

## Quasineutral simulations of plasma response to the lower hybrid antenna electric field\*

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### 1 Introduction

A particle-in-cell (PIC) simulation technique suitable for describing phenomena evolving on time and spatial scales much shorter than respectively  $1/\omega_{pe}$  and  $\lambda_D$  was proposed by Joyce et al.[1]. In this method, which we henceforth refer to as quasi-neutral PIC or QPIC, the self-consistent electrostatic field  $E_s$  is not determined from the Poisson equation but rather at each time step of the simulation  $E_s$  is calculated from the electron momentum equation and the requirement of quasi-neutrality. The QPIC technique is thus suitable where the governing temporal and spatial scales in the problem do not allow large departures from quasi-neutrality. The QPIC approach significantly reduces the simulation CPU time and mainly avoids swamping of the physically induced charge separation by statistical fluctuations in the electron and ion densities. Joyce et al. [1] emphasize the need for a QPIC technique, as opposed to a much simpler and faster hybrid quasi-neutral technique where electrons are described as a fluid, in problems where the electron and/or ion distribution function can be anticipated to be non-Maxwellian. The QPIC technique was adapted and successfully applied to tokamak edge plasma problems, specifically for the interpretation of data from Langmuir probes and field retarding analyzers in Ref. [2]. In the present work we concentrate on applying QPIC to the study of tokamak edge plasma behavior in the vicinity of a lower hybrid (LH) antenna. The QPIC simulations are faster by a factor of about 50 than the corresponding PIC simulations. Quasi-neutrality is maintained throughout the LH grill region and without plasma sources other than thermal plasma inflow along magnetic field lines. The plasma density in the grill region suffers a depression as a result of electron heating and acceleration. Under the same plasma source conditions the ion heating and power flow due to coupling via the charge separation field  $E$  is negligible, in agreement with previous MHD calculations [3].

### 2 QPIC simulation technique

We consider electrostatic fields and a one-dimensional case specified by the Cartesian coordinate  $z$ . We assume global quasi-neutrality, i.e. the total number of electrons equals the total number of ions. The boundary conditions imposed on the simulation region must reflect the global quasi-neutrality condition, i.e. for each particle leaving the region a like particle must enter. We further assume that the scale length and time scale to be resolved in the problem are large compared with

$\lambda_D$  and  $1/\omega_{pe}$ , respectively. Thus the self-consistent electric field  $E_z$  always takes the value necessary to maintain quasi-neutrality. This field  $E_z$  is at each time step and grid point along the simulation region determined from the electron momentum fluid equation [1]

$$-eE_z = \frac{1}{n_e} \frac{\partial}{\partial t} (m_e n_e \langle v \rangle) + \frac{1}{n_e} \frac{\partial P_e}{\partial z} + e \langle E_{rf} \rangle \quad (1)$$

where the last term represents momentum transfer between the rf field and the electrons.  $E_z$  is the self-consistent field also felt by the ions and  $E_{rf}$  is the lower hybrid antenna electric field only acting on the electrons. Further,  $P_e = m_e n_e \langle v^2 \rangle$  is the electron pressure. An essential part of the QPIC technique is a method of dealing with Eq. (1) to yield a solution  $E_z$  which enforces quasi-neutrality. This is achieved by replacing the electron density  $n_e$  in the pressure term of Eq. (2) by the ion density  $n_i$ . This causes stable unphysical rapid oscillations  $\omega_h$  of  $n_e$  around  $n_i$  which, as shown by Joyce et al. [1], are much slower than  $\omega_{pe}$  but having  $1/\omega_h$  much slower than ion time scales. The the first term on the right hand side of Eq. (1) only exhibits the rapid oscillations of  $n_e$  around  $n_i$  and is omitted. Finally then, Eq. (1) for  $E_z$  reduces to

$$-eE_z = \frac{1}{n_i} \frac{\partial (m_e n_i \langle v_e^2 \rangle)}{\partial z} + e \langle E_{rf} \rangle \quad (2)$$

With  $E_z$  given by (2) we solve the particle equations of motion

$$\dot{v}_e = -\frac{e}{m_e} (E_z + E_{rf}) \quad , \quad \dot{v}_i = \frac{e}{m_i} E_z \quad (3)$$

