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A New Treatment of the Heat Transport Equation with a Transport Barrier and Applications to ECRH Experiments in Tore Supra

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1) Introduction

Experimental results[1-3], as well as direct magnetic turbulence measurements [4,5] have shown the existence, in plasmas with strong additional heating, of a critical electron temperature gradient beyond which the turbulence is triggered and the associated electron heat diffusivity is strongly enhanced. This diffusivity presents a marked step at the space location where the critical gradient is reached. The step-like diffusivity has also been observed in the improved confinement regimes with internal transport barrier.

Comparison between theory and experiments for the transport problem is usually performed numerically with a transport code. This approach has the obvious advantage of allowing to include in the comparison secondary effects, such as simultaneous density, current and ion temperature evolutions, radiative losses, etc. Nevertheless, analytical solutions of simplified versions of this problem provide deeper insight in the physical nature of the phenomena involved and could also prove to be more efficient in carrying on extensive comparisons with the experimental data. This is particularly true for the analysis of perturbative electron transport experiments in tokamaks, such as the ECRH power modulation experiments.

2) Heat diffusion equation in a cylinder with a transport barrier

Consider the diffusion equation for the electron temperature T_e, in an infinite cylinder of radius a. The use of this geometry, a standard assumption for transport problems in tokamaks, corresponds to the hypothesis of symmetry in the toroidal and poloidal directions, the only space variations being related to the co-ordinate along the cylinder radius r. The diffusion equation can be written as

$$\frac{\partial T_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa \frac{\partial T_e}{\partial r}\right) - bT_e + S(r,t) \qquad (1)$$

where $\kappa = (2/3)\chi_e$, $b = 1/\tau_d$, $S = (2/3)P/n_e$. Here n_e is the electron density, χ_e is the electron heat diffusivity, P is the power density (i.e., per unit volume) absorbed by the plasma from an external heating system (usually, Ohmic, rf-waves or neutral beams injection), τ_d is the damping time which characterize all losses in the plasma as the radiation, the electron-ion energy exchange, etc. For simplicity, the following initial and boundary conditions are chosen

$$T_e(r,t=0) = 0\ , \quad T_e(r=a,t) = 0\ , \quad \frac{\partial T_e}{\partial r}(r=0,t) = 0\ . \qquad (2)$$

2.1) κ–constant

As shown in [6], the solution of Eq. (1) with boundary conditions (2) for a constant κ is given by

$$T_e(r,t) = \frac{2}{a^2}\sum_{n=1}^{\infty}\frac{J_0(\alpha_n r)}{J_1^2(\alpha_n a)}\int_0^t dt'\left[\exp\left(-(\kappa\alpha_n^2 + b)(t-t')\right)\int_0^a r'S(r',t')J_0(\alpha_n r')dr'\right] \qquad (3)$$

where J_0 , J_1 are Bessel functions of order 0 and 1, respectively, and α_n are the roots of the equation $J_0(\alpha_n a) = 0$. As discussed in Ref. 6, Eq. (3) represents in fact a decomposition of the heat source in eigenmodes $E_n(r) \equiv J_0(\alpha_n r)$, which constitute a set of orthogonal functions. This means that for a source term $S(r) = E_n(r)H(t)$, where H is the Heaviside function, the temperature increases in time keeping the same spatial shape, i.e.,

$$T_e(r,t) = \lambda_n(t)E_n(r) \quad , \qquad \lambda_n(t) = \frac{1-\exp\left(-(\kappa\alpha_n^2+b)t\right)}{\kappa\alpha_n^2+b} \tag{4}$$

The eigenvalues λ_n represent in fact an effectiveness of the response in temperature of the plasma for each eigenmode. For $t << 1/(\kappa\alpha_n^2+b)$, we have $\lambda_n(t) \approx t$ *i.e.* the effectiveness of the temperature response is the same for each eigenmode, and the response in temperature, having the same form as the source, is local. In the stationary phase (for $t >> 1/(\kappa\alpha_n^2+b)$), $\lambda_n(t) \approx 1/(\kappa\alpha_n^2+b)$. As α_n is a series increasing as n^2, the higher the order, the lower the effectiveness. Thus the final temperature profile is dominated by the first modes. In this case the response in temperature appears as non-local. This effect partly explains the temperature profile resilience phenomenon observed in tokamaks. Additional effects, as the heat pinch, critical gradient, etc, can reinforce this resilience. It should be noted that the damping time can strongly affect the profile resilience.

