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Fast and Slow Nonlinear Tearing Mode Reconnection

N. F. Loureiro,[a] S. C. Cowley,[a,b] W. D. Dorland,[c]
M. G. Haines[a] and A. A. Schekochihin[d]

[a]*Department of Physics, Imperial College, London SW7 2BW, UK*
[b]*Department of Physics and Astronomy, UCLA, Los Angeles, CA 90024, USA*
[c]*IREAP, University of Maryland, College Park, MD 20742-3511, USA*
[d]*DAMTP, University of Cambridge, Cambridge CB3 0WA, UK*

1. Introduction. The standard theory of the tearing-mode evolution identifies three stages. The first is the linear stage described by the Furth–Killeen–Rosenbluth (FKR) theory [1]. During this stage, the island width W grows exponentially in time until it reaches the width of the resistive dissipation layer, $\ell_\eta \propto \eta^{2/5}\Delta'^{1/5}$, where η is the resistivity and Δ' is the instability parameter [see Eq. (3)]. Once $W \sim \ell_\eta$, nonlinear terms are sufficiently large to replace inertia as the force opposing the inflow pattern. A slow down of the growth ensues, from exponential to linear in time: $dW/dt \sim \eta\Delta'$. This is the second stage of the tearing-mode evolution, known as the Rutherford regime [2]. Finally, the third, saturated, stage is reached when the island width becomes comparable to the equilibrium shear length [3].

In this paper, we find the tearing-mode evolution to be, in fact, a four-stage process: the FKR regime, the Rutherford regime, a regime of fast nonlinear island growth that we identify as Sweet–Parker (SP) reconnection, and saturation. We carry out a set of numerical simulations that demonstrate two main points. First, we show that, given sufficiently small η, the Rutherford regime always exists; larger values of Δ' require smaller values of η. Rutherford's negligible-inertia assumption is validated and the asymptotically linear dependence of dW/dt on η and Δ' is confirmed. Second, we find that, at large Δ', the Rutherford regime is followed by a nonlinear stage of fast growth linked to X-point collapse and formation of a current sheet. This causes the reconnection to become SP-like. The signature $\eta^{1/2}$ scaling of the effective island growth rate is, indeed, found in this nonlinear stage. The SP stage culminates in the saturation of the mode, which can, thus, be achieved much faster than via Rutherford regime.

2. The Model. We use the conventional Reduced MHD set of equations [8] in 2D for a plasma in the presence of a strong externally imposed magnetic field B_z:

$$\frac{\partial \omega}{\partial t} + \boldsymbol{v}_\perp \cdot \boldsymbol{\nabla}\omega = \boldsymbol{B}_\perp \cdot \boldsymbol{\nabla} j_\parallel, \tag{1}$$

$$\frac{\partial \psi}{\partial t} + \boldsymbol{v}_\perp \cdot \boldsymbol{\nabla}\psi = \eta \boldsymbol{\nabla}^2\psi, \tag{2}$$

where the total magnetic field is $\boldsymbol{B} = B_z\boldsymbol{e}_z + \boldsymbol{B}_\perp$, all gradients are in the (x, y) plane, the in-plane magnetic feld is $\boldsymbol{B}_\perp = \boldsymbol{e}_z \times \boldsymbol{\nabla}\psi$, the in-plane velocity is $\boldsymbol{v}_\perp = \boldsymbol{e}_z \times \boldsymbol{\nabla}\phi$, and the parallel components of the vorticity and current are $\omega = \boldsymbol{e}_z \cdot (\boldsymbol{\nabla} \times \boldsymbol{v}) = \boldsymbol{\nabla}^2\phi$ and $j_\parallel = \boldsymbol{e}_z \cdot (\boldsymbol{\nabla} \times \boldsymbol{B}) = \boldsymbol{\nabla}^2\psi$. Eqs. (1–2) are solved in a box with dimensions $L_x \times L_y$. All lengths can scaled so that the width of the box is $L_x = 2\pi$.

We impose an initial equilibrium defined by $\psi^{(0)} = \psi_0/\cosh^2(x)$ and $\phi^{(0)} = 0$. We choose $\psi_0 = 1.3$ so that the maximum value of the unperturbed in-plane magnetic field $B_y^{(0)} = d\psi^{(0)}/dx$ is 1. Time is scaled by the in-plane Alfvén time. The equilibrium is perturbed with $\psi^{(1)} = \psi_1(x)\cos(ky)$, where $k = mL_x/L_y$. In our simulations, the initial

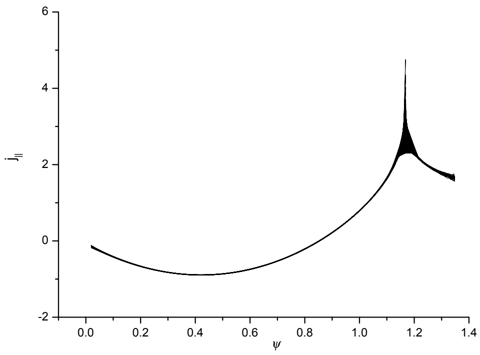

FIG. 1: Scatter plot of $j_{\|}$ vs. ψ. Run parameters are $\Delta' = 8.15, \eta = 2.8 \times 10^{-4}$. Data extracted at $W = 0.9$ [cf. Fig. 2]. At the separatrix, $\psi \simeq 1.17$.

