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Error field amplification in the presence of a resistive wall

V.Igochine[*], S.Günter, K.Lackner, E.Strumberger

Max-Planck Institute für Plasmaphysik, EURATOM-Association, D-85748 Garching, Germany

Introduction. In advanced tokamak scenarios, the plasma performance is strongly limited by the external kink mode. This mode can be stabilized by conducting walls and/or plasma rotation. At the same time, external error fields (for instance asymmetric perturbations produced by the magnetic field coils) can be strongly amplified and stop the plasma rotation. Thus, investigation of the error field amplification (EFA) is one of the key issues for the stability of advanced tokamak discharges. In this paper, we investigate this error field resonance numerically. It allows us to include different effects which are not considered in the analytical theories[1,2,3,4] (different viscosity models, sound wave coupling, real plasma shape, etc.).

Toroidal torque and absorbed power. The tokamak plasma rotates with respect to the chamber and static error fields. The problem of plasma slowing down by external error fields is equivalent to the acceleration of the plasma via externally applied rotating fields. The torque due to the error fields can then be calculated directly from the power absorbed by the plasma as shown below. The total power flow out of external coils (and thus the energy needed to produce plasma perturbations) is connected to the toroidal torque, which is transferred from the coils to the plasma[5]. This result was obtained from the matrix circuit equation under the assumption of linear plasma instabilities. The power required to drive an arbitrary surface current J is the following:

$$P = \frac{d(\delta W)}{dt} = -\int \vec{j}\cdot\vec{E}dV = \frac{1}{4}\frac{d}{dt}(\vec{J}^{+}\cdot\vec{\Phi} + \vec{\Phi}^{+}\cdot\vec{J}) \qquad (1)$$

with J^{+} being the Hermitian conjugate of J. The torque required to drive the antenna current J is

$$\tau_\varphi = -\int (j\times B)\cdot\frac{\partial\vec{x}}{\partial\varphi}dV = i\cdot\frac{n}{2}(\vec{J}^{+}\cdot\vec{\Phi} - \vec{\Phi}^{+}\cdot\vec{J}) \qquad (2)$$

where Φ is the plasma flux and E is the vacuum electric field. Then the torque transferred from the external antenna onto the plasma can be calculated from the relation:

$$\tau_\varphi = 2n\cdot\mathrm{Re}(P/\omega), \qquad (3)$$

where n is the toroidal mode number, and P the absorbed power.

[*] e-mail: vgi@ipp.mpg.de

The linear MHD CASTOR_FLOW code is used to calculate the power absorbed by the plasma for a given frequency ω of the current flowing in the external antenna. This frequency dependent perturbation is equivalent to a rigid rotation of the plasma with the same frequency ω in a static error field. For these investigations, the CASTOR_ANTENNA[6] code has been integrated into the recently developed CASTOR_FLOW code. The CASTOR_FLOW code, which is an extended and modified version of the CASTOR[7] code, includes plasma rotation, viscosity and a resistive wall.

<u>Assumptions of the model</u>

1. Linear resistive MHD approach.
2. The thin shell approximation is used for the resistive wall.
3. Landau damping is approximated by a parallel viscosity force $\vec{F}_{visc} = -\kappa_{\|} \left| k_{\|} \right| v_{th,i} \rho \vec{v}_{\|}$.
4. Perpendicular viscosity is implemented as a constant damping rate ν to the Lagrangian pressure perturbation.
5. Instantaneous plasma response to the antenna perturbations is assumed.
6. Plasma rotation is limited to 0.3Mach if it is not included in the equilibrium.

Our CASTOR_FLOW code is similar to the linear resistive MHD code MARS[8] which allows to make benchmark calculations in the future. One of the differences between these two codes is the more advanced semikinetic model to describe Landau damping in the MARS code[9]. On the other hand, the CASTOR_FLOW code allows calculating the toroidal torque.

<u>Comparison with analytical theory.</u> A simple analytical model for error field amplification has been developed by R.Fitzpatrick[1]. This theory assumes only one external resonance in cylindrical geometry. In order to compare our calculations with the predictions of that model, we have neglected sound wave coupling and viscosity. We have constructed a set of JET shape equilibria stable to the external kink mode (β below the “no wall” limit) with the same q_a-values at the plasma edge, but different β-values. The resulting resonance frequencies (strongest EFA) are shown in figure 1. These results are very similar to those derived by Fitzpatrick. Such a resonance appears in the absence of a resistive wall, and we call it the “ideal branch” of the error field resonance as it measures the energetic distance to ideal marginal stability.

