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Drift Ordered Short Mean Free Path Fluid Equations

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Abstract

The complete short mean free path description of magnetized plasma in the drift ordering has recently been derived and used to obtain a reduced fluid description appropriate to the edge of a tokamak. The complete description is also being used to study the radial transport of toroidal angular momentum that determines the radial electric field and flows in a tokamak. The simplest case is an up-down asymmetric tokamak since there is a large up-down asymmetric term in the gyro-viscosity. Up-down asymmetric are considered here to highlight the impact of the magnetic field topology on the ion flow in the Pfirsch-Schlüter regime.

1. Short Mean Free Path Closure

The short mean free path description of magnetized plasma as originally formulated by Braginskii [1] in 1957 assumes an ordering in which the ion mean flow is on the order of the ion thermal speed. Mikhailovskii and Tsypin [2] realized that this ordering is not the one of most interest in many practical situations in which the flow is weaker and on the order of the ion heat flux divided by the pressure. In their ordering the ion flow velocity is allowed to be on the order of the diamagnetic drift velocity - the case of interest for most fusion devices in general, and the edge of many tokamaks in particular. Their drift ordering retains heat flow modifications to the viscosity that are missed by the MHD ordering of Braginskii. Indeed, some short mean free path treatments of turbulence in magnetized plasmas use some version of the Mikhailovskii and Tsypin results to retain temperature gradient terms in the gyro-viscosity. However, the truncated polynomial expansion solution technique of Mikhailovskii and Tsypin made two assumptions that we remove to obtain completely general results [3]. First, they neglected contributions to the viscosity that arose from the non-linear part of the collision operator. We find that removing this assumption gives rise to heat flux squared terms in the pressure anisotropy and perpendicular collisional viscosity that are the same size as terms found by Mikhailovskii and Tsypin. Second, their truncated polynomial expansion of the ion distribution function is an inadequate approximation to the gyro-phase dependent portion of the ion distribution function. We find that their approximate form is not accurate enough to completely and properly evaluate all of the terms in the perpendicular collisional viscosity. The modifications to the pressure anisotropy and perpendicular collisional viscosity that we evaluate are valid for turbulent and collisional transport, and also allow stronger poloidal density, temperature, and electrostatic potential variation in a tokamak than the standard Pfirsch-Schlüter ordering. We have also evaluated the electron pressure anisotropy and gyro-viscosity.

2. Reduced Collisional Description for Tokamak Edge Plasma

Starting with the corrected Mikhailovskii-Tsypin fluid equations, we derived a system of non-linear reduced moment equations suitable for numerical modeling that describe field-aligned fluctuations in low-beta collisional magnetized edge plasma [4]. These equations advance the plasma density, electron and ion energies (or, equivalently, temperatures), parallel ion flow velocity, parallel current, vorticity (or, equivalently, electrostatic potential), perturbed parallel electromagnetic potential, and perturbed magnetic field. The equations locally conserve particle number and total energy, and insure that perturbed magnetic field and total plasma current are divergence-free. In addition, while intended primarily for modeling plasma edge turbulence, they contain the neoclassical results for plasma current, parallel ion flow velocity, and parallel gradients of equilibrium electron and ion temperatures.

3. Angular Momentum Transport in the Pfirsch-Schlüter Regime

In an up-down symmetric tokamak the radial transport of toroidal angular momentum is quite complex so here we limit ourselves to the special case on an up-down asymmetric tokamak. In this case, the lowest order result from the gyro-viscosity does not vanish and inside the separatrix leads to the contribution [2, 3, 5]

$$\langle R^2\nabla\zeta\cdot\vec{\pi}_g\cdot\nabla\psi\rangle \approx -\frac{2c^2MIp}{5e^2\langle B^2\rangle}\frac{\partial T}{\partial\psi}\langle R^2\vec{B}\cdot\nabla\ell nB\rangle \ , \qquad (1)$$

where p and T are the ion pressure and temperature, and ψ is the flux function associated with the magnetic field $\vec{B} = I\nabla\zeta + \nabla\zeta\times\nabla\psi$, with ζ toroidal angle and R the cylindrical radial distance from the axis. We assume the toroidal magnetic field is in the direction of the plasma current I_p with ψ increasing outward from the magnetic axis. We remark that Hinton [5] only considers the ion flow contribution to Eq. (1) since he works in the high flow ordering of Braginskii. As a result, have used the more general gyro-viscosity expression of references [2] and [3] to obtain the full expression for (1). For an up-down symmetric tokamak Eq. (1) vanishes and the gyro-viscosity must be evaluated to higher order - a calculation beyond the scope of the present discussion.

