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Blowout regime for laser wakefiled in plasma channels

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Abstract. Laser wakefield in a channel provides the most efficient plasma accelerating structure: (i) the presence of channel prevents the diffraction of the laser, (ii) the blowout regime, in which electrons are radially expelled from the axis due to the ponderomotive force, is reached with a significantly lower initial a_0 and (iii) the onset of wave breaking happens in shorter time scales [1].
In this work we develop a theoretical model for the blowout regime, inspired in the seminal work on wave breaking by Dawson [2], generalized with the inclusion of both the channel and the laser. We obtain estimates for the wave breaking time and minimum a_0 required for injection.

1. INTRODUCTION

PIC simulations in OSIRIS[1] have shown that the presence of a parabolic density profile in the plasma can make both the injection and the acceleration processes more effective, in comparison to an uniform plasma density. They have shown that present day laser technology and actual experimental apparatus can effectively accelerate electrons to high energies via laser wakefield excitation in a channel.

In order to optimize the injection acceleration processes, it is fundamental to understand the role of the channel in these mechanisms. In this work we generalize the seminal work on wave breaking by Dawson[2] including both the laser and channel in the analysis. Wave breaking occurs when there is change in the order, or crossing, of two electron sheaths that oscillate back and forth in the plasma. In ref.[2], free oscillations of cylindrical plasma have been examined, while in our study we consider forced cylindrical oscillations of the electron sheaths, driven by the ponderomotive force of the laser, in a non-uniform plasma, corresponding to a parabolic density profile. When the laser interacts with a plasma, two main forces act on each electron sheath: (i) the force due to the ion channel left behind the laser, (ii) the laser ponderomotive force. For certain combinations of the laser and channel parameters, electrons near the propagation axis can be radially expelled, leaving a region totally evacuated of electrons where the laser is sitting – this is the blowout regime of wakefield excitation. Whenever two sheaths with $v \approx v_{wake} \approx c$ oscillating near the blowout radius cross each other, there is local wavebreaking, thus leading to particle injection into a cavity with a strong longitudinal electric field which will accelerate the electrons.

We will study the requirements of both the laser and the channel that allow injection to occur.

2. WAVE BREAKING – FUNDAMENTALS

We follow here reference [2]. Consider a cylindrical uniform plasma column where a cylindrical sheath of electrons with an initial position R_0 has been pushed away, leaving an ion column in the middle. The equation of motion for this oscillating sheath is given by:

$$\frac{d^2 -}{dt^2} = -\frac{1}{2} -_p^2 \frac{(-+1)^2 - 1}{(-+1)} \tag{1}$$

where ρ is the displacement of the column from R_0 normalized to R_0. Eq. (1) describes an anharmonic oscillator; sheaths will have different periods of oscillations whenever they have different oscillation amplitudes. If two or more sheaths oscillate back and forth with different periods they will cross and the wave will break. The typical time for wavebeaking is:

$$t^1_{cross} = \frac{1}{4}\frac{\tau_1^2}{R_{\max}}\frac{dR_0}{d\tau_0} \tag{2}$$

where R_{max} is the amplitude of oscillation and ρ_1 is the period of the inner sheath is determined by:

$$\pi_1 = \frac{2\pi}{\pi_p}\pi 1 \pi \frac{\pi_{\max}{}^2}{12} + \cdot\cdot\pi \tag{3}$$

From (2) we can identify three main dependences of the wavebreaking time: (i) as the period of oscillation increases so does the t^1_{cross}, (ii) an increase of the rate of change of the oscillation period with the initial radius, leads to decreasing t^1_{cross} and (iii) high R_{max} leads to lower t^1_{cross}.

3. OSCILLATIONS IN AN NON-UNIFORM PLASMA

We now assume the plasma to have a parabolic density profile as given by:

$$n(\rho) = n_0 \Delta 1 + \frac{\Delta n}{n_0}\frac{R_0{}^2}{w_0^2}(\Delta + 1)^2 \Delta \tag{4}$$

where Δn is the depth of the channel, w_0 the width of the channel and n_0 the density at the bottom of the channel.

The period of oscillations of the sheaths in the channel is:

$$\pi_c = \frac{2\pi}{\pi_p}\pi\frac{n_0}{n(0)}\pi^{1/2}\pi\, \pi 1 + \pi\frac{5}{48}\pi 1 \pi \frac{2n_0}{n(0)}\pi^2 \pi \frac{3}{16}\pi\pi_{\max}^2 + \cdot\cdot\pi \tag{5}$$

where n(0) is the initial starting density of the sheath.