2.2) κ with transport barrier

In tokamaks, the heat diffusivity is not radially constant, and presents sometimes a drastic change in its radial profile. Consider the following expression for κ:

$$\kappa(r) = \kappa_1 \ \text{ for } 0 \le r \le x \qquad \text{and} \qquad \kappa(r) = \kappa_2 r^N \ \text{ for } x < r \le a \tag{5}$$

with $\kappa_1 = 0.2m^2/s$, $\kappa_2 = 2m^2/s$, $x/a = 0.4$, $N = 1.5$.

As shown in Ref. 7, the solution of equation (1) with a step-like heat diffusivity given by (5) can be written under the following form:

$$T_e(r,t) = \int_0^t dt' \left(\int_0^a 2\pi r' G(r,r',t,t')\, S(r',t')dr' \right) \tag{6}$$

where the Green function is given by

$$G_1(r,r',t,t') = \frac{x^{-1-N/2}}{2\pi\kappa_1} \sum_{n=1}^{\infty} \frac{J_0(\gamma_n r)J_0(\gamma_n r')}{J_0(\gamma_n x)} \Pi_\nu^{x,a}(s_n) \left[\frac{dD}{ds}(s_n)\right]^{-1} \exp\left(-(\kappa_1\gamma_n^2+b)(t-t')\right) \tag{7}$$

for $0 < r < x$ and $0 < r' < x$

$$G_2(r,r',t,t') = \frac{x^{-1}r^{-N/2}}{2\pi\kappa_1} \sum_{n=1}^{\infty} J_0(\gamma_n r') \Pi_\nu^{r,a}(s_n) \left[\frac{dD}{ds}(s_n)\right]^{-1} \exp\left(-(\kappa_1\gamma_n^2+b)(t-t')\right) \tag{8}$$

for $x < r < a$ for $0 < r' < x$

$$G_3(r,r',t,t') = \frac{x^{-1}r'^{-N/2}}{2\pi\kappa_1} \sum_{n=1}^{\infty} J_0(\gamma_n r) \Pi_\nu^{r',a}(s_n) \left[\frac{dD}{ds}(s_n)\right]^{-1} \exp\left(-(\kappa_1\gamma_n^2+b)(t-t')\right) \tag{9}$$

for $0 < r < x$ and $x < r' < a$

$$G_4(r,r',t,t') = \frac{x^{N/2-1}(rr')^{-N/2}}{2\pi\kappa_1} \sum_{n=1}^{\infty} J_0(\gamma_n x) \frac{\Pi_\nu^{r',a}(s_n)\Pi_\nu^{r,a}(s_n)}{\Pi_\nu^{x,a}(s_n)} \left[\frac{dD}{ds}(s_n)\right]^{-1} \exp\left(-(\kappa_1\gamma_n^2+b)(t-t')\right) \tag{10}$$

for $x < r < a$ and $x < r' < a$

In the expressions (7-10), the function $D(s)$ is given by

$$D(s) = q_1 x^{-N/2} I_1(q_1 x) \Pi_\nu^{x,a} + \frac{k_2}{k_1} q_2 x^{-N} I_0(q_1 x) \Sigma_\nu^{x,a} \tag{11}$$

with $\nu = N/|N-2|$, $q_1 = (s/\kappa_1)^{1/2}$, $q_2 = (s/\kappa_2)^{1/2}$.