In order to fully exploit the computational advantage offered by QPIC we now replace the rapid rf electron dynamics ( $\omega_{LH} \approx \omega_{pe}$ ) by the much slower diffusive Langevin process  $\Lambda$

$$(\Delta v)_\Lambda = \Delta t F + \sigma \sqrt{2D \Delta t} \quad (4)$$

where  $\Delta t$  is the time step,  $F$  and  $D$  are, respectively, the dynamic friction and diffusion coefficients derived for the electron-LH grill interaction in Ref [4], and  $\sigma$  is a normally distributed stochastic variable having  $\langle \sigma \rangle = 0$  and  $\langle \sigma^2 \rangle = 1$ . A complete time step cycle consists of pushing the particles to new positions in phase space  $(v, z)$  using Eqs (3), subsequently finding the new distributions  $f_e$  and  $f_i$  and finally calculating the integrals in (2) to update  $E_z$ .

### 3 The 2<sup>nd</sup> order area-preserving Runge-Kutta integration scheme

In PIC and QPIC simulations with many particles and evolving on long time scales it is essential to use a sufficiently simple, accurate and stable integration scheme for the electron and ion equations of motion. For forces which are only a function of time and particle position, the time-centered and second order accurate leapfrog (LF) method can be used [5]. The LF method is time-reversible, area-preserving and conditionally stable [5]. Unfortunately, LF cannot be used here

since  $F$  and  $D$  in (4) depend on velocity. Instead we therefore use the following area-preserving (i.e. unity Jacobian) form of the the 2<sup>nd</sup> order Runge-Kutta (RK) method for equations of motion [6]:

$$\begin{aligned} v_{n+1} &= v_n + \frac{\Delta t}{2}(a_n + a_{n+1}^*) \quad , \quad z_{n+1} = z_n + \frac{\Delta t}{2}(v_n + v_{n+1}) \\ a_{n+1}^* &= a_n(t_n + \Delta t, v_n + \Delta t a_n, z_n + \Delta t v_n) \end{aligned} \quad (5)$$

The subscripts in (5) indicate time-levels. Area-preservation of the mapping (5) is achieved by first evaluating the new velocity  $v_{n+1}$  and using that instead of the usual RK anticipated velocity  $v_{n+1}^* = v_n + \Delta t a_n$  in the position equation. The scheme (5) is approximately time-centred and possesses the same numerical stability properties as the LF scheme [6].

#### 4 Simulations of plasma response in the vicinity of an LH grill

We consider the effect of a 16 wave-guide Tore Supra-like LH grill (wave-guide+septum width  $d=1.05$  cm) generating a parallel (to  $\vec{B}$ ) electric field

$$E_{rf} \cong \omega v_q \cos[\omega t - \varphi(z)] \quad (6)$$

Here,  $\omega=2\pi f$ ,  $f=3.7\text{GHz}$ ,  $v_q=eE_0/m\omega$  is the electron quiver velocity,  $E_0\cong 3.5$  kV/cm is the antenna electric field strength, and  $\varphi(z)$  signifies the  $\pi/2$  phasing between the 16 wave-guides. In the PIC simulations, shown in Fig 1 (with the self-consistent field  $E_z$  from the Poisson equation), the particle equations of motion are solved using the leapfrog method with full electron trajectories in the rf field (6). In the QPIC simulations, shown in Fig. 2 [with  $E_z$  from Eq. (3)] we use the Runge-Kutta scheme (5) and the electron Langevin process (4) where  $D \cong v_q^2 |v_{||}|/2d$ ,  $F = \partial D/\partial v_{||}$  [4]. The simulation region is made up of the grill region extended on each side by 8 cm plasma regions with  $E_{rf}=0$ . The boundary condition in both cases is a Maxwellian plasma influx to compensate for the plasma outflow. The PIC simulation with 30 cells per wave guide, 100 electrons and ions per cell, and a time step  $\Delta t=0.1/f$  took 40 hours. The QPIC simulation with 4 cells per wave-guide, 600 electrons and ions per cell and a variable time step  $\Delta t=0.1 dz/v_{max}$ , where  $v_{max} \cong 2 \times 10^7$  m/s is the upper bound of the electron velocity space stochastic region [4], took 1 hour. It is interesting to note that unlike the PIC code in which the grid spacing must be smaller than the Debye length to maintain stability, the choice of grid spacing in QPIC is limited only by the need to resolve the expected macroscopic gradients.