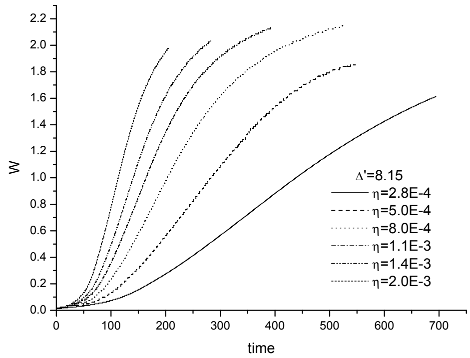

FIG. 2: Island width vs. time for $\Delta' = 8.15$ and several values of η.

perturbation has $m = 1$. The island width is then approximately $W = 4\sqrt{\psi_1(0)/\psi^{(0)\prime\prime}(0)}$. However, in what follows, W is measured directly from the numerical data.

With the equilibrium configuration we have chosen, the instability parameter is [9]:

$$\Delta' = \frac{\psi_1'(+0) - \psi_1'(-0)}{\psi_1(0)} = 2\left[\frac{6k_1^2 - 9}{k_1(k_1^2 - 4)} - k_1\right], \quad (3)$$

where $k_1^2 = k^2 + 4$. The equilibrium is tearing unstable if $k < \sqrt{5}$. We vary the value of Δ' by adjusting the length of the box L_y.

3. The Rutherford Regime. Rutherford's analysis depends on the assumption of negligible inertia, which reduces the vorticity equation to $\boldsymbol{B}_\perp \cdot \boldsymbol{\nabla} j_{\|}{=}0$. This implies that $j_{\|} = j_{\|}(\psi)$ everywhere except at the separatrix (the in-plane magnetic field vanishes at the X-point). Fig. 1 proves the validity of this assumption. At the separatrix, $\psi \simeq 1.17$. Larger values of ψ correspond to the interior the island. The variation of ψ in that region is relatively small, supporting the "constant-ψ approximation" used by Rutherford.

Fig. 2 shows the time evolution of the island width W at constant $\Delta' = 8.15$ and varying resistivity. After the exponential growth stage, a distinct period of linear in time growth is manifest in all curves. Figs. 3 and 4 show the dependence of dW/dt during the linear in time period on Δ' and η, respectively, demonstrating in both cases the linear relation predicted by Rutherford. At fixed finite η, the Rutherford scaling breaks down at large Δ'. However, for a given Δ', it is recovered asymptotically at sufficiently small η.

Although Rutherford-like island growth has been observed in earlier numerical work, no parameter scan showing the linear scaling of dW/dt with η and Δ' has previously been performed. White *et al.* [4] verified the linear in time growth of the island in their numerical simulations of the $m = 2$ mode performed in tokamak geometry, a result later confirmed by Park *et al.* [5]. Biskamp [6] demonstrated the Rutherford behaviour in a numerical experiment done in slab geometry and with a current-dependent resistivity. Biskamp's simulations had a relatively small value $\Delta' = 3$. Recently, Jemella *et al.* [7] carried out a Δ' parameter scan with $\Delta' \in [0.92, 20.93]$ (and constant η). Their results cast doubt upon the validity of Rutherford's analysis by failing to produce the linear in time behaviour: except for the smallest values of Δ', the island growth was exponential at all times. They argued that, instead of Rutherford's X-point configuration, a Sweet–

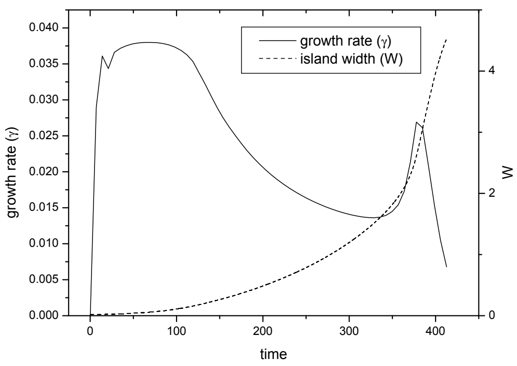

FIG. 6: Effective growth rate γ and island width W vs. time, for a run with $\Delta' = 12.2$ and $\eta = 2.8 \times 10^{-4}$.

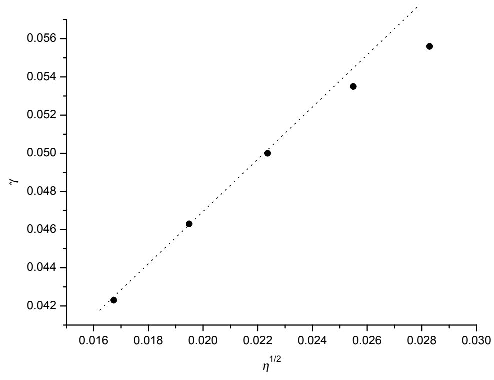

FIG. 7: Dependence of the peak effective growth rate $\gamma_{\max}$ in the fast nonlinear regime on $\eta^{1/2}$ at constant $\Delta' = 17.3$.

X-point configuration assumed in Rutherford's analysis is, in fact, unstable [10]. As time goes on, the effective island growth rate in the Rutherford regime decreases ($\gamma \sim 1/t$) and eventually becomes so low that the X-point configuration can no longer be sustained over the time scale $\sim \gamma^{-1}$. X-point collapse leads to the formation of the current sheet.

Note that, for large Δ', if η is not sufficiently small for the Rutherford stage to occur, the SP stage nearly immediately follows the FKR stage, as is the case in Jemella *et al.* simulations. In the opposite case of small Δ' and small η, the saturation can be reached directly from the Rutherford stage, with the SP stage never having time to materialise.

We leave a more detailed theoretical description of the nonlinear speed-up effect to a forthcoming paper.

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