*31st EPS Conference on Plasma Phys. London, 28 June - 2 July 2004 ECA Vol.**28G**, P-4.205 (2004)*

# Sheath in front of a negatively biased collector that emits electrons and is immersed in a two electron temperature plasma

Tomaž Gyergyek[1,2], Milan Čerček[2]

*1) University of Ljubljana, Faculty of electrical engineering, Tržaška 25, 1000 Ljubljana, Slovenia*
*2) Jožef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia*

**Abstract**

We present a one dimensional fluid model of the sheath formation in front of a negative electrode that emits secondary electrons and is immersed in a plasma that contains energetic electron population.

## 1 Introduction

Plasmas with electron velocity distributions that contain energetic tails are readily produced in plasma devices for material processing. Also in fusion machines such electron populations appear due to strong rf fields during ion cyclotron and lower hybrid wave heating and rf current drive. The presence of energetic electrons has a remarkable effect on potential formation in the plasma and consequently on particle losses to the wall. In theoretical models energetic electrons are often treated as monoenergetic or thermal beams [1,2], or as additional group of maxwellian distributed electrons with lower density and higher temperature than the basic electron population [3,4].

Sheath formation in front of electron emitting electrodes is also an important problem that has been treated by many authors [5,6]. Usually it is assumed that electron emitting electrode is immersed in a plasma with one group of maxwellian electrons and singly charged cold ions. Flux of emitted electrons is usually considered to be proportional to the flux of incoming electrons with a constant proportionality factor called emission coefficient. Secondary electron emission caused by ions is taken into account by very few authors [7].

Recently Ye et al. [8,9] have presented a one dimensional fluid model of a sheath that forms in front of a negative electrode that emits electrons. In this work we extend their model in three directions. First, we assume that additional high temperature maxwellian electron population is present in the plasma. Second, we allow that emitted electrons have a non-zero initial velocity $v_C$ at the collector surface and third we introduce electron *and ion* emission coefficients.

## 2 Model

We consider an infinite plane material surface (collector) in contact with plasma that contains two electron populations and singly charged positive ions that are assumed to be monoenergetic and at rest at a very large distance from the collector. At a large from the collector the plasma potential is assumed to be zero $\Phi(x \to \infty) = 0$ and also the electric field there is zero. As we approach the collector the plasma potential gradually decreases, so that a finite electric field exists in the plasma which accelerates positive ions towards the collector. At a certain distance $x = d$ from the collector, the value of the potential is $\Phi_0$ and there the ions have velocity $v_0$ towards the collector. Plane at $x = d$ is called the sheath edge. There the plasma is still quasineutral, but immediately beyond this point a sheath with large electric field is formed. The potential profile is determined by the Poisson equation:

$$\frac{d^2\Phi}{dx^2} = -\frac{e_0}{\varepsilon_0}\left(n_i(x) - n_{e1}(x) - n_{e2}(x) - n_{e3}(x)\right), \qquad (1)$$

where $n_(x)$ is the ion density, $n_{e1}(x)$ is the density of basic plasma electron population with lower temperature (cool electrons), $n_{e2}(x)$ is the density of the hot electron plasma population and $n_{e3}(x)$

is the density of emitted electrons (called also secondary electrons). Cool and hot electrons are assumed to obey the Boltzmann relation:

$$n_{e1}(x) = n_{e1S} \exp\left(\frac{e_0(\Phi(x) - \Phi_0)}{kT_{e1}}\right), \quad n_{e2}(x) = n_{e2S} \exp\left(\frac{e_0(\Phi(x) - \Phi_0)}{kT_{e2}}\right). \tag{2}$$

Density of ions and secondary electrons is obtained from the assumption that energy and flux of both particle species are conserved:

$$n_i(x) = \frac{n_S}{\sqrt{1 - \frac{2e_0(\Phi(x)-\Phi_0)}{m_i v_0^2}}}, \quad n_{e3}(x) = n_{e3S}\sqrt{\frac{1 + \frac{2e_0(\Phi_0-\Phi_C)}{m_e v_C^2}}{1 + \frac{2e_0(\Phi(x)-\Phi_C)}{m_e v_C^2}}}. \tag{3}$$

Here $n_S$, $n_{e1S}$, $n_{e2S}$, and $n_{e3S}$ are the respective particle densities at the sheath edge, $\Phi_C$ is the collector potential and $m_i$ and $m_e$ are ion and electron masses. We assume that the ratio between hot and cool electron density at the sheath edge is a given parameter:

$$\beta = \frac{n_{e2S}}{n_{e1S}}. \tag{4}$$

We also assume that the flux of secondary electrons $j_{e3}$ from the collector is proportional to the incoming flux of ions and cool and hot electrons in the form:

$$j_{e3} = \gamma_e(j_{e1} + j_{e2}) + \gamma_i j_i, \tag{5}$$

where $\gamma_e$ and $\gamma_i$ are electron and ion emission coefficients and fluxes are given by:

$$j_i = e_0 n_S v_0 \tag{6}$$

$$j_{e1} = e_0 n_{e1S}\sqrt{\frac{kT_{e1}}{2\pi m_e}} \exp\left(\frac{e_0(\Phi_C - \Phi_0)}{kT_{e1}}\right), \tag{7}$$

$$j_{e2} = e_0 n_{e2S}\sqrt{\frac{kT_{e2}}{2\pi m_e}} \exp\left(\frac{e_0(\Phi_C - \Phi_0)}{kT_{e2}}\right), \tag{8}$$

$$j_{e3} = e_0 n_{e3S} v_C \sqrt{1 + \frac{2e_0(\Phi_0 - \Phi_C)}{m_e v_C^2}}. \tag{9}$$

