High $\beta$ electron micro-stability in Spherical Tokamaks

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Introduction

High $\beta$ values achieved in spherical tokamaks, make experimental devices such as MAST [1] an attractive alternative route to fusion energy. Whether this can be realised depends on the underlying anomalous transport, and so understanding the behaviour of the micro-instabilities responsible for transport in these plasmas is very important. Electron scale micro-instabilities are investigated using the gyro-kinetic code, GS2 [2]. Variations of $\beta' = -\beta/L_p$ and global magnetic shear $\delta$ are treated by a perturbative local equilibrium expansion [3] around MAST equilibrium flux surfaces [4]. In this way, linear stability is studied in regimes of steep pressure gradient and high local $\beta$.

Equilibrium Effects

A scan of the normalised pressure gradient $\alpha = -q^2 R \beta'$ is carried out for various values of the magnetic shear $\delta$. The $\rho = \psi/\psi_{LCS} = 0.4$ surface is used for analysis, and the parallel magnetic perturbation is taken to be zero. Ions are treated as adiabatic, and the plasma is taken to be collisionless. Taking $k_y \rho_e = 0.6$ and decreasing the pressure gradient scale length (increasing the pressure gradient $\alpha$) consistently between the equilibrium and gyro-kinetic equation, for the cases of finite $\eta_e = \text{const.}$ and $\nabla n = 0$, reveals the destabilisation and subsequent stabilisation of multiple electron scale micro-instabilities (Figure 1), with $\beta_e = \text{const.} = 0.05$ and $\delta = 2.0$. Discontinuities in the plots of the real frequency indicate a change to a different mode or branch. Examination of the real frequency spectrum, for the $\nabla n = 0$ scenario, in Figure 2 shows that an increase in $\alpha$ brings about a more dominant mode at progressively longer perpendicular wavelengths.

The eigenfunctions of the two branches reveal a possible explanation for the branch change in Figure 1. Figure 3 shows the eigenfunctions for the electrostatic potential at $\alpha = 2.0$ and $3.0$. The increased pressure gradient can...
destabilise higher parallel harmonics, giving the eigenfunction more parallel structure. Figure 4 shows that this effect occurs at different values of the global magnetic shear, for both the finite $\eta_e$ case and the $\nabla n = 0$ case. The dispersion relation has been examined for a wide range of $\delta$ and $\alpha$, and shows a characteristic ETG curve. This work shows that there can be several different ETG branches characterised by the stability of their parallel components. The two competing effects of increasing $\alpha$ are: first to move the longer parallel wavelengths into a regime of second stability, and secondly to provide the energy to destabilise shorter parallel wavelengths.

Similar research on a NSTX plasma [5] shows one well-defined peak in the growth rate of the ETG mode in a $\beta'$ scan for the fully electromagnetic case, and two peaks when $\delta B || = 0$ (The form of which bears a strong resemblance to the $\delta = 1.5$ curve in the left diagram of Figure 4). The parameters used by Bourdelle et al [5] are $\beta = 0.3$, $\delta = 1.3$, $\eta_e = 3$, $k_y \rho_e = 0.3$. Figure 2 suggests that their use of lower $k_y$ than the current work, may reduce the sensitivity of the pressure gradient on the growth rate.

**$\beta$ effects**

In this section the effect of $\beta$ on the linear modes is investigated. We begin by varying $\beta$ inconsistently with the equilibrium, in order to establish the electromagnetic effects on the ETG mode. Keeping to the $\rho = 0.4$ surface of the MAST equilibrium, using the fully electromagnetic terms of GS2, figure 5 shows that $\beta$ has a stabilising influence on the ETG mode.
Contrary to the conventional wisdom, that $\delta B_{\parallel}$ can be neglected at low $\beta$, the $\beta$ effect comes from $\delta B_{\parallel}$ even at $\beta_c = 0.025$ (Figure 6).

The lack of $\beta$ effect from the $A_{\parallel}$ terms can be explained to some extent by considering the local dispersion relation. This can be derived \cite{6} with $\delta B_{\parallel} = 0$, using the gyro-kinetic equation, the quasi-neutrality condition and the parallel component of Ampère's law, giving,

$$1 + \tau - P_0 + \frac{P_1^2}{2k_\perp^2/\beta_e + P_2} = 0$$  \hspace{1cm} (1)

where,

$$P_m = \frac{1}{\sqrt{2\pi}} \int (v_{\parallel})^m \omega - \omega_* [1 + \eta_e (v^2/2 - 3/2)] \omega d\Omega_{v_{\parallel}}$$

$$\tau = T_e/T_i, \ \beta_c = 8\pi n_0 T_e/B^2, \ \omega_* = k_y \ and \ \omega_D = \epsilon_n \omega_* (v_{\parallel}^2 + v_{\perp}^2).$$

The wave numbers $k_{\perp}$ and $k_{\parallel}$ are normalised to the electron gyro-radius $\rho_e = v_{te}/\Omega_{te}$ and density gradient scale length $L_n$ respectively. Frequencies are normalised to $v_{te}/L_n$.

As noted by Kim and Horton \cite{6}, when $k_{\parallel} = 0$, the integrand of $P_1$ is odd in $v_{\parallel}$, and so $P_1 = 0$ and the electromagnetic contribution disappears.

We have examined the $k_{\parallel}$ power spectra for these eigenfunctions, and see that they are dominated by small parallel wavenumbers, which are unaffected by the electromagnetic term of equation 1. This illustrates how a local analysis at high $k_{\parallel}$ \cite{6} may overestimate the impact of $\beta$.

We have used the analytic methods of Kim and Horton \cite{6} to obtain a dispersion relation for the ETG mode with finite $\delta B_{\parallel}$ and $A_{\parallel} = 0$, and find that the electromagnetic term in this case is significant at low $k_{\parallel}$.

The above findings, have also been seen on the $\rho = 0.6$ and $0.8$ surfaces, and in simulations on high aspect ratio, low $\beta$, analytic equilibria.

**Micro-tearing**

Electromagnetic modes with tearing parity in the eigenfunctions have been observed to occur for $s \gtrsim 1$ and $a/L_p \gtrsim 3$ at $k_y \rho_e \sim 0.01-0.15$. The importance of these collisionless electron-tearing modes is illustrated in a conceptual Spherical Tokamak Power Plant (STPP) \cite{7}, where the ETG mode is found to be absolutely stabilised, leaving
micro-tearing modes as the dominant instabilities. A Poincaré sectioning diagnostic (Figure 7) has been developed to show the evolution of the confining magnetic field to stochasticity from magnetic island growth.

**Nonlinear Simulation**

A nonlinear study has begun, to establish the effect of high $\beta$ on turbulent ETG transport (Figure 8), and the significance of the micro-tearing mode, using EPSRC’s HPCx and USDOE’s Cheetah high performance computers.

The early results show experimentally significant transport, though convergence studies of the result in Figure 8 with respect to the $x$ and $y$ wavenumbers and flux-tube width have yet to be completed. Preliminary studies of the tearing mode show that magnetic transport is the dominant mechanism.

![Figure 7: Islands produced by the micro-tearing modes visualised by Poincaré section](image)

![Figure 8: Nonlinear MAST simulation. Thermal diffusivity vs time (left) and contours of electrostatic potential ($\theta - 0$) (right)](image)

### References


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