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# Effect of magnetic field ripples on fast ion non-adiabaticity

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## 1.Introduction

The magnetic moment of fast ions is known to experience non-adiabatic variations when the ions cross the minimum of the magnetic field [1]. However, in the case of smooth spatial dependence of *B*, these variations are relatively small (typically less then 1%), like for NBI ions in the plasma core of spherical tori (NSTX [2], CHS [2-4]), even in the case of a relatively large non-adiabaticity parameter $\varepsilon \sim 0.1$ that gives the ratio of gyro radius to curvature radius of *B*. Recent investigations [4-5] have shown that small-scale poloidal and helical ripples of *B* can result in a significant enhancement of non-adiabatic variations of the magnetic moment (up to ~10-20%) in the plasma periphery of spherical tori. Here the extent of magnetic moment variations is calculated as a function of the ripple magnitude as well as of the ripple scale along the magnetic field line for a modelled toroidal and mirror-like rippled magnetic field. Strong enhancement of the jumps of the magnetic moment is seen near the minimum magnetic field as the ripple scale decreases and the ripple magnitude increases. The numerical results obtained for ripple induced non-adiabatic changes of the magnetic moment $\mu$ agree satisfactorily with qualitative analytical estimations.

## 2. Effect of field ripples on non-adiabatic jumps of $\mu$

The mirror-like magnetic field model used for our investigation of non-adiabatic variations of the particle magnetic moment is of the form

$$A_\varphi = A_\varphi^{coils} + A_\varphi^{ripple} , \tag{1}$$

where $A_\varphi^{coils}$ represents the vector potential for the field between two coils situated at a distance *2L*, and

$$A_\varphi^{ripple} \propto \begin{cases} K_o\left(mR_*\right) I_1\left(mR\right)\cos\left(mZ\right) , & R < R_* \\ -I_o\left(mR_*\right) K_1\left(mR\right)\cos\left(mZ\right) , & R > R_* \end{cases} \tag{2}$$

is the vector potential of the field induced by the surface current in the plasma at $R=R_*<a$ with *a* the plasma radius, *K* and *I* are Bessel functions and $m=k_{ripple}\pi/L$ where $k_{ripple}$ is a parameter characterizing the periodicity of the surface current along the Z-axis (inverse longitudinal scale of ripple perturbation). Further we assume that, near the plane of symmetry (Z=0), the ripple magnitude is rather weak, $\delta=|B_{ripple}/B|<<1$, and is independent of *m*. The typical variation of the ripple perturbation near the plane of symmetry is shown in Fig.1 for different *k*.

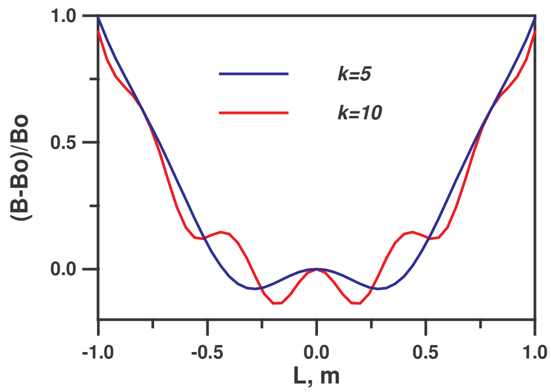


*Fig.1. Variation of the magnetic field along the field line for different $k=\alpha L/\pi$ at R=0.7 m, $B_o=B(Z=0)$.*

The variation of the magnetic moment of a charged particle over successive bounce periods ($\tau_b$) in the considered magnetic field may be described as

$$\bar{\mu} - \mu = D\mu \cdot \cos\bar{\vartheta}, \quad \bar{\vartheta} - \vartheta = \Delta\vartheta(\bar{\mu}) \equiv \int_0^{\tau_b} dt\omega_B = 2\pi\langle\omega_B\rangle / \omega_b, \tag{3}$$

where $D\mu$ is the total variation and $\omega_B$ and $\omega_b$ denote the gyro and bounce frequency, respectively. In the following we will consider in Eq.(3) only the non-adiabatic contribution $\Delta\mu$ to $D\mu$. In accordance with [1], $\Delta\mu$ is given by

$$\frac{\Delta\mu}{\mu} = -\pi\frac{V}{V_\perp}\mathrm{Re}\left\{M\exp\left(i\vartheta - \frac{\alpha}{\varepsilon}\right)\right\}, \quad \frac{\alpha}{\varepsilon} = -i\int_0^{Z_s} dZ\omega_B / \dot{Z}\ ; \tag{4}$$

here $V$ and $V_\perp$ are the total velocity and, respectively, the transverse velocity component of the particle, $\vartheta$ is the corresponding gyro-phase at the plane of symmetry ($Z=0$), $\varepsilon$ = ratio of ion gyro-radius to the gradient scale length of the magnetic field, $Z_s$ is the longitudinal coordinate at the stationary point $B(Z_s) = 0$, and $M \sim 1$ is a value weakly dependent on $\varepsilon$ and the magnetic field geometry [1]. Magnetic field ripples contribute to non-adiabatic jumps of the magnetic moment, as given by Eq.(4), mainly due to the their strong effect on the imaginary part of $Z_s$. Following [3], the lowest values of $Z_s$ may be estimated as a function of ripple amplitude ($\delta$) and space scale ($k$)

$$Z_{lowest} \approx -\frac{1}{k}\ln\delta, \quad \Delta\mu(\delta) \cong \Delta\mu(0)\left(\frac{\delta}{\delta_o}\right)^\eta \quad , \quad \eta \approx \frac{\alpha(Z_{s0}/L)L}{k\varepsilon Z_{s0}}, \tag{5}$$

In Eq.[5] the root $B_0(Z_{s0})=0$ for the unperturbed magnetic field determines the critical value of the ripple perturbation, $\delta_o$. If the perturbation exceeds this critical value, non-adiabatic variations of $\mu$ rapidly increase as the longitudinal scale decreases. We point out that the radial scale of the perturbation is supposed to be in the order of the particle gyro-radius that introduces the upper limit of $k_{ripple}$.

