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Electromagnetic Waves in Periodically Magnetized Plasma

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Abstract: We present theoretical studies of the propagation of electromagnetic waves in periodically magnetized plasma. This is motivated by a proposed scheme for Laser-driven Undulator Radiation in the Infrared, which uses a plasma-filled undulator. The electric fields may be represented by a Bloch / Floquet series, leading to an infinite coupled set of linear equations. The dispersion, determined by the solvability of these equations, shows a band structure. A so-called dense spectrum, as occurs in similar periodic systems – corrugated wave guides – is not found here.

1. INTRODUCTION

The LURI scheme [1] has been proposed as an alternative to the free electron laser. A short intense laser pulse propagates along the axis of a plasma-filled undulator, giving the plasma electrons inside the pulse longitudinal momentum. The magnetic field of the undulator, with period λ_u, accelerates them transversely, making them radiate along the axis. Coherent emission is achieved by quasi-phase matching: $(k + nk_u)v_g = \omega$, where k and ω are the wave number and frequency of the electromagnetic wave, respectively, $k_u = 2\pi/\lambda_u$, v_g is the group velocity of the laser pulse, and n is an integer. For a sinusoidal undulator field, $n = 1$. If one assumes the dispersion for unmagnetized plasma, $\omega^2 = c^2k^2 + \omega_p^2$, with the plasma frequency $\omega_p = \sqrt{n_e e^2/\varepsilon_0 m}$ (where n_e is the electron density, $-e$ and m their charge and mass, respectively, and ε_0 the permittivity of free space), one finds approximate solutions $\omega_1 = \omega_p^2/2ck_u$, $\omega_2 = 2ck_u\,\omega_0^2/\omega_p^2$, where ω_0 is the laser frequency.

2. PROPAGATION IN MAGNETIZED PLASMA

In the proposed LURI scheme, the periodicity of the undulator is taken into account only in the *generation* of radiation, *via* the quasi-phase, $k \to k + nk_u$. Here, we want to study the effect on the *propagation*. The linearized equations for the electric field $\vec{E}$ and the current density $\vec{j}$ of a monochromatic plane wave of frequency ω read:

$$\omega^2\vec{E} - c^2\nabla\times\left(\nabla\times\vec{E}\right) = -i\omega\,\vec{j}/\varepsilon_0 \qquad -i\omega\,\vec{j} = \varepsilon_0\omega_p^2\vec{E} - \vec{\Omega}\times\vec{j} \tag{1}$$

with c the speed of light, and $\vec{\Omega}(z) = -e\vec{B}_u(z)/m$ the electron cyclotron frequency in the external magnetic field $\vec{B}_u$. For an electric field propagating along the z-axis, with $\vec{E} \perp \vec{B}_u \perp \vec{e}_z$, we find a "Schrödinger" equation:

$$\boxed{\left[c^2\, \partial^2/\partial z^2 + V(z)\right]\vec{E}(z) = 0} \qquad (2)$$

with "potential" $V(z) = \left[\left(\omega^2 - \omega_p^2\right)^2 - \omega^2\Omega^2\right] / \left(\omega^2 - \omega_p^2 - \Omega^2\right)$

2.1 Homogeneous Magnetization

In homogeneously magnetized plasma, replace $\partial/\partial z \to ik$, to find the dispersion relation: $c^2 k^2 = V = \left[\left(\omega^2 - \omega_p^2\right)^2 - \omega^2\Omega^2\right] / \left(\omega^2 - \omega_p^2 - \Omega^2\right)$. The dispersion curve shows two branches, the upper corresponding to a mainly transverse field (thus similar to the hyperbolic dispersion of unmagnetized plasma), and the lower corresponding to a mainly longitudinal field.

