Estimation of positive radial electric field created by ECRH-pump out in magnetic confinement devices.

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1.- Introduction and Motivation

Previous work shows the existence of an outward particle flux when Electron Cyclotron Resonance Heating (ECRH) is applied to plasmas confined in TJ-II stellarator [1]. This flux is manifested experimentally through hollow density profiles that are usually accompanied by peaked temperature ones [2]. This extra flux, called pump-out usually, can be explained in terms of the increasing of particles that enter the loss cone in momentum space due to the enhancement of their perpendicular momentum. Those particles are lost in short time giving an enhancement of outward flux. The same type of profiles has been observed in other stellarators (see [3] and [4]) and tokamaks (see e.g. [5] and [6]), although the explanation does not have to be same for the two types of devices. The enhanced electron flux will create a positive electric field for which a first estimated is given in this work.

2.- The Langevin Equations

The enhanced electron flux is due to the combined effect of heating and transport. ECRH can be understood as particle diffusion in momentum space along the vector \( \mathbf{d} = Y_s \mathbf{e}_\perp + N_i u_i \mathbf{e}_i \), where \( Y_s = s \omega_i / \omega \) and \( \mathbf{u} = p / m c \). The estimation of this flux is difficult since it implies the resolution of the 5D kinetic equation (2D in momentum space and 3D in real space) [7]. The problem admits an alternative approach based on Langevin equations, which give the microscopic dynamics of particles in phase space [8].

As the heating is usually performed in the quasi-perpendicular regime it is clear that the diffusion in momentum space is mainly in the perpendicular direction. Therefore, the fraction of particles than enter the loss cone is increased, resulting in an enhancement of outward radial flux. The ambipolar condition implies the onset of a radial positive electric field that is able to stop the electron flux and to reduce the heat flux, appearing a peaked temperature profile.

The trajectories in momentum space of particles embedded in a wave field are given by the equations:

\[
\frac{d\mathbf{u}}{d\tau} = F_i(\mathbf{u}) + D_{ik}(\mathbf{u}) \frac{\partial^2}{\partial \tau^2} \xi_k, \quad i, k = \perp, \parallel;
\]

\[
\langle \xi_k(t) \rangle = 0, \quad \langle \xi_i(t) \xi_k(t + \tau) \rangle = \delta_{ik} \delta(\tau)
\]
These equations are composed of a deterministic and a stochastic part. The deterministic part is given by:

\[
F_\perp = \frac{1}{2} Y_s \left( u_{\perp} N_{\parallel} \frac{\partial D_{cy}}{\partial u_{\parallel}} + u_{\perp} N_{\parallel} \frac{\partial D_{cy}}{\partial u_{\perp}} \right) = \frac{1}{2} d_{\perp} (\vec{d} \cdot \nabla) D_{cy}
\]

\[
F_{\parallel} = \frac{1}{2} (u_{\perp} N_{\parallel}) \left( Y_s \frac{\partial D_{cy}}{\partial u_{\perp}} + u_{\perp} N_{\parallel} \frac{\partial D_{cy}}{\partial u_{\parallel}} \right) = \frac{1}{2} d_{\parallel} (\vec{d} \cdot \nabla) D_{cy}
\]

The stochastic one is the product of a random vector, whose components vary between –0.5 and 0.5, times the following symmetric matrix:

\[
D = \sqrt{2} \mathcal{D}_{cy}^{1/2} \left( \begin{array}{cc} Y_s^2 & Y_s u_{\perp} N_{\parallel} \\
Y_s u_{\perp} N_{\parallel} & (u_{\perp} N_{\parallel})^2 \end{array} \right) = \sqrt{2} \mathcal{D}_{cy}^{1/2} \left( \begin{array}{cc} \frac{d_{\perp}^2}{d^2} & \frac{d_{\perp} d_{\parallel}}{d^2} \\
\frac{d_{\perp} d_{\parallel}}{d^2} & \frac{d_{\parallel}^2}{d^2} \end{array} \right)
\]

The coefficient \( \mathcal{D}_{cy} \) comes from the quasi linear diffusion in momentum space and is proportional to the wave power density, \( w \), and to spectral density \( \Gamma(N_{\parallel}) \):

\[
\mathcal{D}_{cy}(\vec{u}) = \int dN_{\parallel} \frac{w}{|u_{\parallel}|} \left| \vec{r} \cdot \nabla \right|^2 \Gamma(N_{\parallel}) = \frac{w}{|u_{\parallel}|} \left| \vec{r} \cdot \nabla \right|^2 \Gamma(N_{\parallel} R)
\]

Langevin equations are integrated numerically and the electron trajectories in momentum space are estimated. As the deterministic term is much larger than the stochastic one, except in the vicinity of the resonance, we will neglect the stochastic part in this work.