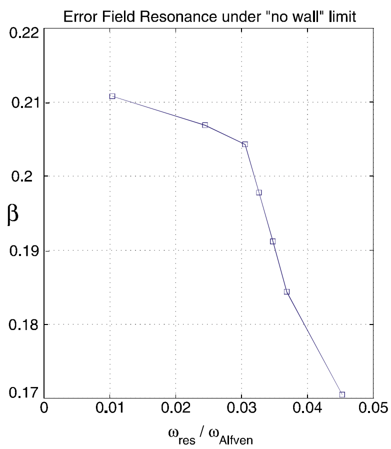

Figure 1. Error field resonance without a resistive wall. The ideal wall is at infinity, the external kink mode is stable. Plasma viscosity and compressibility (sound wave coupling) are neglected.

Influence of a resistive wall and plasma dissipation on the "ideal branch" of error field amplification. In this section we investigate the influence of a resistive wall, sound wave coupling and plasma viscosity on the "ideal branch" of EFA resonance. In the following, we change the stability of the plasma for kink perturbations by shifting the wall instead of changing the β-value. This is more convenient for calculations near the external kink stability boundary. Our equilibrium has JET shape and an external resonant surface (*q=3*) just outside the plasma boundary. If the wall distance corresponds to marginally stability, the resonant frequency drops to zero (see figure 2 (8)). Replacing the ideal wall by a resistive one (width of the wall *5mm*, $\eta_{wall}=10^{-7}$ *Ohm·m*) is destabilizing and thus the EFA frequency does not drop to zero at the original marginal stable wall distance (figure 2 (1)). The influence of sound wave coupling and viscosity on the resonance is even stronger. They completely change the resonance behaviour (figure 2 (2-7)). In this case, frequency and torque amplitude slightly increase with the wall distance. The value of Landau damping has only a small influence on the torque amplitude once it is above a small threshold value (figure 2 (4-6)).

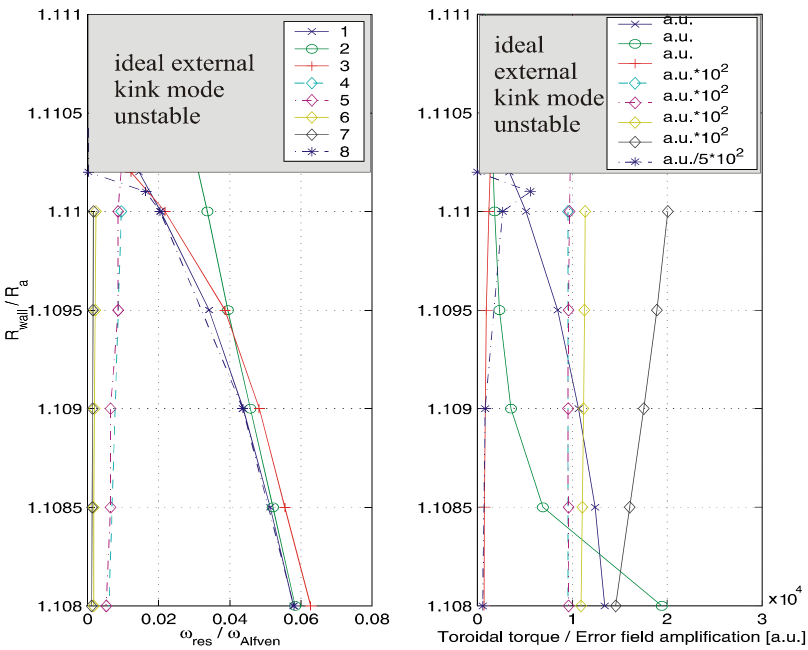

Figure 2 Influence of different factors on the error field resonance. Calculations 1-7 use the resistive wall ($\eta_{wall}=10^{-7}$ Ohm·m). **(1)** ν=0, $\kappa_{||}$=0, Γ=0 **(2)** ν=0.01, $\kappa_{||}$=0, Γ=0 **(3)** ν=0, $\kappa_{||}$=0.1, Γ=0 **(4)** ν=0.01, $\kappa_{||}$=1.5, Γ=1.667 **(5)** ν=0.01, $\kappa_{||}$=0.1, Γ=1.667 **(6)** ν=0.01, $\kappa_{||}$=0.001, Γ=1.667 **(7)** ν=0.001, $\kappa_{||}$=0, Γ=1.667 **(8)** η_{wall}=0, ν=0, $\kappa_{||}$=0, Γ=0. **Antenna perturbations have the following toroidal and poloidal mode numbers: *n=1, m=-2…8*.**

Resistive wall branch of the error field resonance.

Due to the existence of a resistive wall, in addition to the ideal EFA resonance, a lower frequency EFA resonance appears at about the inverse resistive wall time. In the following calculations we use the same equilibrium as for the results shown in figure 2 and include in addition toroidal plasma rotation ($v_{tor} = 0.051 \cdot v_{Alfven}$). This rotation is sufficient to stabilize the resistive wall mode (RWM). Due to the low frequencies involved in the problem the influence of sound wave coupling and viscosity for this resonance is negligible. Calculations show that the maximum toroidal torque and maximum EFA ($EFA=(B_{pl}/B_{vac})-1$) behave differently for this resonance. This difference is due to the phase shift between the induced currents in the wall and the antenna current. As expected one finds the maximum EFA amplitude at about the inverse resistive wall time frequency (see figure 3(b)). The exact value of the resonance frequency as well as the amount of EFA depends on the wall distance (and normalized plasma pressure).

