Scaling of the marginal $\beta_p$ of the (2/1) neoclassical tearing mode and
NTM onset conditions with extreme profiles in ASDEX Upgrade, DIII-D and JET

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**Introduction**

The onset condition and the behaviour of neoclassical tearing modes (NTMs) is governed by the generalized Rutherford equation [1,2]. This theory provides an expression for the marginal $\beta_p$ at the resonant surface required for a NTM to exist and the saturated island size $W_{sat}$ for $\beta_p \geq \beta_{p,marg}$ describing the time evolution of the saturated island. Recently a large effort on several tokamaks has been started to evaluate $\beta_{p,marg}$ [2–4] for the (3/2)-NTM [5] as well as for the (2/1) NTM [3,4]. In the following the common scaling of $\beta_{p,marg}$ for the (2/1)-NTM in a combined scaling for ASDEX Upgrade, DIII-D and JET is considered.

As in previous works the importance of correctly including the different influence of the local temperature gradient $\nabla T_e$ and the density gradient $\nabla n_e$ on the bootstrap current $j_B$ has been shown [1,3,4] is also taken into account here. This plays a significant role for pellet triggered NTMs, where $n_e$ and $T_e$ behave significantly different before the onset. This effect has been taken into account in detail and an extended onset scaling including the pellet triggered cases has been derived.

**Generalised Rutherford equation and predictions**

The generalised Rutherford equation with additional terms due to the neoclassical driving term, the polarisation current term and the $\chi_\perp/\chi_\parallel$-term provides a widely accepted description of neoclassically driven tearing modes [1,2]. The equation and terms are used here in the form as in [3,4]. The driving term for the NTM is the lack of the bootstrap current within the island, as the pressure profile is flattened and therefore the bootstrap current is reduced. For the unperturbed bootstrap current $j_{BS} \sim \nabla \beta / B_p$ holds. The driving term is therefore proportional to $\sim \beta_{p,marg}/L_p$.

Under the assumption that the polarisation current term is the only stabilising term, the following expression for the minimal $\beta_p$ required for the mode’s onset can be derived

$$\beta_{p,marg}^{pol., curr} = \frac{3\sqrt{2}}{2} \sqrt{\frac{L_p}{L_q}} \cdot \frac{a q g(\xi, v_{ii}, \omega^*, m)}{a_2^2} \cdot (-\Delta') \rho_{pi}. \quad (1)$$

The function $g(\xi, v_{ii}, \omega^*, m) = \xi^{3/2} / 2$ is known from theory only in the two extreme cases of $v_{ii} = v_{ii}/m_\perp \omega^* \ll \mathcal{C}$ or $v_{ii} \gg \mathcal{C}$ respectively and governs the dependence on collisionality.

**Figure 1:** Typical (2/1)-NTM rampdown discharges for ASDEX, JET and DIII-D (from left to right). The JET case shows the typical early ICRH preheating for triggering the NTM with an early large sawtooth. The large increase of the NBI heating would have not been required in this case.

Considering the $\chi_\perp/\chi_\parallel$-model alone ($a_3 = 0, a_4 = 0$) as a stabilising term for small islands one finds for $\beta_{p,marg}$ (Eq. (2a))

$$\beta_{p,marg}^\chi = \frac{2W_0 \Delta'}{a_2 \sqrt{\xi}} \cdot \frac{L_p}{L_q} \quad (a) \quad \rightarrow \quad \beta_{p,marg}^\chi \sim v_{ii}^{0.5} \rho_{pi}^* \quad (b). \quad (2)$$
At the onset of the island and especially at the point where the island reaches its marginal size the condition of a small island is by definition fulfilled. Taking $W_0 = 5.1(\chi_\perp/\chi_||)^{1/2}(t_{res}L_{qT}/em)^{1/2}$, assuming a gyro-Bohm scaling ($\chi_\perp = T^{3/2}/B^2$) and taking the Spitzer formula $\chi_{Spit}$ for the parallel transport, Eq. (2b) is obtained. Within the island, the parallel transport $\chi_||$ is reduced due to the heat flux limit [6] leading to $\beta_{p,marg} \sim \rho_{pi}^9$ [7].

It is important for the calculation of the bootstrap current to include the different influence from the density gradient $\nabla n$ and the temperature gradient $\nabla T$ in the calculation of $L_p$ [8,3]. The corrected pressure gradient length according to [1] results in $1/L_p^{corr} \approx 1/3 \cdot 1/L_T + 2/3 \cdot 1/L_m$, which has been used within this work. The $T \cdot \nabla n$ term contributes stronger to the bootstrap current than the $n \cdot \nabla T$ term.

In order to fit $\beta_{p,onset}$ for the onset or the marginal $\beta_{p,marg}$, a simple power-law ansatz has been used. This ansatz is strictly only appropriate for fitting the data according to the $\chi_\perp/\chi_||$-model as well as for the model by Poli [9] which both predict a power-law scaling.