3.- Transport and electric field estimates: Linear approximation.

The outward particle flux due to the pushing of electrons into loss cone is related to the flux in momentum space through the expression:

\[
\nabla \cdot \Gamma_E^{ECH} = \left( \frac{\partial n}{\partial t} \right)_{ECH} = \int_\delta f(\vec{u}) \frac{du}{dt} \cdot d\vec{s}
\]

\( \delta \) is the border of loss cone in momentum space and \( f \) is the electron distribution function. In this expression it is necessary to know the distribution function and the exact structure of loss cone in momentum space. Knowing these two elements is equivalent to having solved the problem, but we can introduce some approximations on that expression in order to do a quick calculation that allows us to extract the main properties of the ECRH induced particle flux. First of all we consider that the momentum distribution function of the electrons is Maxwellian, i.e., the deformation of the distribution function is small (we perform a linearization of the problem); second, we assume that all the particles that enter the loss cone escape from the magnetic surface; and third, the structure of the loss cone is simple (given by a cone) and does not change during the process. Of course, the distribution
function will be modified by the interaction of the electrons with the waves and by the escaping particles, and the structure of loss cone is modified by the electric field. We also disregard the effect of collisions that tend to diminish the particle flux inside loss cone, therefore we are overestimating the flux that can be written as:

\[
(V)_{ECH} = \left(\frac{\partial n}{\partial t}\right)_{ECH} = 2\pi \int_{-\infty}^{\infty} du_{1/} u_{\perp} \left(-\cos \theta \frac{du_{1/}}{dt} + \sin \theta \frac{du_{1/}}{dt}\right) f(\vec{u}) + 2\pi \int_{-\infty}^{0} du_{1/} u_{\perp} \left(\cos \theta \frac{du_{1/}}{dt} + \sin \theta \frac{du_{1/}}{dt}\right) f(\vec{u})
\]

Figure 1 shows the structure of the flux in momentum space versus the parallel momentum. The electron trajectories along the surface of loss cone are inwards for momentums lower than resonance condition and outwards for larger momentums. Despite of this fact that total flux in momentum space is positive because the particle density decreases strongly with momentum.

Assuming a temperature, a magnetic field and a power deposition profiles the total flux can be estimated: \(\Gamma = 1/r \int r' dr' (V)_{ECH}\)

Figure 2 shows the divergence of the flux that gives the local contribution to the integrated flux, which is also plotted. The divergence of the flux can change its sign along from LFS (flux directed inwards) to HFS (outward flux). The total integrated flux is, not surprisingly, directed outwards and the most important contribution comes from the plasma core, where the absorbed power is maximum. The high value of the flux that has been obtained can be explained considering that collisions are disregarded and, especially, that we have obtained an instantaneous flux that appears before the electric field is established. The application of the ambipolar condition will imply that a radial positive electric field must be created to keep the plasma quasi-neutrality [9]. Once the
instantaneous flux has been obtained the evolution of the field can be estimated by solving the following 1st-order equations:

\[ m \frac{d\Gamma}{dt} = -enE - \rho'; \quad \frac{dE}{dt} = \frac{e}{\varepsilon} \Gamma; \quad \frac{3}{2} \frac{d}{dt}(p') = -\left( q' + q'/r - q/r^2 \right) + w' \]

Where \( p \) is the plasma pressure, and \( q \) is the heat flux \((q = (5/2)\Gamma(p/n) - \chi p')\). In absence of collisions and viscosity, an oscillating behaviour of particle flux and electric field appears (see figure 3). The frequency of the oscillations is just the plasma frequency, therefore the typical time scale for the modification of the field is \( \tau \sim 1/\omega_p \) that is much shorter than the observed experimentally (of the order of tens of ms) [10]. The radial profiles of the flux and the field in times marked with I II and III are plotted in fig. 4.

**References**