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Transport and Zonal Flow Energetics in Plasma Edge Turbulence

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The model equations describing the non-linear evolution and saturation of drift-Alfvén turbulence in flux tube geometry under gyro-Bohm normalization may be written as [1, 2]

$$\frac{\partial \Omega}{\partial t} + \mathbf{v}_E \cdot \nabla \Omega = \mathcal{K}(n) + \nabla_{\shortparallel} J + \mu_\Omega \nabla_\perp^2 \Omega, \tag{1a}$$

$$\frac{\partial n}{\partial t} + \mathbf{v}_E \cdot \nabla (n_{\text{eq}} + n) = \mathcal{K}(n - \phi) + \nabla_{\shortparallel}(J - u) + \mu_n \nabla_\perp^2 n, \tag{1b}$$

$$\frac{\partial}{\partial t}\left(\widehat{\beta}\psi + \widehat{\mu}J\right) + \widehat{\mu}\mathbf{v}_E \cdot \nabla J = \nabla_{\shortparallel}(n_{\text{eq}} + n - \phi) - CJ, \tag{1c}$$

$$\widehat{\varepsilon}\left(\frac{\partial u}{\partial t} + \mathbf{v}_E \cdot \nabla u\right) = -\nabla_{\shortparallel}(n_{\text{eq}} + n), \tag{1d}$$

governing the vorticity $\Omega = \nabla_\perp^2 \phi$ with ϕ the electrostatic potential, the deviation n of the plasma particle density from its equilibrium $n_{\text{eq}}(x)$, the magnetic potential ψ and parallel current $J = -\nabla_\perp^2 \psi$, and the parallel ion velocity u. The electric drift and magnetic field perturbations are given by $\mathbf{v}_E = \mathbf{b} \times \nabla \phi$ and $\mathbf{B} = -\mathbf{b} \times \nabla \psi$. Perpendicular and parallel spatial lengths are scaled by ρ_s and $2\pi q R_0$, while time is normalized by $L_\perp / c_s$ where $L_\perp = \left|\partial n_{\text{eq}}/\partial x\right|^{-1}$ is the equilibrium density length scale. Furthermore, we have introduced the differential operators

$$\nabla_\perp^2 = \left(\frac{\partial}{\partial x} + \frac{\hat{s}s}{qR_0}\frac{\partial}{\partial y}\right)^2 + \frac{\partial^2}{\partial y^2}, \quad \nabla_{\shortparallel} = \frac{\partial}{\partial s} - \widehat{\beta}\,\mathbf{b} \times \nabla\psi \cdot \nabla, \quad \mathcal{K} = -\omega_B\left(\sin s\,\frac{\partial}{\partial x} + \cos s\,\frac{\partial}{\partial y}\right),$$

and defined the parameters $\widehat{\varepsilon} = (qR/L_\perp)^2$ being the scale ratio of parallel and perpendicular dynamics, $\widehat{\beta} = (2\mu_0 P/B_0^2)\widehat{\varepsilon}$ representing magnetic induction, $\widehat{\mu} = (m_e/m_i)\widehat{\varepsilon}$ describing finite electron inertia effects, $C = 0.51 L_\perp \widehat{\mu}/\tau_e c_s = 0.51\widehat{\nu}\widehat{\mu}$ governing resistive relaxation, $\hat{s} = \partial \ln q / \partial \ln r$ the magnetic shear, and finally $\omega_B = 2L_\perp/R_0$ describing magnetic field inhomogeneity and curvature. The notation is otherwise standard with τ_e is the electron collision time and R_0 the magnetic field radius of curvature. In the computations to be presented here the equilibrium, satisfying $\mathcal{K}(n_{\text{eq}}) = \partial J_{\text{eq}}/\partial s$ and $\partial \phi_{\text{eq}}/\partial s = -CJ_{\text{eq}}$, is allowed to evolve self-consistently under the influence of the turbulent fluctuations.