**Common discharge scenarios to determine the marginal $\beta_p$ scaling** For all three machines, AUG, DIII-D and JET, H-mode discharges with a JET-like low triangularity shape have been used. The main difference compared to the usual ASDEX Upgrade shape is at AUG a positive upper triangularity $\delta_{top} \approx 0.24$ which is typically around 0 at ASDEX Upgrade. The lower triangularity $\delta_{bottom} = 0.38$ is only slightly higher as in the usual ASDEX Upgrade case. The ellipticity $\kappa \approx 1.7$ is similar for both shapes. For ASDEX Upgrade and DIII-D purely NBI heating has been used, whereas for JET only at low $I_p \approx 1$ MA and $B_t \approx 1$ T NBI provided enough heating power to trigger a NTM reliably. At higher $I_p$, $B_t$ values additional ICRH heating was applied in the early discharge phase to trigger the NTM with an early large sawtooth. After the onset of a (2/1)-NTM, which eventually locks, either by a $n = 1$ and/or locked mode detector for JET and DIII-D a preprogrammed NBI waveform for the power ramp has been triggered. At ASDEX Upgrade an entirely programmed waveform has been used (see fig. 1).

With increasing heating power, decreasing $q_{95}$ or decreasing feedback controlled density and hence collisionality $\nu_i$ the (2/1)-NTM gets excited. The otherwise appearing (3/2)-NTM enters the database $\beta_{p,marg}$-scaling [5]. Still at high heating power the (2/1)-NTM locks to the vessel and stops the entire toroidal plasma rotation. With the reduction of the NBI heating power the mode gets smaller and starts rotating again. At JET the locked mode caused a problem due to the strong loss of particle confinement leading to heating power trips avoiding the unlocking and reaching $\beta_p^{marg}$.

**Figure 2:** Combined $\beta_p^{marg}$-scaling for ASDEX Upgrade (black crosses) and JET-like (red diamonds) ASDEX Upgrade type of discharges. Fig. (a) shows the parameter space in $\rho_{pi}$ and $\nu_{ii}$, fig. (b) the dependence of $\beta_p^{marg}/L_p^{corr}$ as function of $\rho_{pi}^2$ and fig. (c) the full scaling with respect to $\rho_{pi}^2$ and $\nu_{ii}$.

**Importance of the calculation of $L_p$** The uncorrected $L_p$ can be used for the onset, as long as only discharges with negligible $n_e$-peaking at the time of the onset are considered. During the rampdown the ratio between the different terms significantly differs and $L_p^{corr}$ gives the best results for the fitting of the temporal evolution of the mode amplitude as shown in fig. 3a and 3b [3]. For pellet triggered NTMs, either (3/2) or (2/1), a strong peaking of the density due to the pellets happens. This makes the use of $L_p^{corr}$ also necessary for the NTM onset.

**Scaling of the marginal $\beta_p$ for the (2/1)-NTM** The first data set for a $\beta_p^{marg}$-scaling for the (2/1)-
NTM at ASDEX Upgrade gave the scaling $\frac{\beta_{p,marg}}{L_{p,corr}^{marg}} \left(\frac{2}{1}, AUG\right) = 714 \cdot \rho_{pi}^{1.60\pm0.48} \cdot \nu_{ii}^{0.254\pm0.095}$.[3,4] Adding the new JET-shaped data gave a clear extension of the parameter space (see fig. 2a), due to the fact, that the higher triangularity $\delta \approx 0.4$ compared to $\delta \approx 0.3$ results in a higher particle confinement time $\tau_p$ resulting in higher achievable densities and collisionalities $\bar{\nu}_{ii}$ and lower $\rho_{pi}^*$. With this combined data set, also spanning a wider range in $\chi_{qs} = 3.8...5.5$, the following scaling (see fig. 2c) is obtained

$$\frac{\beta_{p,marg}}{L_{p,corr}^{marg}} \left[2/1, AUG\right] = 121 \cdot \nu_{ii}^{0.106\pm0.076}. \quad (3)$$

It must be noted that the $\rho_{pi}^*$ dependence alone on the other hand gives a much smaller exponent, than the exponent found for the (3/2)-NTM (see fig. 2b).

Both for JET and DIII-D $\beta_{p,marg}$ is typically reached just at the H to L backtransition. As this transition happens on a faster timescale than the typical decay time of the NTM, it is very difficult to fix the exact value of $\beta_{p,marg}$ and the marginal island size $W_{marg}$. Additionally the profiles change rapidly. For ASDEX Upgrade the same situation occurs for low $\chi_{qs} < 3.6$. For higher $\chi_{qs}$ the marginal $\beta_p$ is reached well in H-mode.

