (a_1^2 - a_2^2)\Big\{ f_0'\left[p_\perp^2,(p_\parallel^+)^2\right] + f_0'\left[p_\perp^2,(p_\parallel^-)^2\right] \Big\} J_0(\chi) J_2(\chi) \Big] . \end{aligned} \tag{1}$$

Here f_0 is the particle distribution and $f_0' = \partial f_0/\partial p^2$, where $\mathbf{p} = (p_\perp, p_\parallel)$ is the particle momentum. The directions in the subscripts refer to a uniform magnetic field $\mathbf{H}_0 = (0, 0, H_0)$. The plasma is situated in a reference frame that is moving with velocity v_R with respect to the observer, and we are working in the limit $(v_R/c)^2 \ll 1$, with c the speed of light. The notation q contains the sign and size of the particle charge. The electromagnetic perturbation is of the form

$$\mathbf{A} = \mathbf{A}_k e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}, \quad \mathbf{A}_k = (a_1, ia_2, 0), \tag{2}$$

with wavevector $\mathbf{k} = (k_1, 0, k_3)$, and frequency ω determined by the dispersion relation of the type of wave under consideration. The two poles

$$p_\parallel^\pm = \frac{\mathcal{E}}{c^2 k_3}(\omega \pm \Omega) \tag{3}$$

are associated with the Cauchy integrals deformed according to the Landau prescription. $\mathcal{E} = mc^2/\sqrt{1-(v/c)^2}$ is the relativistic energy of a particle with rest mass m moving at velocity v. The cyclotron frequency is given by $\Omega = qcH_0/\mathcal{E}$.

An axisymmetric plasma in thermal equilibrium has a Maxwellian distribution function in the rest frame of the observer given by

$$f_0 = \frac{N}{(2\pi m k_B T)^{3/2}} \exp\left(-\frac{p^2}{2mk_BT}\right), \tag{4}$$

where N is the number of particles per unit volume, k_B the Boltzmann constant, and T the absolute temperature. Its derivative, as defined earlier, is

$$f_0' = -\frac{N}{\sqrt{\pi^3(2mk_BT)^5}} \exp\left(-\frac{p_\perp^2 + p_\parallel^2}{2mk_BT}\right). \tag{5}$$

This becomes

$$f_0' \mid_{p_\parallel^\pm} = -\frac{N}{\sqrt{\pi^3(2mk_BT)^5}} \Theta^\pm \exp\left(-\frac{p_\perp^2}{2mk_BT}\right) \tag{6}$$

at resonances $p_\parallel^+$ and $p_\parallel^-$, where

$$\Theta^\pm = \exp\left(-\frac{(p_\parallel^\pm)^2}{2mk_BT}\right). \tag{7}$$

A substitution into (1) gives the available energy as

$$\begin{aligned} W &= \frac{\pi N q^2 \omega}{2c^2\sqrt{\pi(2mk_BT)^5}} \left(\frac{\omega}{k_3} - v_R\right) \int_0^\infty dp_\perp p_\perp^3 e^{-p_\perp^2/2mk_BT} \\ &\quad \cdot \Big\{ \left[(a_1^2 + a_2^2)(\Theta^+ + \Theta^-) + 2a_1a_2(\Theta^+ - \Theta^-)\right] J_0^2(\chi) \\ &\quad + (a_1^2 - a_2^2)(\Theta^+ + \Theta^-) J_0(\chi) J_2(\chi) \Big\}. \end{aligned} \tag{8}$$

Using the values of the momentum at the two poles as given by (3), we obtain

$$\Theta^+ + \Theta^- = 2e^{-\gamma(\omega^2+\Omega^2)}\cosh(2\gamma\omega\Omega), \tag{9}$$

$$\Theta^+ - \Theta^- = -2e^{-\gamma(\omega^2+\Omega^2)}\sinh(2\gamma\omega\Omega), \tag{10}$$