To diagnose kinetic energy transfer and zonal flow generation by electromagnetic drift wave turbulence we further define the following energy integrals,

$$P(t) = \int d\mathbf{x}\, \frac{1}{2}\widetilde{n}^2, \qquad K(t) = \int d\mathbf{x}\, \frac{1}{2}\left(\nabla_\perp \widetilde{\phi}\right)^2, \qquad U(t) = \int d\mathbf{x}\, \frac{1}{2} v_0^2.$$

Here we denote flux-surface averages by a zero index and the deviation from this average by an over-tilde. The evolution of the zonal flow energy level is readily obtained from equation (1a),

$$\frac{dU}{dt} = T_{\mathrm{RS}} + T_{\mathrm{MS}} + T_{\mathrm{GAM}} + \Delta, \tag{2}$$

where the kinetic energy transfer terms due to Reynolds stress (RS), Maxwell stress (MS) and geodesic acoustic modes (GAM) are given respectively by

$$T_{\mathrm{RS}} = \int d\mathbf{x}\, \widetilde{v}_x \widetilde{v}_y \frac{\partial v_0}{\partial x}, \qquad T_{\mathrm{MS}} = -\widehat{\beta} \int d\mathbf{x}\, \widetilde{B}_x \widetilde{B}_y \frac{\partial v_0}{\partial x}, \qquad T_{\mathrm{GAM}} = -\omega_B \int d\mathbf{x}\, v_0 n \sin s.$$

The last term in Eq. (2) represents collisional damping due to viscous diffusion. Flow generation by Reynolds stresses is well known to result from an average phase correlation between the velocity fluctuations in the drift plane spanned by the x and y coordinate axes. The tendency of convective structures to be tilted with a seed sheared flow makes the transfer term T_{RS} generally positive, draining energy from the fluctuating motions to the zonal flows [3, 4].

Neglecting the toroidicity of the equilibrium magnetic field, the vorticity equation (1a) gives the linear spectral relation $(\omega/k_{\parallel})\widehat{\phi}_{\mathbf{k}} = \widehat{\psi}_{\mathbf{k}}$. If the parallel phase velocity $\omega/k_{\parallel}$ is close the Alfvén speed we have $\widehat{\phi}_{\mathbf{k}} = \widehat{\beta}^{1/2}\, \widehat{\psi}_{\mathbf{k}}$, indicating a possible cancellation of electrostatic and magnetic stresses [5]. The latter effect is most likely to be important at large $\widehat{\beta}$.

The transfer term T_{GAM} in Eq. (2) shows that sidebands of the axisymmetric density perturbations may alter the evolution of the zonal flow energy. If we for the moment assume adiabatic electron motion and and neglect dissipation, the poloidally mean flows are given by

$$\frac{\partial v_0}{\partial t} + \frac{\partial}{\partial x}\left(v_x v_y - \widehat{\beta} B_x B_y\right)_0 + \omega_B (n \sin s)_0 = 0. \tag{3}$$

From the plasma continuity equation (1b) we find the evolution of the density sidebands,

$$\frac{\partial}{\partial t}(n \sin s)_0 + \frac{\partial}{\partial x}\left(\sin s\, n \frac{\partial \phi}{\partial y}\right)_0 + \omega_B \left(\sin^2 s \frac{\partial n}{\partial x}\right)_0 = \omega_B \left(\sin^2 s \frac{\partial \phi}{\partial x}\right)_0 - \left(\sin s \frac{\partial u}{\partial s}\right)_0. \tag{4}$$

The contribution of the flow $v_0 = \partial\phi_0/\partial x$ in the first term on the right hand side of Eq. (4), describing the up-down asymmetric plasma compression due to poloidal rotation, couples with the zonal flow equation (3) and results in geodesic acoustic modes (GAMs) at frequency $\omega_B/\sqrt{2}$ (Refs. 6–9). Other terms in Eq. (4), along with coupling to the ion flow side-bands, may cause an acceleration of zonal flows in the presence of poloidally asymmetric particle fluxes, known as Stringer-Winsor spin-up [7–10]. In this connection we also note that the transfer to the energy K of the fluctuating motions due to toroidal geometry is given by

$$-\int d\mathbf{x}\,\widetilde{\phi}\,\mathcal{K}(n) = -\omega_B \int d\mathbf{x}\left(\sin s\, n\,\frac{\partial\phi}{\partial x} + \cos s\, n\,\frac{\partial\phi}{\partial y}\right).$$

This indeed indicates the tendency towards a ballooning structure of the fluctuations since this drives velocity fluctuations as far as the turbulent plasma transport is radially outwards from the torus axis and poloidally towards the out-board mid-plane. This geodesic transfer process was recently revisited in Ref. 1 where it was claimed that the GAM transfer is generally from the zonal flows through the density side-bands to the turbulent fluctuations.

