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Determination of Internal Fields from Frequency Sweeping Observation

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Introduction

Comparisons between numerical models and present day tokamak devices rely upon accurate diagnostic measurements of the key plasma parameters. In assessing the effect of energetic particle-driven instabilities upon their confinement, it is extremely important to measure the amplitude of internal modes. This is generally difficult to do accurately as it usually involves extrapolating field measurements at the edge to determine the mode amplitudes in the plasma core. In this work we demonstrate how it may be possible to calculate the absolute value of the amplitude of these modes from the measured spectrum of frequency sweeping that has been observed in the excitation of toroidal Alfvén eigenmodes (TAE) [see Ref. 1 and references therein]. In particular, numerical simulations of the bump-on-tail kinetic instabilities near marginal instability have shown how a hole and clump spontaneously appear in the particle distribution function and how this process supports a set of long-lived Bernstein, Greene, Kruskal (BGK) nonlinear waves that shift up and down in frequency. A similar nonlinear kinetic process of hole-clump production also occurs for TAE modes driven unstable by the radial gradient of the fast ion pressure. This mechanism is a primary candidate to explain the fast frequency sweeping observed in several experiments. Figures 1 and 2 show two such experimental examples in the JET and MAST tokamaks respectively.

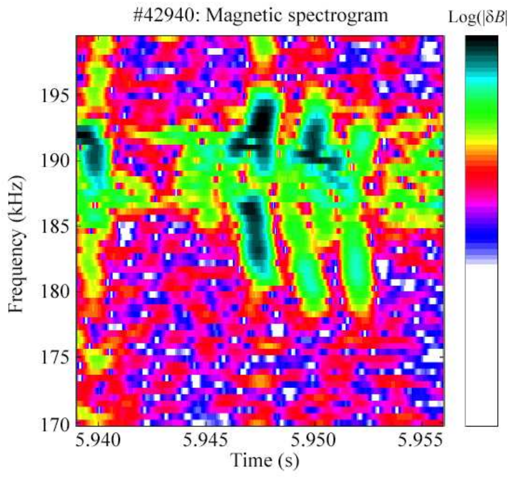

Figure 1: Example of frequency sweeping mode during a shear optimised D-T pulse in the JET tokamak. In this case, $\delta\omega/\omega_0 \sim 5\%$ in $\delta t \sim 1$ ms.

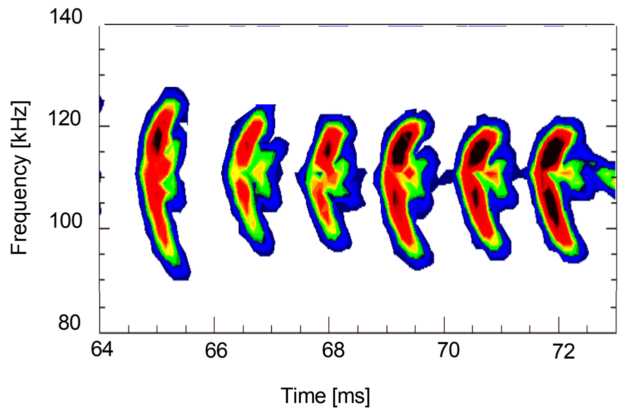

Figure 2: Magnetic spectrogram showing frequency sweeping $n = 1$ core-localised mode in MAST #5568. The first event is seen to sweep by 18 kHz in a time of 0.8 ms