where the definition

$$\gamma = \frac{1}{2mk_BT}\left(\frac{\mathcal{E}}{c^2k_3}\right)^2 \tag{11}$$

has been employed. Consequently, expression (8) becomes

$$\begin{aligned} W &= \frac{\pi N q^2\omega}{c^2\sqrt{\pi(2mk_BT)^5}}\left(\frac{\omega}{k_3}-v_R\right)e^{-\gamma(\omega^2+\Omega^2)}\int_0^\infty dp_\perp p_\perp^3 e^{-p_\perp^2/2mk_BT} \\ &\quad .\Big\{\Big[(a_1^2+a_2^2)\cosh(2\gamma\omega\Omega)-2a_1a_2\sinh(2\gamma\omega\Omega)\Big]J_0^2(\chi) \\ &\quad +(a_1^2-a_2^2)\cosh(2\gamma\omega\Omega)J_0(\chi)J_2(\chi)\Big\}. \end{aligned} \tag{12}$$

Integrals over the perpendicular component of the momentum are evaluated using the expression

$$\begin{aligned} &\int_0^\infty x^{\lambda+1}e^{-\alpha x^2}J_\mu(\beta x)J_\nu(\beta x)\,dx \\ &= \frac{1}{2\alpha^{1+\lambda/2}}\left(\frac{\beta^2}{4\alpha}\right)^{(\mu+\nu)/2}\sum_{n=0}^{\infty}\left(-\frac{\beta^2}{4\alpha}\right)^n\frac{\Gamma[n+1+(\mu+\nu+\lambda)/2]\,\Gamma(2n+\mu+\nu+1)}{n!\,\Gamma(n+\mu+1)\,\Gamma(n+\nu+1)\,\Gamma(n+\mu+\nu+1)} \end{aligned} \tag{13}$$

with the constraint $\nu \neq -1, -2, -3, ...$ With the abbreviations

$$\alpha = \frac{1}{2mk_BT} \tag{14}$$

and

$$\beta = \frac{ck_1}{qH_0}, \tag{15}$$

the results

$$\int_0^\infty p_\perp^3 e^{-\alpha p_\perp^2}J_0^2(\beta p_\perp)\,dp_\perp = 2(mk_BT)^2\sum_{n=0}^{\infty}\frac{(n+1)(2n)!}{(n!)^3}(-y)^n \tag{16}$$

$$\int_0^\infty p_\perp^3 e^{-\alpha p_\perp^2}J_0(\beta p_\perp)J_2(\beta p_\perp)\,dp_\perp = 2(mk_BT)^2 y\sum_{n=0}^{\infty}\frac{(2n+2)!}{(n!)^2(n+2)!}(-y)^n \tag{17}$$

follow, where

$$y = \frac{\beta^2}{4\alpha} = \frac{k_BT/2mc^2}{\Omega_0^2/(ck_1)^2} = \left(\frac{ck_1}{\Omega_0}\right)^2\frac{k_BT}{2mc^2}\,. \tag{18}$$

The frequency Ω_0 is the Larmor frequency in the nonrelativistic approximation. Keeping only the first two terms in the summation series, equation (12) becomes

$$\begin{aligned} W &= \frac{\pi N q^2 \omega}{2c^2\sqrt{2\pi m k_B T}} \left(\frac{\omega}{k_3} - v_R\right) e^{-\gamma(\omega^2+\Omega^2)} \\ &\quad . \left\{ \left[(a_1^2 + a_2^2) + (a_1^2 - a_2^2) y \right] \cosh(2\gamma\omega\Omega) - 2a_1 a_2 \sinh(2\gamma\omega\Omega) \right\} (1 - 4y). \quad (19) \end{aligned}$$

The number of terms to be included is determined by the value of y. For $y = 0.01$ we need 5 terms in integral (16) and 4 terms in integral (17) for the numerical results to change less than one unit at the sixth decimal place. For $y = 0.1$ the same accuracy requires 7 terms in both integrals, while $y = 1$ requires 21 terms in (16) and 20 terms in (17). The physics behind the numerical values reflects the interplay of the thermal perturbations against the perturbations of the wave mode propagating in the plasma, as shown in (18). It can also be shown that integral (13) is absolutely convergent, and therefore, can be used for any combination of the plasma parameters entering expression (18).

Bibliography

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