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Hamiltonian dynamics investigated in a travelling wave tube

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The Hamiltonian which describes the motion of a charged particle in two electrostatic waves can be considered as a paradigm for Hamiltonian dynamics. A small cold electron beam with a kinetic energy much lower than the wave(s) energy propagates along the axis of a long Traveling Wave Tube (TWT) [1] and interacts non-self-consistently with the wave(s). The velocity distribution function of the electron beam is investigated with a trochoidal energy analyzer[2] which records the beam energy distribution at the output of the TWT. An arbitrary waveform generator is used to launch a prescribed spectrum of waves along the slow wave structure (a 4 m long helix) of the TWT. For the first time, the resonant velocity domain associated to a single wave is observed, as well as the breaking of invariant KAM tori and the subsequent transition to large scale stochasticity when the resonant domains of two waves overlap. Even for the case when a single frequency is excited, chaos can occur through the overlap of the helix mode and a beam mode. Detailed structures of phase space associated to secondary resonances can also be observed. Control of Hamiltonian chaos has been successfully achieved by launching a wave associated to the beating of the two modes with a proper phase.

The apparatus is shown schematically in Fig.1 and was described in details in [1,2]. It consists of a traveling wave tube in which an electron beam interacts with the waves on the slow wave structure. The slow wave structure is made long enough to allow non linear processes to develop. It consists in a 4 m long wire helix that is rigidly held together by three threaded alumina rods and is enclosed by a glass vacuum tube. A resistive rf termination at each end of the helix serves to reduce reflections. The glass vacuum jacket is in turn enclosed by an axially slotted cylinder that defines the rf ground. Inside this cylinder but outside the vacuum jacket are four axially movable probes which are capacitively coupled to the helix. The dispersion relation closely resembles that of a finite radius, finite temperature plasma. But, unlike a plasma, the helix does not introduce any appreciable noise. This allows defining and controlling the input wave spectrum by launching a discrete spectrum of modes with prescribed amplitude and phase on one probe.

The electron beam is produced at one end of the helix and is confined along its axis by a strong axial magnetic field (500 gauss). The central part of the electron gun consists of the grid-cathode subassembly of a ceramic microwave triode and the anode is replaced by a Cu plate with an on-axis hole whose aperture defines the beam diameter (3 mm).

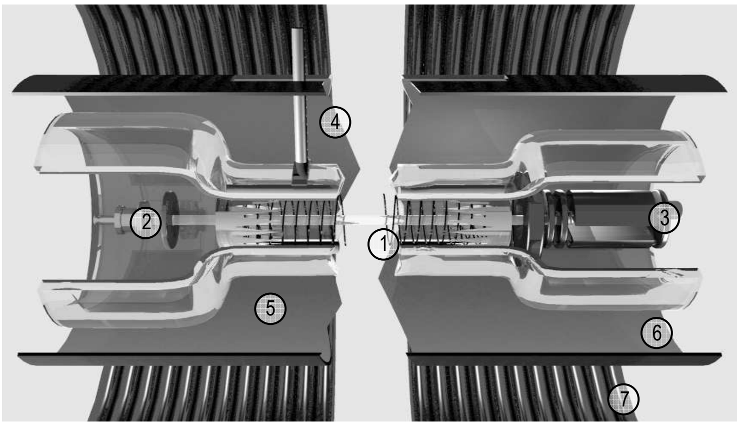

Figure 1: Rendering of the Travelling Wave Tube: (1) helix, (2) electron gun, (3) trochoidal analyzer, (4) antenna, (5) glass vacuum tube, (6) slotted rf ground cylinder, and (7) magnetic coil.

A trochoidal analyzer is set at the output of the slow wave structure. It works on the principle that charged particles undergo an E×B drift when passing through a region in which an electric field E is perpendicular to a magnetic field B. Electrons that can escape the drift region are characterized by a specific energy w_p given by the dimensions of the analyzer and the amplitude of the electric and magnetic field [1]. The electron energy distribution is determined by retarding the electron beam by means of entrance electrodes and selecting electrons having the correct drift energy w_p determined by the potential difference on two deflector plates. Retarding potential and measured current are computer controlled allowing an easy acquisition and treatment.

