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Parametric "Nonresonant" Instability of Cyclotron Radiation Enhanced by Self-consistent Drawing of Wave Frequency to the Cyclotron Resonance

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1. Abstract. In the present paper new traits of parametric "nonresonant" instability of cyclotron radiation are revealed that raise interest to this mechanism of interaction between cyclotron radiation and plasma in many respects. This regime investigated in papers [1,2] represents the amplification of bichromatic high-frequency radiation in the presence of low-frequency modulation of electrons that is possible even if there is no energy exchange between waves and particles without modulation. Such a medium is reactive with respect to the monochromatic radiation - due to the absence of resonant particles. Now it is shown that the regime of "nonresonant" amplification can be realized without premodulation of the electron ensemble. The efficiency of the amplification process cardinally depends on the electrons density. The parametric instability can be essentially enhanced by the self-consistent drawing of wave frequency to the resonance.

2. The theoretical model. We consider here the most optimal scheme where two waves propagate along the magnetic field in opposite directions. $\mathbf{E}(z,t) = \mathrm{Re}\,\mathbf{e}_+ \sum_{j=1}^{2} E_j \exp\left(ik_j z - i\omega_j t\right)$.

Here $\omega_1 \approx \omega_2$, $k_1 \approx -k_2$, $\mathbf{e}_+ = \mathbf{x}_0 + i\mathbf{y}_0$. This field interacts with beam of electrons that are distant from the resonance with these two waves, so that cyclotron synchronism detunings $\Delta_{1,2} = \omega_{1,2} - eB/(mc\gamma) - k_{\|1,2} V_{\|} = \Delta_0 - k_{\|1,2} V_{\|}$ are large compared with the reversed time of interaction. At the same time the fulfilment of parametric synchronism condition is required: $\omega_1 - \omega_2 \approx \left(k_1 - k_2\right) V_{\|}$. The interaction of one of two waves with this electron ensemble results in no energy exchange between wave and electrons. The main idea proposed in [1,2] is the following. Since the partial synchronism detunings Δ_1 and Δ_2 include the Doppler shifts of different sign, the modulation of electrons longitudinal velocity provides the oscillation of Δ_1 and Δ_2 and correspondingly oscillations of electrons reactive responses to these two waves in opposite phases. Then the equations of waves coupling admit the simultaneous amplification of two waves. The required type of modulation of electrons corresponds to the following form of the distribution function $f\left(p_{\|}, p_{\perp}\right) = f_0 + f_M \cos(\varphi_M + (k_1 - k_2)z - \left(\omega_1 - \omega_2\right)t)$, where

$f_M(p_{\|}, p_{\perp}) = -f_M(-p_{\|}, p_{\perp})$. Being settled initially (at t=0) such type of modulation is maintained during sufficiently long time period if the parametric synchronism condition is fulfilled.

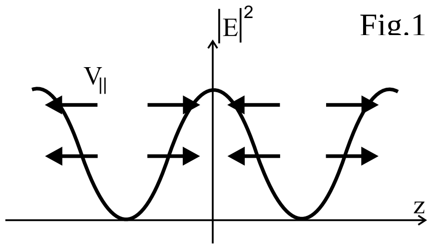

Fig.1

The mechanism of energy exchange between two waves and nonresonant electrons was explained in [2] in the frame, where $\omega_1=\omega_2$. It corresponds to the definite correlation of longitudinal motion of particles with spatial dependence of the field amplitude (see fig.1), so that according to the expression (1) for change of electron energy the energy of electron ensemble decreases.

$$\frac{d}{dt}\langle w\rangle = -\frac{e^2}{m\gamma_0}\frac{1}{2\omega\Delta_0}\left(1-\frac{V_\perp^2}{c^2}\frac{\omega}{\Delta_0}+\frac{\omega}{\Delta_0}\left(2\frac{V_\perp^2}{c^2}\frac{\omega}{\Delta_0}-1\right)\right)\left(V_{\|}\frac{\partial}{\partial Z}|E|^2(Z(t))\right) \quad (1)$$