### 3.Numerical results

For numerical modelling of non-adiabatic variations of the magnetic moment we take $B(R=0,Z=0)=4.5\ KG$, $B(R=0,Z=L)/B(R=0,Z=0)=6$, $L=2\ m$ and $a=1\ m$. Full gyro-orbit following calculations were performed for 15 keV deuterons during more then 100 bounce periods. Starting points were chosen just below the surface current radius $R_*$ with initial velocities that prevent trajectory intersecting with this radius. Further, the ripple perturbation was taken small ($B_{ripple}/B_o<<1$) and independent on $k$ at $R=R_*$ in the range $4<k<20$. In all calculations the particle's energy and angular momentum were conserved with a relative accuracy < $10^{-8}$. The results obtained for different ripple perturbations are presented in Fig.2.

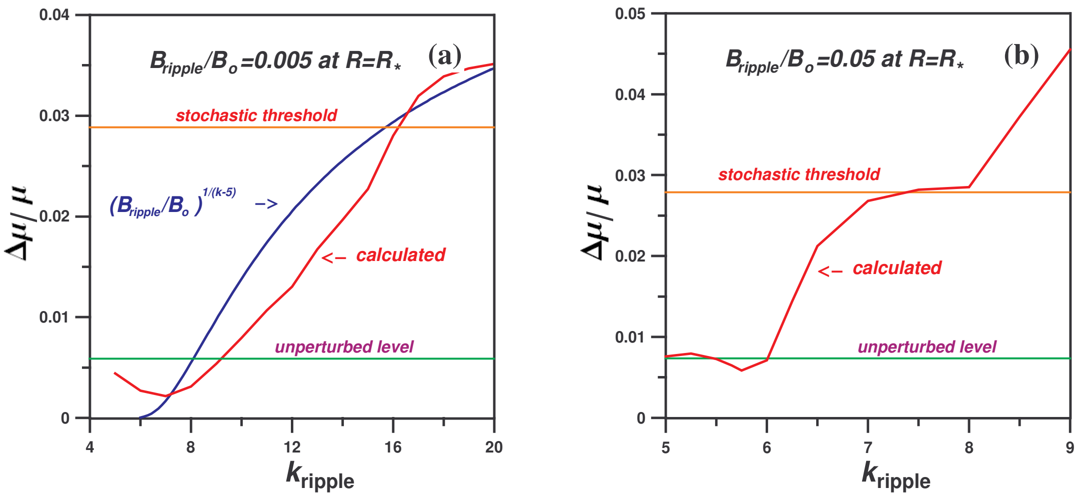


*Fig.2. Non-adiabatic variation of the magnetic moment vs inverse longitudinal scale of the ripple perturbation. Also shown are the corresponding levels for the unperturbed magnetic field and the stochastic threshold.*

From Fig.2.a it is seen that a ripple scale decrease results in a more than 10-fold increase of $\Delta\mu$, which agrees with the qualitative analysis of Sec.2. Moreover, even for the diminutive perturbation amplitude assumed one may expect to exceed the stochasticity threshold at a small longitudinal scale of the perturbation. In this region the calculation accuracy of non-adiabatic variations decreases, which may be a reason for the irregular behaviour of $\Delta\mu$, i.e. the more rapid increase for larger $k_{ripple}$). For higher perturbation levels, $B_{ripple}$=0.05$B_0$, Fig.2.b demonstrates a similar tendency of $\Delta\mu$: the expected increase is observed again, though now in a narrow interval of $k$ in correspondence with the predictions of Sec.2. In the low-$k$ range a competition can be detected between the unperturbed-$B$ contribution to the non-adiabatic variation of $\mu$ and that caused by the ripple perturbation. To illustrate the transition to stochastic behaviour of $\mu$ with increasing $k$ we depict in Fig.3 the complete time evolution of the magnetic moment of a 15 keV deuteron with $\frac{V_{\|}}{V} = 0.5$.

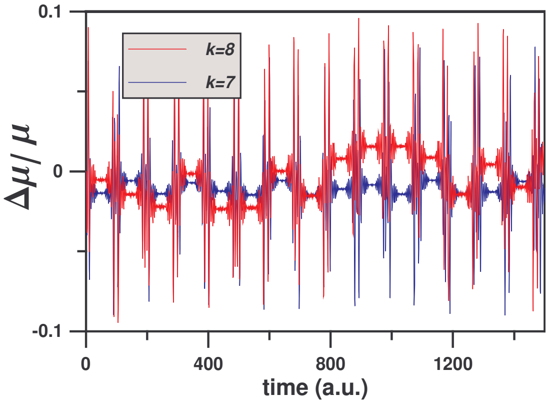


*Fig.3. Time dependence of the magnetic moment of a 15 keV deuteron with $V_{\|}/V = 0.5$ for $B_{ripple}/B$=0.05, k=8 and k=7 showing the transition from quasi-periodic to stochastic behavior.*

### 4.Conclusions and discussions

The calculations performed demonstrate an extremely strong enhancement of the jumps of the magnetic moment near the minimum magnetic field as the ripple scale decreases and the ripple magnitude increases. The numerical results are in satisfactory agreement with qualitative analytical estimations of ripple induced non-adiabatic changes of $\mu$. Periodic small-scale perturbations of the magnetic field may result in a locally unstable behaviour of $\mu$. Further investigations are required to account reliable magnetic field perturbations in toroidal plasmas.

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