The functions $\Pi_\nu^{r,a}$ $\Sigma_\nu^{r,a}$ are combination of Bessel functions I_ν and K_ν:

$$\Pi_\nu^{r,a} = K_\nu\left(\lambda r^{1/(1+\nu)}\right) I_\nu\left(\lambda a^{1/(1+\nu)}\right) - K_\nu\left(\lambda a^{1/(1+\nu)}\right) I_\nu\left(\lambda r^{1/(1+\nu)}\right) \quad (12)$$

$$\Sigma_\nu^{r,a} = K_{\nu+1}\left(\lambda r^{1/(1+\nu)}\right) I_\nu\left(\lambda a^{1/(1+\nu)}\right) + I_{\nu+1}\left(\lambda r^{1/(1+\nu)}\right) K_\nu\left(\lambda r^{1/(1+\nu)}\right) \quad (13)$$

where $\lambda = (1+\nu)q_2 = q_2/(1-N/2)$.

Note that s_n are the zeros of the function $D(s)$ *i.e.* $D(s_n) = 0$, and it is found that these zeros s_n are real and negative, thus can be defined as $s_n = -\kappa_1 \gamma_n^2$.

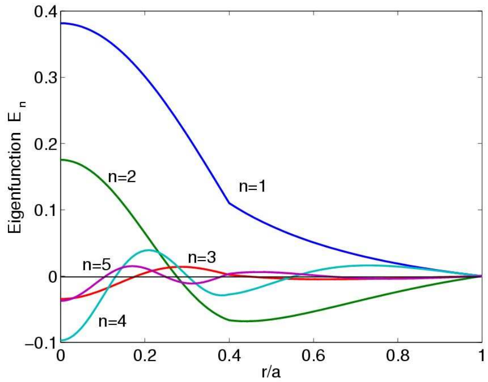

Fig. 1 Eigenfunctions for the heat diffusivity κ given by (5).

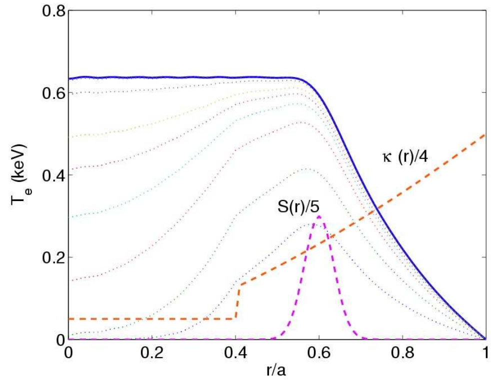

Fig.2 Time evolution of the electron temperature profile for an off-axis heat source.

As demonstrated in Ref. 7, the following functions :

$$E_n(r) = x^{-N/2} J_0(\gamma_n r) \Pi_\nu^{x,a}(s_n) \qquad \text{for } r < x$$
$$E_n(r) = r^{-N/2} J_0(\gamma_n x) \Pi_\nu^{r,a}(s_n) \qquad \text{for } r > x \quad (14)$$

constitute an orthogonal set of eigenmodes. Fig.1 displays the first 5 eigenfunctions. Note that all of the eigenfunctions have a marked discontinuity of their derivatives at the κ step location. As for the κ-constant case, $\lambda_n \sim t$ for vanishing t, thus all the eigenmodes contribute to the first phase of the evolution. However, higher order eigenmodes saturate quickly and the memory of the shape of the source function is eventually lost. This is related to the well-known phenomenon of resilience of the temperature profiles during the off-axis heating.

Fig.2 displays the time evolution of the electron temperature profile for a source deposited at *r/a*=0.6. The discontinuity of the gradient is not visible in the steady state, since the temperature is flat in all the region inside the power deposition, but it is well pronounced in the evolution phase: this property could be exploited for an experimental determination of the critical gradient or the threshold. As in the κ-constant case, when the damping time is short, i.e. the heat losses are large, the heat can reach the plasma center with more difficulty, and the temperature profile perturbation is more localized. Thus a strong heat loss in the plasma can affect the profile resilience effect.