Since the density has a minimum on axis, the estimated period (5) is always smaller than (3). Furthermore, as the channel gets deeper or as w_0 gets smaller the difference between ρ_l and ρ_c grows.

We can compare directly the change of ρ with R_0 for the uniform and non-uniform cases, for large $n(0)/n_0$ and assuming that the amplitudes of oscillation are the same in both situations. The later assumption lies on the fact that the unknown initial perturbation of the plasma can make the amplitudes equal on both cases. These assumptions lead to:

$$\frac{d\approx_c/dR_0}{d\approx_1/dR_0} \approx \approx \frac{n_0}{n(0)} \approx^{1/2} \approx \frac{5}{4} \approx 1 \approx \frac{2n_0}{n(0)} \approx^{2} \approx \frac{9}{4} \approx \tag{6}$$

It is not obvious that the presence of the channel will contribute to a faster wave breaking at this point. In fact eq. (6) has two regions of interest: one for high n(0)/n0, in which eq. (6) is lower than unity and a second region, where it is greater than unity. However, under the approximations that lead to (6) we can compute the relation between wave breaking times with channel, t^c_{cross} and without channel t^1_{cross}, given by:

$$\frac{t^1_{cross}}{t^c_{cross}} \approx \approx \frac{n(0)}{n_0} \approx^{1/2} \approx \frac{5}{4} \approx 1 \approx \frac{2n_0}{n(0)} \approx^{2} \approx \frac{9}{4} \approx \tag{7}$$

where we conclude that the presence of the channel contributes to faster wave breaking times.

4. LASER PONDEROMOTIVE FORCE- Forced anharmonic oscillator

We now include a the laser with a flat top profile into the analysis. We assume that the ponderomotive force is of the type:

$$F_{pond} = -m_e c^2 \frac{-\left(a^2/2\right)}{\sqrt{1+a^2}} \qquad a(-) = \frac{qA}{m_e c^2} = a_0 e^{\frac{-2R_0^2}{w_0^2}(-+1)^2} \tag{8}$$

where a(ρ) is the normalized vector potential of the laser. While writing the equation of motion we have neglected relativistic effects due to the transverse electronic motion, which play only a minor role in the transverse dynamics of the electron sheaths, but the relativistic mass correction of the electrons due to the laser field in included in our analysis.

We can estimate the equilibrium radius of an electron sheath, linearizing the motion equation about ρ=0 and considering high a_0. The resulting equation of motion describes an harmonic oscillator in which the frequency depends on R_0. For ρ_{eq} we obtain:

$$\rho_{eq} = \left[1 + \frac{8R_0^2}{w_0^2} - \frac{k_p^2 w_0^2 n(0)}{2a(0)n_0}\right]^{-1} \qquad (9)$$

showing: (i) the amplitude of oscillation decreases with increasing n(0)/n0. (or the force to pull away electrons from the axis increases with $n(0)/n_0$ (ii) smaller laser spot-sizes, and high a_0, increase ρ_{eq} because they lead to a stronger radial ponderomotive force.

Applying (2) to the new periods of oscillation, considering that $\rho_{max}=2\rho_{eq}$, and making the assumption that the time for wave breaking equals a period of the sheath oscillation, we can obtain estimates for the a_0 required for wave breaking. The following equation gives an implicit estimate in the absence of channel:

$$1 = \frac{3}{8}\frac{1}{k_p^2 R_0^2 \rho_{max}^4}\left(\frac{1}{a(0)} - \frac{8}{k_p^2 w_0^2}\right)^{-1} \qquad (10)$$

In the case of deep channels, we obtain the following:

$$1 = \frac{1}{8} + \frac{R_0^2}{w_0^2} - \frac{k_p^2 w_0^2}{16a(0)}\left(\frac{n(0)}{n_0} - 1\right) \qquad (11)$$

5. CONCLUSIONS

We have obtained approximate estimates for a_0 that enables injection in the laser wakefield blowout regime. We conclude that channels contribute to a faster sheath crossing in the sense they allow faster periods of oscillation in comparison to uniform plasma profile. However the maximum radius achieved by the sheaths, when excited by the ponderomotive force of the laser, is smaller in the presence of the channel. This effect balances with the period of oscillation, and in the case of a long laser, i.e. a laser with a pulse duration longer than the period of oscillation, it can overcomes it. In the view of this model, in order for the channel to contribute to an enhanced wave breaking it is necessary that the laser pulse duration is smaller than the typical period of oscillation of the sheaths.

REFERENCES

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