2.2 Periodic Magnetization

In a periodic magnetic field $\vec{B}_u(z) = \vec{B}_u(z + \lambda_u)$, waves $\propto \exp(ikz - i\omega t)$ with a single wave number no longer satisfy eqn. (2), but a superposition in form of a Floquet / Bloch wave does: $\vec{E}(z,t) = \sum_{n=-\infty}^{\infty} \vec{E}_n \exp\left(i(k + 2nk_u)z - i\omega t\right)$. Substituting this into eqn. (2) leads to an infinite set of coupled linear equations for the expansion coefficients E_n: $\sum_{n'=-\infty}^{\infty} a_{nn'} E_{n'} = 0, \quad n = -\infty \ldots \infty$. Solvability requires the determinant of the matrix $A = \{a_{nn'}\}$ to vanish. The coefficients $a_{nn'}$ depend on the form and magnitude of $\vec{B}_u$. *E.g.*, for $\vec{B}_u = B_{u0}\cos(k_u z)\,\vec{e}_x$, $\Omega^2 = \Omega_0^2\left(1 + \cos(2k_u z)\right)/2$ (note the period of $\lambda_u/2$), $\Omega_0 = -eB_{u0}/m$, they are given by: $a_{nn} = \left(\omega^2 - \omega_p^2\right)^2 - \omega^2\Omega_0^2/2 - c^2k_n^2\left(\omega^2 - \omega_p^2 - \Omega_0^2/2\right)$, $a_{n\,n-1} = a_{n\,n+1} = \Omega_0^2\left(c^2k_n^2 - \omega^2\right)/4$, with $k_n = k + 2nk_u$, $a_{nn'} = 0$ for $n' \neq n, n \pm 1$.

The solution depends on the "potential": for $\omega^2 \notin \left[\omega_p^2, \omega_p^2 + \Omega_0^2\right]$, $\omega^2 - \omega_p^2 - \Omega^2(z) \neq 0$, $V(z)$ is regular, and the "Schrödinger" eqn. (2) is an example of Hill's eqn. (a generalized Mathieu eqn.) [2]. In this case, the matrix may be normalized so that its determinant $D(k)$ converges. Its k-dependence (for fixed ω) is then given by

$D(k) = D(0) - \sin^2\left(\pi k/(2k_u)\right)/C = 0$, where C and $D(0)$ depend on ω^2, ω_p^2 and Ω_0^2.

To compute the complex solution $k(\omega)$ (Fig. 1), $D(0)$ must be calculated to convergence, increasing the size of the matrix.

In the opposite case, $\omega_p^2 \le \omega^2 \le \omega_p^2 + \Omega_0^2$, $\omega^2 - \omega_p^2 - \Omega^2(z) = 0$ at certain points and $V(z)$ diverges there, leading to (logarithmic) singularities of the solution of eqn. (2). Although these may be taken into account by generalizing the Bloch wave, the determinant of the resulting coefficient matrix, even if normalized, does not converge as a function of matrix size, but oscillates, with a nonzero amplitude. We conclude that there is no solution in this frequency range.

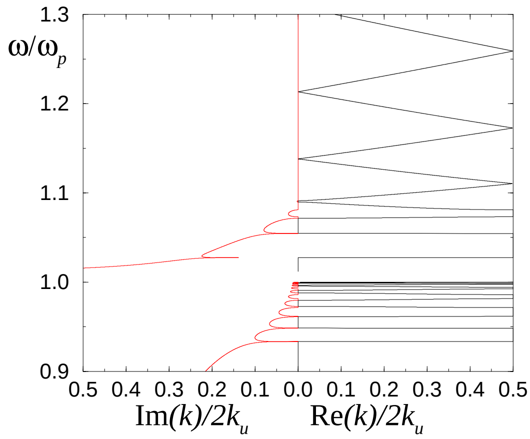

Figure 1: Dispersion curve $\omega(k)$ for periodically magnetized plasma $n_e = 4 \cdot 10^{14}\,\mathrm{cm}^{-3}$, $B_0 = 1\,\mathrm{T}$, $\lambda_u = 1\,\mathrm{cm}$: $\omega_p = 1.13\,\mathrm{THz}$, $\Omega_0 = 0.176\,\mathrm{THz}$, $ck_u = 0.188\,\mathrm{THz}$

These results are confirmed by numerical simulations of the time-dependent versions of eqns. (1,1a). Starting with a sinusoidal modulation which fixes the wave number k (commensurate with the periodic boundary conditions), the spectrum (Fig. 2) of the resulting electric shows peaks at the corresponding frequencies $\omega(k)$.