From equations (4) - (9) $n_{e1S}$, $n_{e2S}$, and $n_{e3S}$ are expressed in terms of $n_S$ and inserted into (2) and (3). Then (2) and (3) are inserted into (1). We get:

$$\frac{d^2\Psi}{dz^2} = \frac{1}{1+\beta+Ge}\left((1-G_i)\exp(\Psi) + \beta(1-G_i)\exp\left(\frac{\Psi}{\Theta}\right) + \frac{G_e + G_i(1+\beta)}{\sqrt{1 - \frac{\Psi}{\Psi_S - \frac{N^2\mu}{2}}}}\right) - \\ - \frac{1}{\sqrt{1 - \frac{2\Psi}{M^2}}}. \tag{10}$$

The following variables were introduced:

$$z = \frac{x}{\lambda_D}, \quad \lambda_D = \sqrt{\frac{\varepsilon_0 k T_{e1}}{n_S e_0^2}}, \quad \Psi = \frac{e_0(\Phi(x) - \Phi_0)}{kT_{e1}}, \quad \Psi_S = \frac{e_0(\Phi_C - \Phi_0)}{kT_{e1}}, \tag{11}$$

$$\Theta = \frac{T_{e2}}{T_{e1}}, \quad \mu = \frac{m_e}{m_i}, \quad v_0 = M\sqrt{\frac{kT_{e1}}{m_i}}, \quad v_C = N\sqrt{\frac{kT_{e1}}{m_i}}, \tag{12}$$

$$G_e = \frac{\gamma_e \left( \exp(\Psi_S) + \beta\sqrt{\Theta}\exp\left(\frac{\Psi_S}{\Theta}\right)\right)}{\sqrt{2\pi(N^2\mu - 2\Psi_S)}}, \quad G_i = \frac{\gamma_i M}{\sqrt{(N^2\mu - 2\Psi_S)}}. \tag{13}$$

The Poisson equation (10) is multiplied by $d\Psi/dz$ and integrated once over $\Psi$ from 0 to $\Psi$. We get:

$$\frac{1}{2}\left(\frac{d\Psi}{dz}\right)^2 = \frac{1}{1+\beta+G_e} \cdot \left[ \begin{array}{c} (1-G_i)\left(\exp(\Psi)-1\right) + \beta\Theta\left(1-G_i\right)\left(\exp\left(\frac{\Psi}{\Theta}\right)-1\right) + \\ +2\left(G_e + G_i\left(1+\beta\right)\right)\left(\Psi_S - \frac{N^2\mu}{2}\right)\left(1-\sqrt{1-\frac{\Psi}{\Psi_S - \frac{N^2\mu}{2}}}\right) \end{array} \right] - $$
$$-M^2\left(1-\sqrt{1-\frac{2\Psi}{M^2}}\right). \tag{14}$$

Equation (14) is expanded in Taylor series. The zero and the first order terms cancel each other out and the second order terms give:

$$\frac{1}{2}\left(\frac{d\Psi}{dz}\right)^2 = \Psi^2\left(\frac{1}{1+\beta+G_e}\left((1-G_i)\left(1+\frac{\beta}{\Theta}\right) + \frac{G_e + G_i(1+\beta)}{2\left(\Psi_S - \frac{N^2\mu}{2}\right)}\right) - \frac{1}{M^2}\right). \tag{15}$$

The left hand side of equation (15) is positive, because it a square of a quantity proportional to the electric field. So also the right hand side must be positive. In the limit, when both sides are zero, we get the following condition for the Mach number $M$:

$$M = \sqrt{\frac{1+\beta+G_e}{(1-G_i)\left(1+\frac{\beta}{\Theta}\right) + \frac{G_e+G_i(1+\beta)}{2\left(\Psi_S - \frac{N^2\mu}{2}\right)}}}. \tag{16}$$

Equation (16) is a modification of the Bohm criterion because of the presence of hot and secondary electrons in the plasma.