3) ECRH modulation experiments

Low-frequency (1-2 *Hz*) ECRH modulation experiments have been performed on Tore Supra tokamak by using two gyrotrons at 118 GHz with a total injected power of 700 kW. Figures 3 & 4 show respectively the amplitude and phase of the Fourier transform of the temperature modulation of 1 *Hz* for the 5th, 7th and 9th harmonics. The input ECRH power is deposited at *r/a*=0.34. A strong asymmetry in gradient for both amplitude and phase has been clearly observed around the ECRH power deposition. This confirms the existence of a drastic change in the heat diffusivity at the location of the ECRH power deposition [2]. In these figures the solid lines represent the simulation by the above analytical model with a step-like diffusivity: $\kappa_1 = 0.5m^2/s$, $\kappa_2 = 4m^2/s$, $N = 0$, and $b = 20s^{-1}$. There is a good agreement between the simulation and experiments around the power deposition layer. The time evolution of the modulated temperature is also well reproduced by the above analytical model. It should be noted that the low frequency ECRH modulation experiments allow to exploit a very broad time scale, from first harmonic to the high harmonics (up to 11th harmonic), on the heat transport property, and to display the transition from χ^{PB} (heat diffusivity in stationary phase estimated by power balance method, lower value) to χ^{HP} (heat diffusivity in transient phase, higher value).

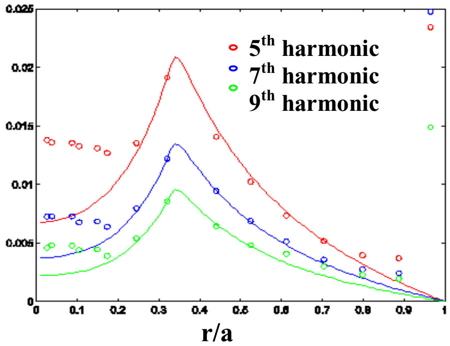

Fig.3 Amplitude of the Fourier transform of T_e. The solid lines represent the simulation by the analytical method.

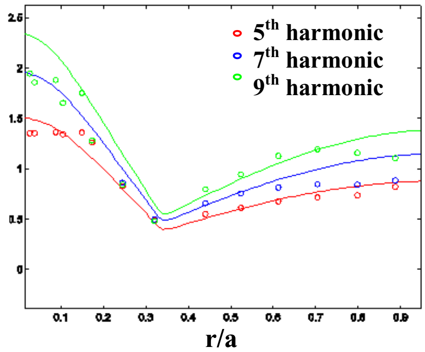

Fig. 4 Phase of the Fourier transform of T_e. The solid lines represent the simulation by the analytical method.

4) Conclusions

An exact analytical solution of the electron heat diffusion equation in a cylinder has been found with a step-like diffusion coefficient, plus a monomial increase in the radial direction and a constant damping term. This model is sufficiently general to describe heat diffusion in the presence of a critical gradient threshold or a transport barrier, superimposed to the usual trend of increasing heat diffusivity from the plasma core to the edge. This solution is an expansion on a basis of eigenmodes of the diffusion-damping operator that generalizes the familiar Fourier-Bessel series, which are the eigenmodes in the constant diffusivity case.

This type of representation allows to see some well-known properties of heat transport phenomena in a different light. For instance, it has been shown that the contributions of the eigenmodes to the time dependent solution grow at speeds that depend on the eigenmode order *i.e.* at the beginning of the heating phase all the eigenmodes are equally involved, whereas at the end only the lower order ones are left. This implies, e.g., that high frequency modulation experiments provide a characterization of transport phenomena that is intrinsically different with respect to power balance analysis of a stationary phase. The analytical solution provides a complementary approach to the analysis and interpretation of transient transport experiments. It is particularly useful to analyse power switch on/off events and whenever high frequency modulations are not technically feasible.

Low-frequency (1-2 *Hz*) ECRH modulation experiments have been performed on Tore Supra. A large jump (a factor of 8) in the heat diffusivity has been clearly identified at the ECRH power deposition layer. The amplitude and phase of several harmonics of the Fourier transform of the modulated temperature, as well as the time evolution of the modulated temperature have been reproduced by the analytical solution. The jump is found to be much weaker at lower ECRH power (one gyrotron). This experiment allows to exploit a very broad time scale, from first harmonic to the high harmonics, on the heat transport property, and to display the transition from the χ^{PB} to the χ^{HP}.

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