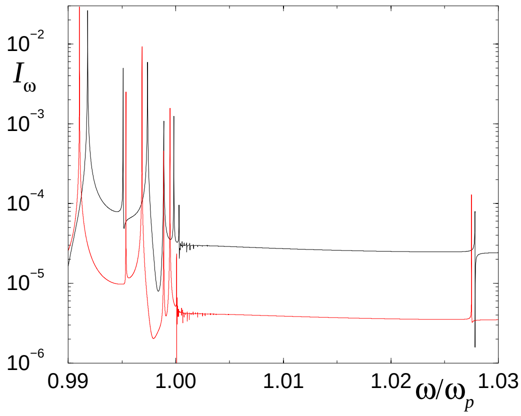

Figure 2: Detail of spectrum of transverse electric field due to an initial modulation with $k = k_u/2$.

3. LASER-DRIVEN RADIATION

In a further simulation we studied laser-driven undulator radiation, *i.e.* the electromagnetic fields generated when the electrons in the periodic magnetic field are driven longitudinally by the ponderomotive force F_p of a laser pulse (with group velocity v_g, and duration τ) (Fig. 3). This shows emission at $\omega_1 = \omega_p^2 / ck_u$ (note the factor 2 due to period $\lambda_u/2$), but also a strong and several weak lines near the plasma frequency ω_p. However, there appears no emission at the further crossings of the laser line $\omega = v_g k$ with the branches of dispersion curve, where the condition for quasi-phase matching is also satisfied.

4. CONCLUSIONS

In conclusion, we have studied the effect of periodic magnetization on wave propagation in plasma, by computing the dispersion curve both analytically and using simulations. As expected for a periodic system, it shows a band structure. A dense spectrum was not found.

Although quasi-phase matching would seem possible for many frequencies, laser-driven radiation is only emitted near the plasma frequency, and at $\omega = \omega_p^2 / ck_u$.

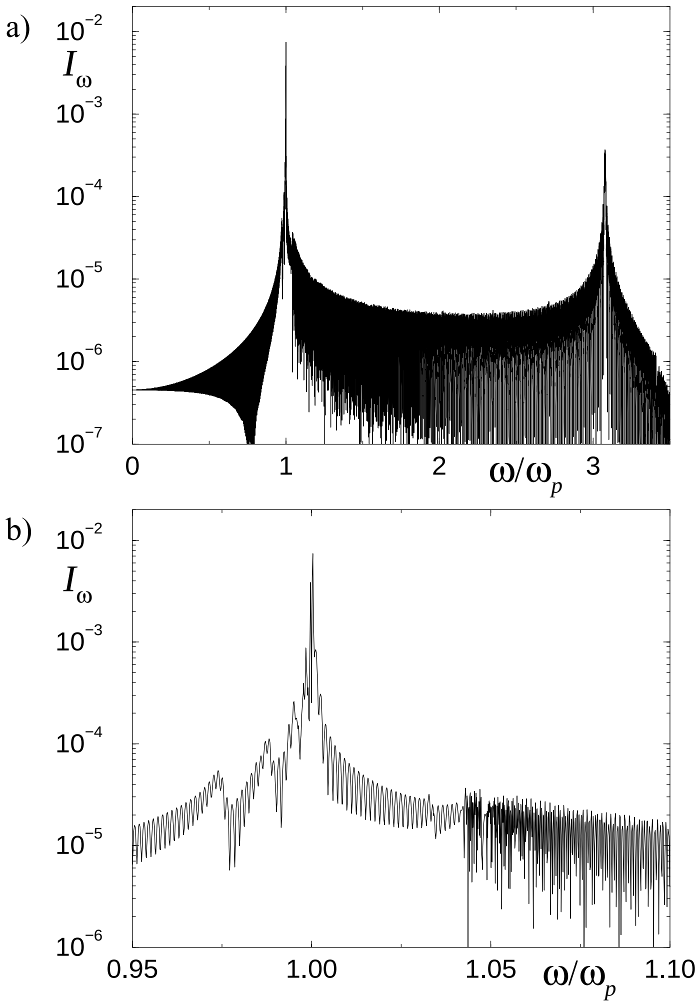

Figure 3: a) Spectrum of transverse electric field due to short laser pulse penetrating periodically magnetized plasma; b) enlarged detail; $n_e = 5.6 \cdot 10^{14}\,\mathrm{cm}^{-3}$, $B_0 = 1\,\mathrm{T}$, $\lambda_u = 1\,\mathrm{cm}$: $\omega_p = 2.26\,\mathrm{THz}$, $\Omega_0 = 0.176\,\mathrm{THz}$, $ck_u = 0.188\,\mathrm{THz}$

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