Theoretical Background

Theoretical analysis and numerical studies have shown that plasma modes destabilized by kinetic drives can induce frequency sweeping. The sweeping results from the spontaneous excitation of phase-space structures when the system is near marginal instability, i.e. when $|\gamma_L - \gamma_d| << \gamma_L$. In this case a phase-space explosion is triggered, which, upon relaxation, leads to the formation of BGK nonlinear modes. The distribution function in the trapping regions form a hole and/or a clump, determined by the property that the trapped distribution has a lower or higher value respectively than the surrounding passing particle distribution function. The background plasma dissipation that is present does not damp the wave, but instead forces the wave frequency to change. This change allows the "rising" holes or "falling" clumps to extract energy from the ambient distribution so that there can be power balance with the dissipative power being extracted by the background plasma. The rate at which the frequency sweeps is determined by the wave-particle nonlinearity that relates the frequency shift $\delta\omega$ to the nonlinear bounce frequency ω_b of a particle trapped in the potential well of the wave. A simplified model of the process gives the scaling as, $\delta\omega = C_1 (\gamma_d/\gamma_L)^{1/2} \omega_b^{3/2} \delta t^{1/2}$, $\omega_b = C_2\gamma_L$ with C_1 and C_2 being constants of proportionality ($C_1 \approx 1$ and $C_2 \approx 0.5$). More general simulations show that the estimates for C_1 and C_2 remain within 10% of these values when phase space structures are spontaneously formed by TAE excitations in a tokamak. Hence, we use these values in the estimation of internal magnetic fields in the frequency sweeping data of the MAST experiment. It is also important to note that phase space structures only spontaneously form when $\gamma_L \sim \gamma_d$, and as the rate of frequency sweeping is proportional to $(\gamma_L/\gamma_d)^{1/2}$ it follows that the internal field amplitude and the rate of frequency sweeping is insensitive to the closeness to marginal stability. Thus, the recording of the time scale of the frequency sweeping determines the bounce frequency of trapped particles. In turn, the bounce frequency depends upon the mode amplitude through the relation, $\delta\omega = C_1 (\gamma_d/\gamma_L)^{1/2} \omega_b^{3/2} \delta t^{1/2}$, $\omega_b = C_2\gamma_L$,where here the mode amplitude is defined in relation to the toroidal magnetic field on axis, B_0, as, $A = \delta B_r/B_0$ and C_3 is another constant of proportionality that generally has to be determined numerically (as discussed here). The time evolution of the frequency sweep described by equation (1), $\omega \sim \delta t^{1/2}$, follows from the theory which shows that the mode amplitude, A, and thus the nonlinear bounce frequency, ω_b, remains constant in time as long as the frequency shift (which is assumed larger than the linear growth rate) is not too large as to break some of the underlying theoretical assumptions. Collisionality, which we neglect in this work, can be expected to cause the phase-space structures to collapse and lead to a decrease in mode amplitude. However, work performed including such effects shows that the frequency shift still follows the $\delta t^{1/2}$ scaling even when the mode amplitude decays.

Thus we have an expression for how the absolutely calibrated mode amplitude in the core of the plasma can be determined from the observed sweeping rate of the TAE frequency, $\frac{\delta B_r}{B_0} = \frac{1}{C_1^2}\left(\frac{\gamma_L}{\gamma_d}\right)\left(\frac{\delta\omega^2}{C_3^2\delta t}\right)^{2/3}$ .The numerical method of obtaining the constants C_1 and C_3 from a numerical code is described below.

Numerical Modelling

Frequency sweeping close to marginal stability has been numerically modelled in the HAGIS code by introducing an additional external damping mechanism, represented by the damping rate, γ_d. Simulations are initially performed with $\gamma_d = 0$ to obtain the mode's linear growth rate, γ_L, without extrinsic dissipation. This then allows the selection of $\gamma_d \approx \gamma_L$, so that the system is near marginal instability. Then the simulations are repeated, but now near marginal instability, where conditions are favorable for the demonstration of spontaneous frequency sweeping.

We consider the case of a circular cross-section plasma with an inverse aspect ratio, a/R_0 = 0.3 and a monotonically increasing safety factor from an on-axis value of q_0 = 1.1 to an edge value, q_a = 3.5. The HAGIS code is used to simulate the case of a distribution of co-passing energetic particles with $\lambda = v_{||}/v = 1$. The growth rate is first numerically measured in the absence of any additional external damping mechanisms, i.e. $\gamma_d = 0$, and for the closest studied cases we use $\gamma_L/\omega_0 = 0.027 \pm 0.002$. The mode is then made marginally unstable by adding an artificial external damping mechanism such that γ_d/ω_0 = 0.02 and the simulation is repeated. Then the mode amplitude saturates at a constant level, $\delta B/B \approx 10^{-4}$ as shown in fig (3). A Fourier spectrogram of the evolving mode reveals a predominantly down-shifted frequency sweeping branch as shown in fig(4). Thus in this case,