Before combining the data from ASDEX Upgrade (AUG and JET shaping) and JET, the normalisation of $\rho_{pi}$ has to be considered. The definitions $\rho^*_{pi}(AUG) = \rho_{pi}/a$ and $\rho^*_{pi}(JET) = \rho_{pi}/r_{res}$ differ typically by a factor of 2, due to the mode location and different machine sizes. For consistency the ASDEX Upgrade definition has been applied to the JET data. The definition $\rho^*_{pi}(AUG)$ does not contain information about the q-profile, whereas $\rho^*_{pi}(JET)$ contains the q-profile through $r_{res}$. For the JET (2/1) NTM data one finally arrives at

$$\frac{\beta_{p,marg}}{L_{p,corr}^{marg}} \left[JET\right] = 39163 \cdot \rho_{pi}^{3.04\pm0.021} \cdot \nu_{ii}^{0.0678\pm0.0057}.$$  

The combined scaling law for the ASDEX Upgrade data and the JET data gives

$$\frac{\beta_{p,marg}}{L_{p,corr}^{marg}} \left[AUG + JET\right] = 27359 \cdot \rho_{pi}^{2.76\pm0.27} \cdot \nu_{ii}^{0.358\pm0.081}.$$  

Both scalings are at present in contrast to previous results and should be treated with great care and should be treated with great care and considered as preliminary, as the number of data point is still very small and the scatter in the data pretty large. Also the DIII-D data has not yet been included in the combined scaling.

Figure 3: Fig. (a) and (b) show the importance of using $L_{p,corr}$ instead of just $\beta_p$ or $\beta_p/L_p$ for a (2/1)-NTM just after unlocking [4]. Fig. (c) shows the extreme case of a pellet triggered (3/2)-NTM, where after the last pellet triggering the NTM the profiles show a strong variation with respect to $L_{p,corr}$.

Several arguments can be found for the unclear (2/1)-NTM scaling: The coupling of the (2/1) mode to the wall might add additional physics and an additional term in the Rutherford equation. With the large $n_e$ peaking at the $q = 2$ surface it is unclear, if one is reaching the threshold-like behaviour of the $g$-function in $\bar{\nu}_{ii}$ [10,11]. This could no longer be described with a power-law assumption. Modes with different mode numbers are not required to have the same exponents in a power-law ansatz.

**Extension of the parameter range in $\rho_{pi}^*$ and $\bar{\nu}_{ii}$ for the NTM onset with pellets**  
As presented in [12,6], pellets injected from the high field side may trigger NTMs. This typically happens in a two step process. The first pellets of a pellet sequence not only increase the density, but also lead to a strong peaking of the density [13]. The increased $n_e$ rises the collisionality $\bar{\nu}_{ii}$ (fig. 4a). The peaking increases
the equilibrium bootstrap current significantly, as this depends strongly on the radial density gradient \( n_r \). The resulting reduction in the temperature lowers \( \rho_p^{*\text{corr}} \) (fig. 4a) and hence \( \beta_p^{\text{marg}} \). A subsequent pellet provides then the triggering seed-island.

\[
\beta_p^{\text{marg}} / L_p^{\text{corr}} [3/2, \text{AUG}] = 395 \cdot \rho_p^{*} 1.13^{\pm 0.19} \cdot \chi_i^{0.0535^{\pm 0.0291}}.
\]

This scaling is consistent with previous results mainly influenced by \( \rho_p^{*} \). The pellet database alone results in a stronger collisionality dependence: \( \beta_p^{\text{marg}} / L_p^{\text{corr}} [3/2, \text{pellets}] = 431 \cdot \rho_p^{*} 1.12^{\pm 0.28} \cdot \chi_i^{0.156^{\pm 0.091}} \). The role and the mechanism for a NTM triggered by pellets needs to be discussed in more detail.

**Summary and conclusions** An attempt to extend the \( \beta_p^{\text{marg}} \)-scalings for the (2/1)-NTM by combining ASDEX Upgrade, DIII-D and JET data has been made. Using ASDEX Upgrade data with AUG and JET shapes extends the accessible parameter range and results in a larger collisionality impact and a higher \( \rho_p^{*} \) exponent for the (2/1)-NTM. Including JET data gave no conclusive results yet and including DIII-D data had to be left as a future task. The high collisionalities \( \chi_i \) question the validity of a power-law ansatz as one might reach the threshold behaviour. Including pellet triggered (3/2)-NTMs in the local scaling confirmed the linear \( \rho_p^{*} \) dependence and a stronger \( \chi_i \) influence. In all cases \( L_p^{\text{corr}} \) instead of \( L_p \) alone had to be used to fit the time evolution of the islands.

**Acknowledgement** O. Sauter has been supported by the Swiss National Science Foundation. The support and cooperation from all three machines is greatly appreciated.

**References**