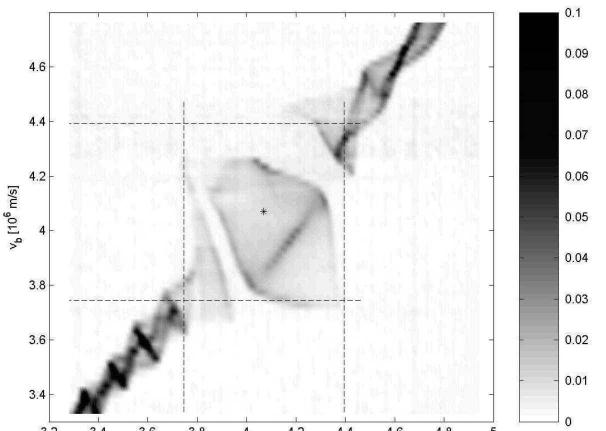

Figure 2 : Measured resonant velocity domain for a single wave at 30 MHz

We work with a beam with such a low intensity that the growth rate of the unstable modes is low enough for the electrons to be considered as test particles in the spectrum of externally excited waves.

For a single wave with constant amplitude launched at the entrance of the helix, Fig.2 is the result of superposing velocity distribution measurements obtained for different

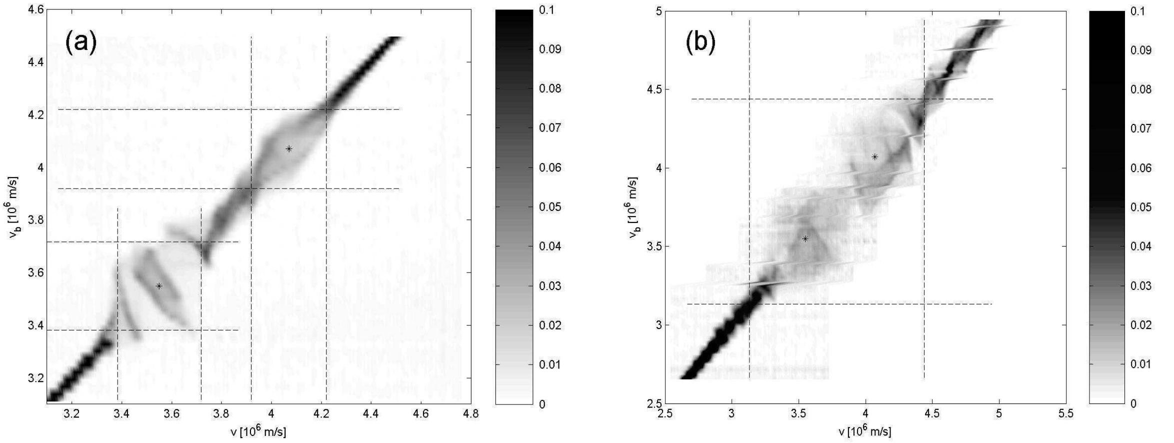

Figure 3 : Measured resonant velocity domain for two waves at 30 MHz and 40 Mhz : a) s = 0.63, b) s = 1.5

entrance energy eV_b (or related test beam mean velocity $v_b = (2\eta V_b)^{1/2}$ with $\eta = e/m$, $-e$ and m are the charge and mass of an electron). Two different regions are clearly apparent in Fig.2. For small or large values of v_b, the distribution remains centered around its initial velocity along the bisectrix with a small spread due to the electrons sloshing around v_b. For intermediate values of v_b, the beam can be trapped in the potential troughs of the wave. The central velocity of the domain where the distribution is significantly spread is indeed the phase velocity of the 30MHz wave and the broken lines in Fig. 2 define the upper and lower limits of the resonant trapping region given by an estimate of the wave amplitude obtained by determining the emitting probe coupling coefficient using 3 probes measurements. We observe that these lines correctly limit the domain where the distribution is no longer symmetrical around v_b and this domain can therefore be seen as the measured resonant zone.