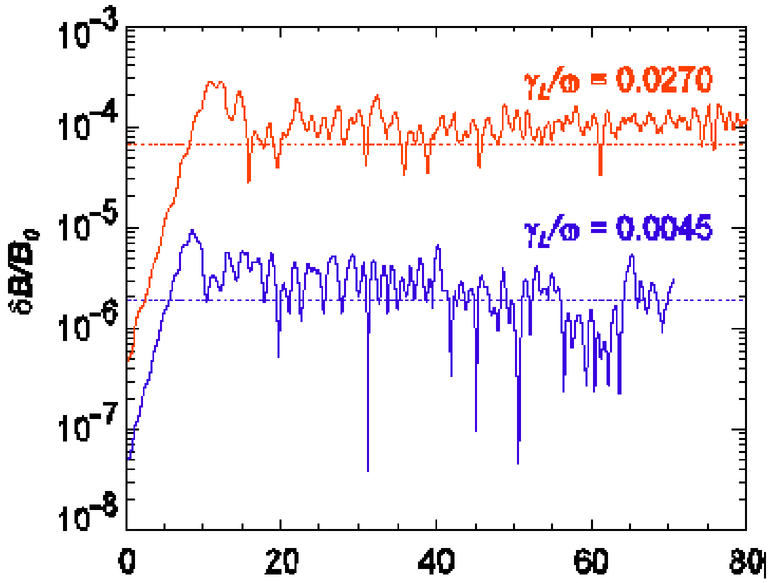

Figure 3: Evolution of mode amplitude in the presence of an artificial external damping mechanism such that γ_d/ω_0 = 0.02 (red) and γ_d/ω_0 = 0.004 (blue)

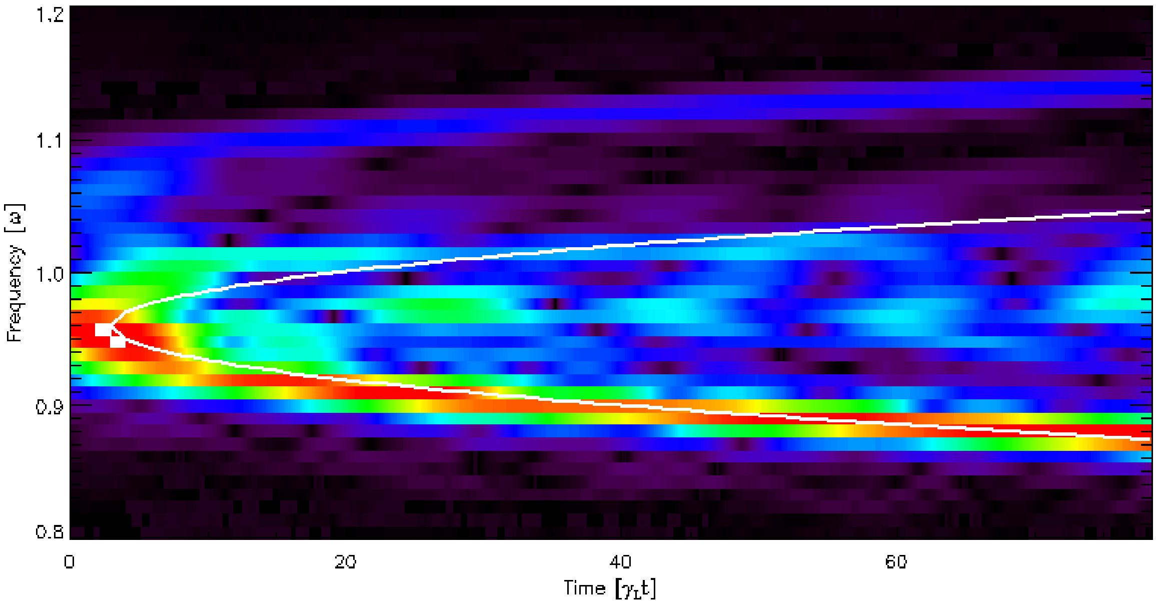

Figure 4: Sliding Fourier spectrum showing frequency evolution of marginally unstable TAE mode in response to kinetic α-particle drive (γ_L/ω_0 = 0.027) and external damping (γ_d/ω_0 = 0.02). The over-plotted white line shows the theoretically predicted frequency sweeping rate of $0.4\gamma_L(\gamma_d t)^{1/2}$