If emission of secondary electrons from the collector increases, eventually the density of secondary electrons and consequently negative space charge in front of the collector becomes so high, that the electric field at the collector becomes zero. This is called the critical emission and the corresponding emission coefficients $\gamma_{ec}$ and $\gamma_{ic}$ are the critical emission coefficients. Is it possible to calculate the values of $\gamma_{ec}$ or $\gamma_{ic}$, if the other parameters $\Psi_S$, $N$, $\mu$, $\Theta$ and $\beta$ are selected? The answer is yes. We put $\frac{d\Psi}{dz} = 0$ and $\Psi = \Psi_S$ into equation (14) and $M$ is given by (16). We get the following equation:

$$0 = \frac{1}{1+\beta+G_e} \cdot \left[ \begin{array}{c} (1-G_i)\left(\exp(\Psi_S)-1\right) + \beta\Theta\left(1-G_i\right)\left(\exp\left(\frac{\Psi_S}{\Theta}\right)-1\right) + \\ +2\left(G_e + G_i\left(1+\beta\right)\right)\left(\Psi_S - \frac{N^2\mu}{2}\right)\left(1-\sqrt{1-\frac{\Psi_S}{\Psi_S - \frac{N^2\mu}{2}}}\right) \end{array} \right] - \tag{17}$$
$$-\frac{1+\beta+G_e}{(1-G_i)\left(1+\frac{\beta}{\Theta}\right) + \frac{G_e+G_i(1+\beta)}{2\left(\Psi_S - \frac{N^2\mu}{2}\right)}}\left(1-\sqrt{1-\frac{2\Psi_S\left((1-G_i)\left(1+\frac{\beta}{\Theta}\right) + \frac{G_e+G_i(1+\beta)}{2\left(\Psi_S - \frac{N^2\mu}{2}\right)}\right)}{1+\beta+G_e}}\right).$$

Equation (17) is a single equation for 2 unknown auxiliary emission coefficients $G_e$ and $G_i$. So we either have to select one of them and solve (17) for the other, or we need one more equation. One possibility is to select the ratio between $G_i$ and $G_e$:

$$G_i = QG_e. \tag{18}$$

Then equations (17) and (18) form a set of 2 equations for 2 unknown auxiliary emission coefficients $G_e$ and $G_i$, with all the other parameters $\mu$, $\beta$, $\Theta$, $\mu$, $\Psi_S$, $N$ and $Q$ selected. Then critical emission coefficients $\gamma_{ec}$ and $\gamma_{ic}$ can be calculated from:

$$\gamma_{ec} = G_e \frac{\sqrt{2\pi(N^2\mu - 2\Psi_S)}}{\left(\exp(\Psi_S) + \beta\sqrt{\Theta}\exp\left(\frac{\Psi_S}{\Theta}\right)\right)}, \tag{19}$$

$$\gamma_{ic} = G_i \sqrt{\frac{(1-G_i)\left(1+\frac{\beta}{\Theta}\right)(N^2\mu - 2\Psi_S) - G_e - G_i(1+\beta)}{1+\beta+G_e}}. \tag{20}$$

Next we calculate the floating potential of the collector. When the collector is floating the total current $J_{tot}$ to the collector is zero:

$$J_{tot} = \frac{j_i + j_{e3} - j_{e1} - j_{e2}}{j_0} = \sqrt{\frac{1+\beta+G_e}{(1-G_i)\left(1+\frac{\beta}{\Theta}\right) + \frac{G_e + G_i(1+\beta)}{2\left(\Psi_S - \frac{N^2\mu}{2}\right)}}} + \tag{21}$$

$$+\frac{1}{1+\beta+G_e}\left[(G_e + G_i(1+\beta))\sqrt{N^2 - \frac{2\Psi_S}{\mu}} - \frac{1-G_i}{\sqrt{2\pi\mu}}\left(\exp(\Psi_S) + \beta\sqrt{\Theta}\exp\left(\frac{\Psi_S}{\Theta}\right)\right)\right] = 0,$$

where

$$j_0 = e_0 n_S \sqrt{\frac{kT_{e1}}{m_i}}$$

and equations (6) - (9) are used. If we select a set of parameters $\mu$, $\beta$, $\Theta$, $\mu$, $N$ and $Q$, equations (17), (18) and (21) form a set of 3 equations for 3 unknown quantities: the floating potential $\Psi_S$ and both auxiliary emission coefficients $G_e$ and $G_i$. Then the critical emission coefficients $\gamma_{ec}$ and $\gamma_{ic}$ are then calculated by (19) and (20).

**References**


1. Schott L., Phys. Fluids, **30**, (1987), 1795
2. Bradley J. W., J. Phys. D:Appl. Phys., **34**, (2001), 3241
3. Takamura S., Phys. Letters A, **133**, (1988), 312
4. Song S. B., Chang C. S., Choi D., Phys. Rev. E, **55**, (1997), 1213
5. Hobbs G. D., Wesson J. A., Plasma Physics, **9**, (1967), 85
6. Anderson N., Int. J. Electronics, **32**, (1972), 425
7. Hantzsche E., Contrib. Plasma Phys., **38**, (1998), 467
8. Ye M. Y., Shimada T., Kuwabara T., Ohno N. Takamura S., Proceedings of the 1998 ICCP&25th EPS Conference on Controlled Fusion and Plasma Physics, Praha, 29 June - 3 July 1998, ECA, **22C**, (1998), 23; Ye M. Y., Takamura S., Phys. Plasmas, **7**, (2000), 3457
9. Takamura S., Ohno N., Ye M. Y., Kuwabara T., Contrib. Plasma Phys., **44**, (2004), 126; Hagino Y., Ohno N., Takamura S., Contrib. Plasma Phys., **44**, (2004), 144