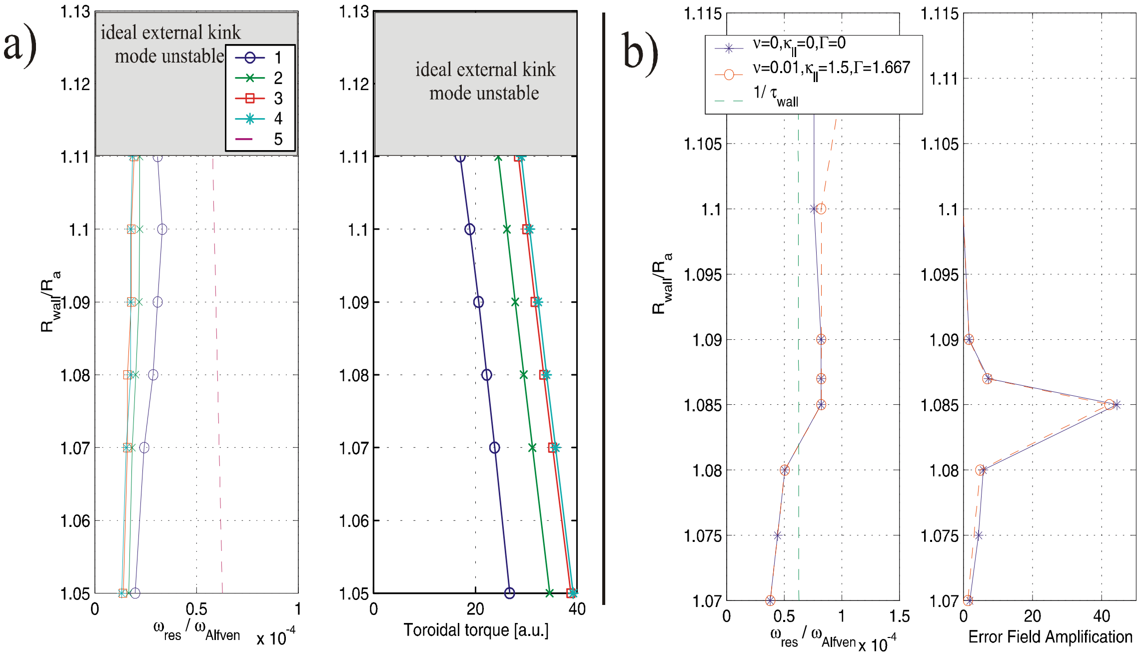

Figure 3 Resistive wall branch of error field resonance ($\eta_{wall}=10^{-7}$ Ohm·m). a) Toroidal torque: 1-ν=0.01, $\kappa_{||}$=1.5, Γ=1.667**, 2**-ν=0.01, $\kappa_{||}$=0.1, Γ=1.667**, 3**-ν=0, $\kappa_{||}$=0, Γ=1.667**, 4**-ν=0, $\kappa_{||}$=0, Γ=0**, 5**-inverse resistive wall time**. b) Error field amplification. In all calculations, antenna perturbations have the following toroidal and poloidal mode numbers: *n=1, m=-2…8.***

Our calculations of the resistive wall resonance are similar to DIII-D experiments. In these experiments RWM was stabilized by plasma rotation and then exited by an external n=1 magnetic field[10]. It was found that the resonance frequency of the EFA is a fraction of the inverse resistive wall time and EFA amplitude/frequency increases with increasing of β. This is similar to our result (see figure 3(b)), but detailed comparison is necessary. This is a subject for further investigations.

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[1] R.Fitzpatrick, Phys.Pl., **9**, (2002) 3459
[2] J.M.Finn, Phys.Pl., **2**, (1995) 198
[3] C.G.Gimblett, Phys.Pl., **11**, (2004) 1019
[4] V.D.Pustovitov, Pl.Phys.Rep., **30**, (2004) 211
[5] A.H.Boozer, Phys.Rev.Lett., **86**, (2001) 5059
[6] G.T.A.Huysmans, Phys.Pl., **2**, (1995) 1605
[7] W.Kerner, J.Comp.Phys., **142**, (1998) 271
[8] A.Bondeson et al, Phys.Fluids B, **4**, (1992) 1889
[9] A.Bondeson et al, Pl.Phys.Contr.Fusion, **45**, (2003), A253
[10] H.Reimerdes et.al., **30th EPS Conference**, 2003, P-4.45