As it was proposed in the paper [1] the required type of modulation is produced in the laboratory frame as result of preliminary short-time action of the longitudinal wave with frequency $\Omega=\omega_1-\omega_2$ and wave number $\kappa=k_1-k_2$. But it is especially worth to note that the same type of modulation is created self-consistently under the action of two high-frequency interacted waves. For establishing this fact it is sufficient to trace the change of averaged longitudinal momentum under the action of two waves in quadratic approximation to the wave amplitude (see for details [2]): $\frac{d}{dt}P_{\|} = \frac{e^2}{m\gamma_0}\frac{1}{2\Delta_0^2}\frac{V_\perp^2}{c^2}\left(\frac{\partial}{\partial Z}|E|^2(Z)\right)$. If the cyclotron detuning Δ_0 is positive the created periodic dependence of longitudinal momentum corresponds to the conditions of energy transfer from electrons to the waves.

The damage of initial modulation leads to the saturation of instability. The reasons for such damage can be the following. First of all it is the ballistic or dynamic transformation of initial modulation of longitudinal velocity to the modulation of space density. In this case the time of saturation can be estimated as $t_{sat}\sim\Delta_M^{-1}$, where $\Delta_M\sim(k_1-k_2)\Delta V_{\|}$ ($\Delta V_{\|}$ -is the spread of longitudinal velocity in the electron ensemble), if the modulation of electrons is settled initially and nonlinear distortion of initial modulation is negligible; otherwise $\Delta_M\sim eEV_\perp\omega/(mc\gamma\Delta_0)$. Besides, the damage of initial modulation can be caused by the destroying influence of excited longitudinal plasma field. In this case the saturation time is defined by plasma frequency ω_p of electron ensemble: $t_{sat}\sim\omega_p^{-1}$.

In papers [1,2] the kinetic theory of this effect was developed in linear, quasi-vacuum approximations under condition $t<<t_{sat}$. The resulting equations of parametric coupling of

two waves has the form:

$$\dot{E}_1 = -\exp(i\varphi_M)(\Gamma t + iD)E_2,\ \dot{E}_2 = +\exp(-i\varphi_M)(\Gamma t + iD)E_1, \qquad (2)$$

where the coupling coefficients are defined by the formulas:

$$\Gamma = -2\pi e^2 V_\perp^2 \omega \big/ (m^2\gamma^2 c^2)\int dp_\perp dp_\| f_M\, 2kp_\| \big/ \Delta_0^2 \ , \qquad D = 2\pi e^2 V_\perp^2 \omega \big/ (m^2\gamma^2 c^2)\int dp_\perp dp_\| f_M\, 2\ kp_\| \big/ \Delta_0^3\ . \qquad (3)$$

This system admits the unstable exponential solution for two waves with the exponent: $\mu = \mu' + i\mu'' = |D| - i\Gamma t(D/|D|)$. It is important to note that due to the parametric interaction the frequency of amplified mode is drawing to the resonance with particles, although this frequency shift just as the increment must be small compared with the initial detuning from the resonance for the approximations of the analytical calculations to be fulfilled.

3. ***The results of numerical calculations.*** Here we present the results of numerical analysis of the “nonresonant” regime of parametric instability, fulfilled on the basis of strict equations of particles motion in the field of two electromagnetic waves with time-dependent amplitude, prescribed circular polarization and spatial structure, and the field of excited longitudinal wave. The waves amplitudes obey the strict equations. Both the examination of analytical results and analysis of the parametric regime out of the frame of the analytical approximations were done. Different scenarios of amplification process were revealed.

The first type of instability process development is realized if the electron density is sufficiently low, so that the following condition is fulfilled:

$$\omega_p, \mu' << \Delta_M \qquad (4)$$

The amplification scenario is well described analytically by equations (2) under this condition if the modulation of electron ensemble is formed preliminary and the initial wave amplitudes are relatively small (the nonlinear effects are negligible). On the Fig.2 the corresponding typical result of numerical calculation for the evolution of electrons energy is presented. The saturation of instability is defined by $\Delta_M \sim (k_1 - k_2)\Delta V_\|$ in this case. If the modulation of electron ensemble is formed under the nonlinear action of the two interacted waves but condition (4) is still fulfilled then the behaviour of the system resembles the previous one with the substitution of parameter Δ_M by the nonlinear one $\Delta_M \sim eEV_\perp\omega/(mc\gamma\Delta_0)$. The corresponding numerical result is presented on the Fig.3. In both cases the efficiency of the amplification process (the ratio of the extracted electrons energy to the initial kinetic energy of electron ensemble) is very low. It can be estimated using the solution of equation (2) in the following way: $\dfrac{\Delta\langle W\rangle}{W_0} = -\left(\dfrac{e|E|}{mc\omega}\right)^2 \dfrac{V_\perp^2/c^2}{2\gamma_R(\gamma_R - 1)}\left(\dfrac{\omega}{\Delta_0}\right)^3 \lesssim \dfrac{\Delta_M^2}{\omega\Delta_0}$. The gain coefficient (the