In the steady state, with no momentum sources in the plasma, the vanishing of the sum of Eq. (1) plus the perpendicular viscous contribution inside the separatrix,

$$\langle R^2\nabla\zeta\cdot\vec{\pi}_\perp\cdot\nabla\psi\rangle \approx -\frac{3c^2M^2\nu p}{10e^2}\left\langle\frac{R^2B_p^2}{B^2}\left(R^2+\frac{3I^2}{B^2}\right)\right\rangle\frac{\partial\omega}{\partial\psi} \ , \qquad (2)$$

determines the radial electric field in an up-down asymmetric tokamak, where we define $\omega = -c[\partial\Phi/\partial\psi + (en)^{-1}\partial p/\partial\psi]$ with Φ the electrostatic potential and n the plasma density and ν the ion collision frequency. Notice that in the absence of ion temperature variation the only solution is the Maxwell-Boltzmann like ion response Φ equals a constant. In the presence of radial temperature variation the resulting heat flow can transport momentum radially according to the simple physical picture given in Ref. [6].

Using $\langle R^2\nabla\zeta\cdot(\vec{\pi}_g+\vec{\pi}_\perp)\cdot\nabla\psi\rangle = 0$ gives the steady state, momentum source free result

$$\frac{d\omega}{d\psi} \approx -\frac{4IdT_i/d\psi}{3M\nu_i\langle B^2\rangle}\frac{\langle R^2\vec{B}\cdot\nabla\ell nB\rangle}{\left\langle R^2B_p^2B^{-4}\left(R^2B^2+3I^2\right)\right\rangle}, \tag{3}$$

where the sign of $\langle R^2\vec{B}\cdot\nabla\ell nB\rangle$ determines the sign of $d\omega/d\psi$.

From expression (3) we can make predictions about how the ion flow components change direction with changes in the up-down asymmetric magnetic topology. To do so we note that in the Pfirsch-Schlüter regime the lowest order ion flow velocity inside the separatrix must be of the form [7]

$$\vec{V} = \omega(\psi)R^2\nabla\zeta + u(\psi)\vec{B}, \tag{4}$$

where $u \approx -(1.8cI/e\langle B^2\rangle)\partial T/\partial\psi$. Using Eqs. (3) and (4) and the expression for u we obtain the following predictions.

(i) If the toroidal current is reversed ($I_p \rightarrow -I_p$) then the poloidal magnetic field must reverse ($\vec{B}_p \rightarrow -\vec{B}_p$) so $\psi \rightarrow -\psi$, and $\vec{B}\cdot\nabla \rightarrow -\vec{B}\cdot\nabla$ giving $\omega \rightarrow -\omega$ and $u \rightarrow -u$. As a result,

$$\vec{V} = (\omega R^2 + uI)\nabla\zeta + u\nabla\zeta\times\nabla\psi \rightarrow -(\omega R^2 + uI)\nabla\zeta + u\nabla\zeta\times\nabla\psi\ . \tag{5}$$

(ii) If the toroidal magnetic field is reversed ($B_t \rightarrow -B_t$ or $I \rightarrow -I$) then $\omega \rightarrow -\omega$ and $u \rightarrow -u$ and we obtain

$$\vec{V} = (\omega R^2 + uI)\nabla\zeta + u\nabla\zeta\times\nabla\psi \rightarrow (-\omega R^2 + uI)\nabla\zeta - u\nabla\zeta\times\nabla\psi\ . \tag{6}$$

(iii) If both the toroidal current and toroidal magnetic field are reversed then $\omega \rightarrow \omega$ and $u \rightarrow u$ so that

$$\vec{V} = \omega R^2\nabla\zeta + u\vec{B} \rightarrow \omega R^2\nabla\zeta - u\vec{B}\ . \tag{7}$$

(iv) Finally, we simply let the poloidal angle change sign so that $\vec{B}\cdot\nabla \rightarrow -\vec{B}\cdot\nabla$ then $\omega \rightarrow -\omega$ and $u \rightarrow u$ and we find

$$\vec{V} = \omega R^2\nabla\zeta + u\vec{B} \rightarrow -\omega R^2\nabla\zeta + u\vec{B}\ . \tag{8}$$

This case corresponds to changing from lower single null to upper single null while retaining the precise shape of the mirror reflected flux surfaces (note: the direction of the currents and magnetic fields do not change). Cases (iii) and (iv) are related. They can be obtained from one another by turning the tokamak over (since this is an awkward exercise in the laboratory we do not give expressions for this case). Since turning over the tokamak changes the toroidal direction and the direction of the magnetic field (when viewed from above), the direction of the ion flows is the same for (iii) and (iv) when one of them is "turned over".

The sensitivity of the directions of various flow components to magnetic topology is observed in Alcator C-Mod and other tokamaks [8]. Indeed, some of the features of the Alcator C-Mod results for cases (iii) and (iv) appear to be in qualitative agreement with the sign changes of Eqs. (7) and (8). However, it must be kept in mind that the preceding expressions are only valid inside the separatrix and for a collisional plasma. When these restrictions are removed the much of the same logic holds but certain flux functions cannot be made explicit and some flux functions may need to be replaced by functions that vary along the magnetic field. Work is underway to determine whether the preceding predictions based on the up-down asymmetric behavior of the lowest order terms in the gyro-viscosity are in quantitative agreement with the lower and upper single null and reverse current and magnetic field experimental observations of flows in C-Mod inside the separatrix; and whether these analytic results or suitable generalizations provide any insight into flows observed in the C-Mod scrape-off-layer.

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