Fig. 3 is obtained in the same way as Fig.2 but in the case when two different frequencies are excited on the emitting probe. One can define the overlap parameter $s = 4 (A_1^{1/2} + A_2^{1/2}) / \Delta v$, where a_i (i =1,2) is the amplitude of each helix mode, Δv is the phase velocity difference between the modes and $A_i = \eta\, a_i$. For s = 0.63, we observe two distinct resonant domains whereas, for s = 1.5 (when the two resonant domains overlap), a single velocity domain can be defined. The broken lines indicate the analytical predictions obtained from probes measurements of the waves amplitudes in good agreement with the measured velocity domains. By using a beam centered at the phase velocity of one of the two helix modes and comparing measurements of the beam velocity spread in the presence of each single wave or of both waves, we can relate the observed behavior to the breaking of

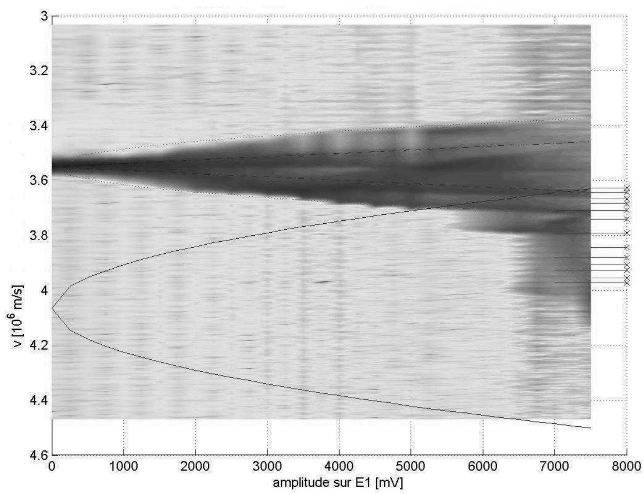

Figure 4 : Transition to chaos by overlap of beam and helix modes at 30 MHz.

invariant velocity barriers, as expected from transition to large scale chaos in a non integrable Hamiltonian system. Even in the case when a single frequency is excited on the emitting probe E1, one can observe transition to large scale chaos by resonance overlap. This is shown on Fig. 4 which plots the measured distribution function at the output of the TWT as the amplitude of a 30 MHz applied signal is varied. The bottom continuous line gives the analytical estimate of the trapping velocity domain of the helix mode centered on its phase velocity equal to $4.07\text{x}10^6$ m/s. When the signal amplitude is small, we observe the velocity modulation of the beam around its initial velocity $v_b = 3.55\text{x}10^6$ m/s as indicated by the symmetric straight broken lines around v_b given by first order perturbation theory close to electrons unperturbed orbits. When increasing the signal amplitude, we first observe a deviation from these predicted lines and then the formation of a velocity domain very similar to a trapping domain as indicated by the dotted lines. This trapping domain can be associated to the existence of an electron beam mode corresponding to a plasma oscillation propagating with the beam. For the maximum signal amplitude, we observe that electrons can even be trapped deeply into the trapping domain of the helix mode. Again transition to chaos occurs through overlap of the helix and the beam modes. Fig. 4 also clearly shows that this transition occurs by jumps associated to the overlap of secondary resonances of the two waves Hamiltonian system. Such a detailed knowledge of the system allowed us to successfully test a method of control[3] of Hamiltonian chaos achieved by launching a wave associated to the beating of the two modes with a proper phase for a minimal energy cost.

[1] G. Dimonte and J.H. Malmberg, Phys. Fluids 21, 1188 (1978); S.I. Tsunoda, F. Doveil and J. H. Malmberg, Phys. Rev. Lett. 58, 1112 (1987); D.A. Hartmann, C.F. Driscoll, T.M. O'Neil and V.D. Shapiro, Phys. Plasmas 2, 654 (1995).

[2] D. Guyomarc'h, and F. Doveil, Rev. Sci. Instrum. 71, 4087 (2000).

[3] C. Chandre, G. Ciraolo, R. Lima, and M. Vittot, to appear in Celest. Mech. (2004).