To address the simultaneous action of all these effects we resort to three-dimensional numerical computations of the four-field model (1) on a grid of $64 \times 256 \times 32$ points, with dimensions $64 \times 256 \times 2\pi$ in x, y and s, respectively. Nominal parameter values are $\widehat{\varepsilon} = 18750$, $\widehat{\mu} = 5$, $\hat{s} = 1$, $\omega_B = 0.05$ and $\mu_\Omega = \mu_n = 0.025$. In Figs. 1–3 we present the variation with $\widehat{\beta}$ and $\widehat{\nu}$ of the energy integrals, the turbulent radial plasma transport Γ_n and zonal flow energy transfer rates averaged over 10^3 time units in the turbulent state of the computations. We observe that the fluctuation energy and turbulent transport increases with the parallel resistivity $\widehat{\nu}$ over the whole range of $\widehat{\beta}$ investigated. On the other hand, at low $\widehat{\beta}$ the saturation level of zonal flow energy decreases with both $\widehat{\beta}$ and $\widehat{\nu}$, and is significantly smaller than the kinetic energy in the fluctuating motions. The branching ratio of kinetic energy into the zonal flows decreases slightly with $\widehat{\nu}$ and $\widehat{\beta}$. Also shown are the zonal flow energy transfer rates. We observe that the Reynolds stress transfer is always such as to drive zonal flows but decreases with increasing $\widehat{\nu}$ and $\widehat{\beta}$. With increasing $\widehat{\beta}$ the Maxwell stress acts to cancel the Reynolds stress drive, first at low $\widehat{\nu}$ but eventually over the whole range of collisionality at large $\widehat{\beta}$. Finally, the geodesic transfer mechanism generally drains energy out of zonal flow modes at low $\widehat{\beta}$ and $\widehat{\nu}$ but drives zonal flows in the opposite limit where the turbulent flux is large.

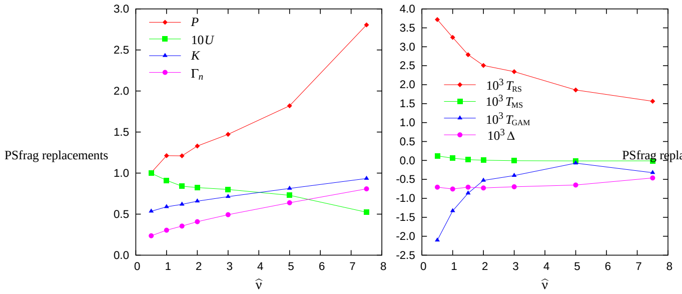

Figure 1: Transport, energy integrals and zonal flow energy transfer terms for $\widehat{\beta} = 1.0$.

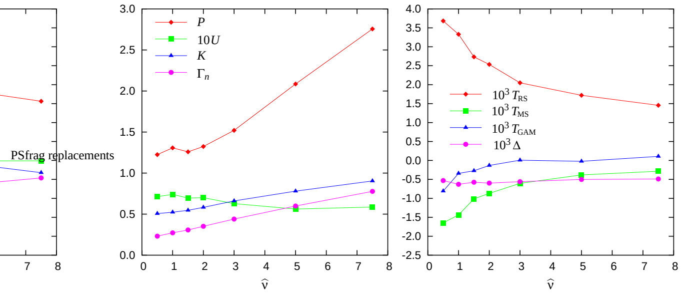

Figure 2: Transport, energy integrals and zonal flow energy transfer terms for $\widehat{\beta} = 7.5$.

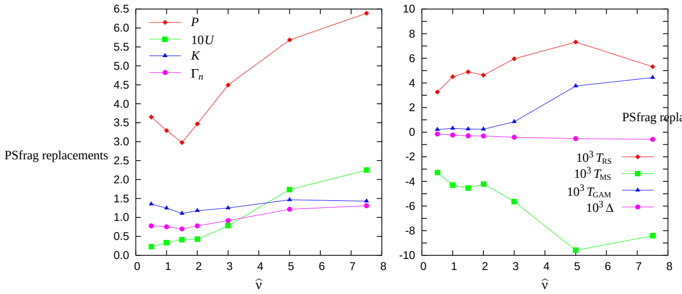

Figure 3: Transport, energy integrals and zonal flow energy transfer terms for $\widehat{\beta} = 30$.

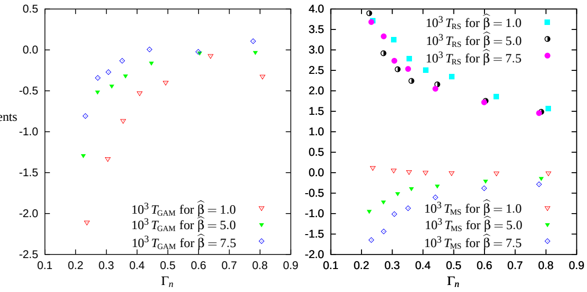

Figure 4: Scatter plot of zonal flow energy transfer rates against the turbulent flux Γ_n.

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