#### References

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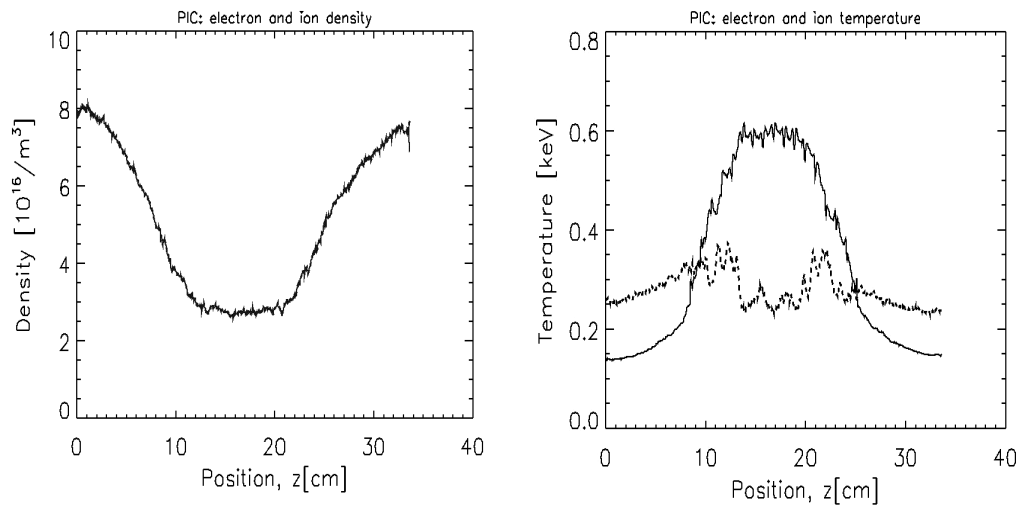


Fig. 1 PIC simulation of plasma response to the LH grill electric field with Newton electron dynamics and the leapfrog integration method. a) The electron and ion densities exhibit a quasi-neutral response. b) Electron and ion temperatures vs position. The electrons are strongly heated within the grill region. The initial conditions are  $T_e = 50$  eV,  $T_i = 200$  eV,  $n_{e,i} = 5 \times 10^{17}$   $1/m^3$ .

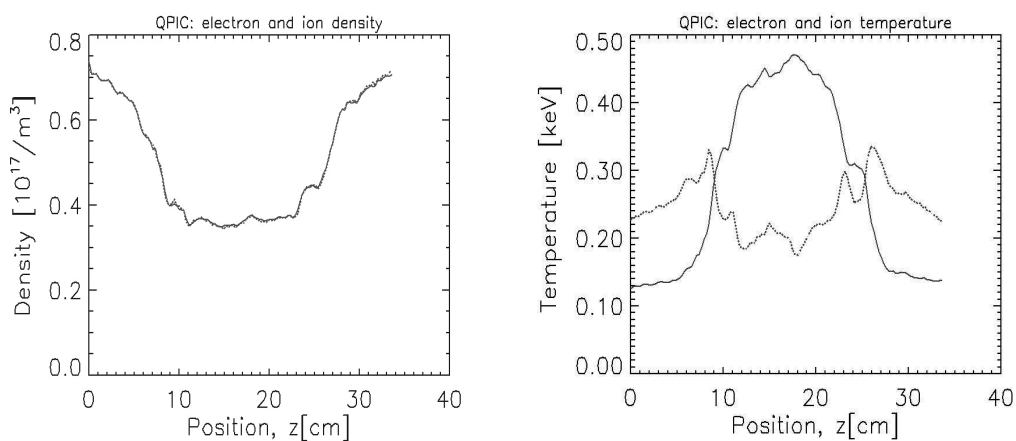


Fig. 2 QPIC simulation of plasma response to the LH grill electric field with Langevin electron dynamics and the Runge-Kutta integration method. a) Electron and ion densities and b) electron and ion temperatures vs position.