Quasilinear electron acceleration in a driven plasma wave

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We study the electron acceleration by driven electron plasma waves. The model consists of the Zakharov equations for the electron plasma and ion acoustic waves driven by an external periodic force. A modification of the electron distribution function is accounted for in the quasilinear approximation. The simulations define the saturation levels and spectra of electron plasma and ion acoustic waves and characterize the shape of the electron distribution in function of the wave number and the amplitude of the pump.

The saturation of electron plasma wave (EPW) amplitudes is an important issue in many plasma applications. It is especially important in the context of inertial confinement fusion by lasers, as wave-particle interactions can accelerate electrons to high energy that induces a preheat of the fusion fuel and reduces the target gain. One of the mechanisms that can generate EPWs is the Langmuir decay instability (LDI) that involves the decay of a the primary EPW into another EPW, propagating in the opposite direction, and an ion acoustic wave (IAW), propagating in the laser direction. This instability can be driven by the Langmuir waves produced by the stimulated Raman scattering (SRS), the two plasmon decay, and the ion acoustic decay instability. In the present work, an external driver is chosen as a source of primary EPW.

The fluid equations and the generalized Zakharov models have been used for a long time to study a nonlinear evolution of the SRS [1, 2]. The Zakharov equations [3] can describe the wave-wave processes that result in a cascade of the EPW energy to low wave numbers as well as the modulational instability and nucleation processes that result in a formation of coherent wave packets and their subsequent collapse in self-generated density wells. However the fluid model ignores the wave-particle interactions that may severely underestimate the saturation level. Vlasov or particle-in-cell (PIC) simulations have been used for a more complete description of the problem. However, these kinetic simulations are very time consuming and have difficulties in studying weak instabilities and a long time behavior because they are forced to resolve very small (electronic) spatial and temporal scales.

A quasilinear-Zakharov (QLZ) model was explored in Ref. [4] in order to account for the wave-wave and wave-particle processes simultaneously without using PIC simulations. This is a fluid model, which includes kinetic effects by coupling the Zakharov equations to the electron quasilinear diffusion equation. Although this model has a limited domain of applicability due to the random phase approximation for the EPW spectrum and the linear treatment of the IAW response, it is simple and flexible enough to investigate the processes in large time and spatial scales and to accommodate additional physical effects.

In this paper, we use the QLZ model with an external periodic driver for studies of an electron acceleration in a broadband spectrum of EPWs. The Langmuir wave electric field $E$ is enveloped over the electron plasma frequency, $\omega_{pe} = (4\pi e^2n_0/m_e)^{1/2}$, where $n_0$ is the average electron density. It depends of the electron distribution function via the Landau damping term and couples to the IAW density perturbations:

$$2i (\partial_t E + v_e * E) + 3 \partial_x^2 E = \delta n E + k_p S e^{i(k_p+\delta k_p)x-\omega(k_p+\delta k_p)t}.$$  (1)
Here we use the dimensionless units where the length is measured in the Debye lengths, \( \lambda_D = \frac{v_{th} \omega_p}{\omega_p} \), the time in \( \omega_p^{-1} \), the electron energy in \( T_e \), the density in \( n_0 \), the electric field in \( \sqrt{4\pi n_0 T_e} \), and the energy density in \( n_0 T_e \). The source of the amplitude \( S \) drives the primary Langmuir wave with the wave number \( k_p \).

The density perturbation, \( \delta n \), is described by a linear IAW equation (without dispersion and with a fixed Landau damping) and it is coupled to the Langmuir field by the ponderomotive force:

\[
\partial_t^2 \delta n + 2v_i \partial_t \delta n - \mu(1 + 3/\Theta) \partial_x^2 \delta n = \frac{1}{4} \mu \partial_x^2 |E|^2
\]

(2)

where \( \mu = Z m_e / m_i \) and \( \Theta = Z T_e / T_i \). The Landau damping terms \( v_e \) and \( v_i \) are known in the Fourier space. The EPW damping

\[
\dot{v}_e(k, t) = -\pi (2k|k|)^{-1} \partial_x F_e(v = 1/k, t)
\]

depends on the electron distribution function \( F_e(v, t) \), which is spatially averaged over the length of the simulation domain \( L \). The IAW damping \( \dot{v}_i(k) \) is fixed and is calculated for Maxwellian distribution functions:

\[
\dot{v}_i(k) = |k| \sqrt{(1 + 3/\Theta) \pi \mu / 8} \left( \sqrt{\mu + \Theta^{3/2} e^{-3/2 - \Theta/2}} \right).
\]

The evolution of the electron distribution function is described by the quasilinear equation

\[
\partial_t F_e = \partial_v (D \partial_v F_e), \quad D(v, t) = (4 \pi L)^{-1} |\dot{E}(1/v, t)|^2.
\]

(3)

Because of strong Landau damping the diffusion coefficient \( D(v, t) \) is zero for sufficiently small velocities, \( v < 2 - 3 \), therefore the quasilinear diffusion involves only the tail of electron distribution. This allows us to neglect the bulk electron heating and consider a constant electron temperature in the Zakharov equations.