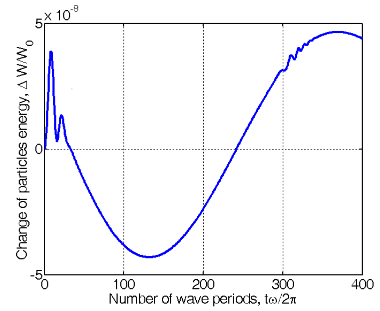

Fig.2

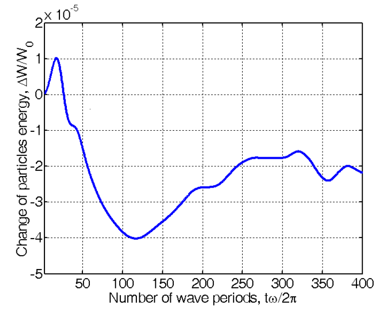

Fig.3

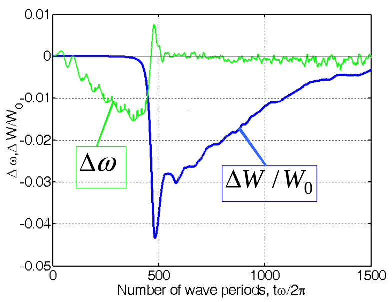

Fig.4

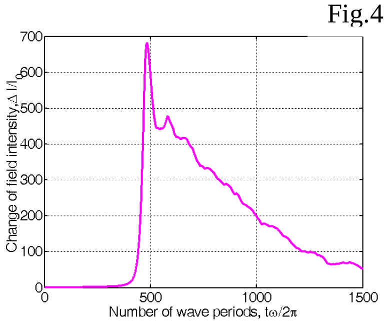

Fig.5

ratio of the change of the waves intensity to the initial value) in this case is also small: $\frac{\Delta I}{I_0} = \frac{2\mu'}{\Delta_M} << 1$.

The much more effective instability process is realized if the other condition is fulfilled:

$$\mu' >> \Delta_M, \omega_p \qquad (5)$$

The typical scenario of the amplification process in this condition is presented on Figs.4,5. The extraction of kinetic energy and gain coefficient reach about 5% and 10^3 correspondingly. The physical mechanism of such enhanced parametric instability is the following. The relatively weak amplification of the bi-component mode caused by the parametric interaction with “nonresonant” modulated electron ensemble is accompanied by the drawing of the waves frequency to the resonance with particles. If the density of electrons is not so small (the condition (5) is fulfilled) this “drawing” process passes faster than the saturation processes and leads to the essential strengthening of the parametric coupling that results in the considerable increase of the energy exchange rate. This process is saturated at the nonlinear stage of interaction. The efficiency and gain coefficients can be estimated by the following formulas: $\frac{\Delta\langle W\rangle}{W_0} \sim \frac{1}{\gamma}\left(\frac{2p_\perp\omega_p}{mc\omega}\right)^{2/3}$, $\frac{\Delta I}{I_0} = \left(\frac{\omega_p^4 p_\perp}{\omega^4 mc}\right)^{2/3}$. The solution presented in the Figs.4,5 corresponds to the following parameters. The intensity of 1mm radiation interacting with the ensemble of relativistic electrons (relativistic gamma factor equal to 1.7) with density $7{*}10^9$cm^{-3} reaches in maximum 500 kW/cm^2. The averaged power per unit volume of energy extraction process comes to 50 kW/cm^3.

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[1] M.A.Erukhimova, M.D.Tokman, JETP, (1997), 85,640

[2] Erukhimova M.A., Tokman M.D., Radiophysics and Quantum Electronics, 2003,**45**,249