only a clump in the fast ion distribution was produced. The over-plotted white line, which is the theoretically predicted frequency sweeping expression, $\delta\omega = 0.4\gamma_L(\gamma_d t)^{1/2}$, shows good agreement with the simulation for the frequency shift of the hole. We also find that good results emerge for a simulation with reduced energetic particle drive, where $\gamma_L/\omega_0 = 4.5 \times 10^{-3} \pm 0.5 \times 10^{-3}$ and an external damping rate, $\gamma_d/\omega_0 = 4 \times 10^{-3}$.

Results

The HAGIS code was used to determine the mode amplitude of a frequency sweeping global mode that arises in the tight aspect-ratio MAST tokamak for the data shown in figure (2). A mode number analysis using a toroidally distributed array of magnetic pick-up coils around the inside of the vessel identifies this mode as an $n = 1$ mode. This discharge was heated by neutral beam injection (NBI) with the injected deuterons having an energy of 40 keV. The magnetic field strength at the magnetic axis was 0.5 T, the major radius was 0.77 m and the minor radius 0.55 m. To identify the constant of proportionality between the nonlinear bounce frequency and the mode amplitude a single particle trajectory was followed in the MAST geometry when there is a toroidal wave field present with the radial displacement of the magnetic field given by, $\xi(\boldsymbol{r},t) = A\,\mathrm{Re}\xi(r,\theta)\exp(in\phi - i\omega t)$. In this case, energy, E, and toroidal canonical momentum, P_ϕ, are no longer conserved quantities, but the quantity $H' = E - (\omega/n)P_\phi$ remains invariant. It is then possible to map out the island structure of fast ions trapped in the TAE and directly determine the relation between ω_b and δB_r. This was done for the value of $H' = 20$ keV which was chosen because the parallel velocity of fast ions at the center (and the peak of the eigenfunction) was equal to the Alfvén velocity, $v_{||} = v_A$. The amplitude of the mode was arbitrarily chosen to be $A = \delta B_r/B_0 = 10^{-3}$. Indeed it was confirmed that the nonlinear bounce frequency scales with the square root of the mode amplitude.

We now have sufficient information to estimate the amplitude of the TAE mode that is seen to sweep in frequency in the MAST discharge discussed above and we find an absolute amplitude of $\delta B_r \approx 2 \times 10^{-4}$ T. We note that in the experiment that the measured amplitude of the perturbed magnetic field at the mid-plane Mirnov coil was 10^{-5} T. Using the MISHKA code with an ideally conducting wall at $R/a = 2$ to calculate the linear mode structure in the plasma and vacuum regions allows the peak amplitude to be determined to be $\delta B_r \approx 5 \times 10^{-4}$ T. This value is in good agreement with that obtained above using the spectroscopic observation of the mode's frequency sweeping rate.

Conclusions

A spectroscopic technique has been formulated for inferring the internal amplitudes of frequency sweeping modes that start from the linear TAE frequency and remain in the TAE gap during the sweep. The example presented is that of a frequency sweeping TAE mode in the core of the MAST tokamak. In this case (MAST #5568 at $t = 65$ ms) the $n = 1$ global TAE amplitude was inferred to be $\delta B_r/B_0 \approx 4 \times 10^{-4}$. This result correlates well with direct coil measurements at the edge and the inference, from MISHKA, for what the fields should then be at the mode center.

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[1] Pinches S. D., Berk H. L., Gryaznavich M. P., Sharapov, S. E., and JET-EFDA Contributors, 2004 *Plasma Physics and Controlled Fusion*, **46** 7 S47 - S57.