The QLZ system conserves the total number of electrons, \( N_e = \int dv F_e \), and increases the electron entropy, \( H = -\int dv F_e \ln F_e, \partial_t H > 0 \). The variation of the total energy density of the system, \( \partial_t (W_E + W_e) = P \), depends on the deposited power,

\[
P = k_p (2L)^{-1} \int dx \text{Im} \left( E^* S e^{i k x - \frac{\omega_p}{2} t} \right)
\]

where \( W_E = (2L)^{-1} \int dx \ |E|^2 \) is the EPW energy density and \( W_e = \frac{1}{2} \int dv v^2 F_e \) is the electron kinetic energy.

The initial electric field and density perturbation in the Zakharov equations are set to zero. The initial electron distribution function is assumed to be a Maxwellian one and we assume periodic boundary conditions. We use a time-splitting spectral approximation for numerical solution of the QLZ equations. On each time step \( \Delta t \) we solve first (1) without coupling term \( \delta n E \) by the Fourier spectral method. Then we solve (2) and account for the coupling term in (1). To evaluate the electric field \( E \) at the new time step, we approximate the integral of \( \delta n E \) on the interval \( \Delta t \) by the trapezoidal rule. Equation (3) is discretized with an implicit difference scheme by using an ad-hoc velocity mesh grid.

The driver excites the primary EPW at \( k = k_p \). The amplitude of this wave grows until it exceeds the LDI threshold. It is a resonant three-wave interaction that transfers
the energy from unstable EPW into the backward propagating EPW at \( k_1 = -k_p + \delta k \) and the IAW at \( k_{iu} = 2k_p - \delta k \). Here \( \delta k = (2/3)\sqrt{\mu} \), accounts for the energy transmitted to the IAW.

As the scattered wave EPW1 grows, it becomes sufficiently energetic to act as a pump for second cascade of LDI. This process continues for a few steps and each time the EPW wave number is reduced by the amount \( \delta k \). This cascade spreads the wave energy to lower wave numbers. Such a weak EPW-IAW turbulence has been observed in numerical simulations of the Zakharov equations [1, 2]. If there is a sufficient dissipation at the low wave numbers, the primary instability can be saturated and the wave-particle interaction is weak. If the dissipation in the domain of small wave numbers is insufficient, the energy builds up at long wavelengths. This provokes the modulational instability that assumes the energy transport toward smaller scales by means of spatially collapsing wave packets. The modulational instability and the EPW collapse are both manifestations of a strong Landauir turbulence.

The collapse causes an energy transfer up to large wave numbers, where the wave-particle interaction provides a sufficient dissipation to saturate the primary instability. This implies that the absorbed energy is transferred to electrons, changes their velocity distribution and the Landau damping. The perturbations of the distribution function spread towards larger velocities and may modify completely the initial spectrum of the EPW turbulence. The purpose of the present study is to obtain an asymptotic state of the electron distribution and the EPW and IAW energy spectrum and investigate how this energy partition between these two subsystems depends on the wavelength and the amplitude of the driver.

One example of evolution is shown in figures 1 and 2. The system is driven with a constant source of the amplitude \( S = 5 \times 10^{-3} \) and we consider three wave numbers \( k_p = 0.09, 0.14 \text{ and } 0.24 \). Other parameters such as the mass ratio, \( \mu = 1/2000 \) and the temperature ratio is \( \Theta = 10 \) remain fixed. The length of interaction box, \( L \approx 2000 \), was adjusted to satisfy the periodic boundary conditions for the driver. We use 512 grid points with the Fourier mode spacing \( \Delta k = 2\pi/L \approx 3 \times 10^{-3} \).

Figure 1 shows the time history of the dimensionless mean EPW energy density, and

![Figure 1: The time history of the spatially averaged Langmuir wave energy (left) \( W_E \) and the mean electron energy \( W_e \) (right) for \( S = 2 \times 10^{-3} \) and \( k_p = 0.09 \).](image)

the mean electron kinetic energy density, \( W_e = W_e - 1/2 \).

In a first stage between \( t = 0 \) and \( t = 2000 \) we have a linear growth where the external
pump excites the primary EPW with $|E| = k_p \lambda_D St$. The stage between $t = 2000$ and $t = 3000$ corresponds to the convective saturation where the EPW energy density grows as $|E|^2 \sim t$.

The third stage begins when the primary EPW exceeds the LDI threshold. We have a few LDI cascades during the characteristic time scale $\Delta t = 1/\Gamma_{LDI}$ where $\Gamma_{LDI} = \frac{1}{4} \sqrt{k_p \sqrt{\mu}}$ is the LDI growth rate. On this time scale, the quasilinear diffusion has a tendency to flatten the electron distribution and the wave-particle process begins. Then the electrons are accelerated, what implies that the Landau damping term increases. So the EPW energy density strongly decreases and we have an asymptotic saturation at $\omega_p t = 6000$. In the contrary, the electron kinetic energy does not change during the first two stages and demonstrates a quick jump at the moment of the final saturation of the EPW energy. Such a lower level saturation of $W_E$ and a steady growth of $W_e$ are due to a combination of wave-wave and wave-particle processes.

In conclusion, the wave-particle interactions make an important contribution to the EPW energy density saturation. That implies that wave-particle interactions would decrease the SRS reflectivity.

Finally, by characterizing the shape of the electron distribution, we can determine the slope of the hot electron tail and determine the number of the accelerated electrons.

References