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Effect of dust adiabaticity on dust acoustic solitons

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Abstract

A comparative study between the adiabatic model and the γ-model in view of the investigation of the existence of dust acoustic solitons, has been conducted. It is shown a perfect agreement between the results derived by both approaches. Dust acoustic soliton amplitude and width as well as Sagdeev potential derived in the framework of both models, are matching well.

Introduction

A study of the conditions of existence of dust acoustic solitons where electrons and ions are taken Boltzmannian, whereas dust grains are adiabatic has been conducted[1]. Two approaches have been adopted and their results compared, namely, the adiabatic model and the γ-model. In the linear regime, the dispersion relation derived in the framework of the two models is found to be the same. However, when the non-linear regime is dealt with, the solitons amplitudes derived by both models show a difference of about 20% for the small amplitude case. Whereas, for the large amplitude case, the Sagdeev potentials are found to be different. As far as the condition on Mach numbers is concerned, it was calculated that soliton solutions do not exist for M>1.06 for the adiabatic model, whereas there are no restrictions in the γ-model; solutions may exist beyond this value. The two approaches are leading consequently to very different predictions. It has been stated that the γ-model validity is confined to lower temperatures. We believe however, that when the two models are redefined, they converge necessarily towards the same conclusions.

Formulation

Let's consider a plasma consisting of electrons and ions that are taken Boltzmannian for low-frequency modes, with their normalized number densities (by Zn_0) given by,

$$n_e = n_{e0}e^{\phi} \tag{1}$$

$$n_i = n_{i0}e^{-\sigma\phi} \tag{2}$$

and adiabatic dust particulates, where n_0 is the equilibrium dust density, Z is the number of charge on the dust grain, $\sigma = \frac{T_e}{T_i}$, T_e and T_i being the electron and ion temperatures. For the adiabatic dust component, the pressure P is related to the number density n by the equation of state,

$$\frac{P}{n^{\gamma}} = const. \tag{3}$$

Furthermore, pressure P is related to temperature T through,

$$P = nT. \tag{4}$$

It should be stressed that in Eq.(4) no γ (adiabatic exponent) is to be added. By virtue of Eqs.(3) and (4), the equation of state may be cast as,

$$\frac{T}{n^{\gamma-1}} = const. \tag{5}$$

In an adiabatic fluid where the density is not uniform, temperature is no longer constant. Consequently, the pressure term in the dust momentum equation may be written otherwise (c.f.Refs.[2,3]), viz.,

$$\nabla P = \gamma\, T \nabla n \tag{6}$$

It should be pointed out that Eq.(6), which is an exact equation, has been derived by way of invoquing dust adiabaticity. Hence, temperature T is not to be considered constant. The term $\gamma\, T$ has not been pulled out of the nabla operator. As a consequence, in both formulations, namely, the one involving P or that one involving T, an equation of state has to be incorporated in the system to meet the system closure requirement. The dynamics of the dust fluid is then given by,

$$\frac{\partial n}{\partial t} + \frac{\partial nv}{\partial x} = 0, \tag{7}$$

$$\frac{\partial n}{\partial t} + v\frac{\partial n}{\partial x} = Z\frac{\partial \phi}{\partial x} - \frac{1}{n}\frac{\partial P}{\partial x}, \tag{8.a}$$

$$\frac{\partial n}{\partial t} + v\frac{\partial n}{\partial x} = Z\frac{\partial \phi}{\partial x} - \frac{\gamma T}{n}\frac{\partial n}{\partial x}, \tag{8.b}$$

$$\frac{\partial P}{\partial t} + v\frac{\partial P}{\partial x} + \gamma P\frac{\partial v}{\partial x} = 0, \tag{9.a}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} + (\gamma - 1)T\frac{\partial v}{\partial x} = 0, \tag{9.b}$$

$$Z\frac{\partial^2 \phi}{\partial x^2} = n_e + n - n_i, \tag{10}$$

where dust density n, dust velocity v, dust pressure P, dust temperature T as well as the electrostatic potential ϕ, are normalized by, n_0, $c_{SD} = \sqrt{T_e/m}$ (m being the dust mass), n_0T_e, T_e and T_e/e respectively. Time and space variables are normalized by the dust plasma period (by 2π) $\omega_{pd}^{-1} = (4\pi n_0 Z^2 e^2 / m)^{-1/2}$ and the Debye length $\lambda_D = (T_e/4\pi\, n_0 Z^2 e^2)^{1/2}$. For small amplitude regime, a KdV equation may be derived for both formulations from the system (7)-(10) along with Eqs. (1) and (2), by means of the reductive perturbation technique. The stretched coordinates are $\xi = \sqrt{\varepsilon}(x - Mt)$ and $\tau = \varepsilon^{3/2}t$, where ε is the smallness parameter and M is the soliton normalized velocity. Thus Eqs.(7)-(10) as well as Eqs. (1) and (2) reduce to,

$$(v-M)\frac{\partial n}{\partial \xi} + \varepsilon\frac{\partial n}{\partial \tau} + n\frac{\partial v}{\partial \xi} = 0, \tag{11}$$

$$(v-M)\frac{\partial v}{\partial \xi}+\varepsilon\frac{\partial v}{\partial \tau}-Z\frac{\partial \phi}{\partial \xi}+\frac{1}{n}\frac{\partial P}{\partial \xi}=0, \quad (12.a)$$

$$(v-M)\frac{\partial v}{\partial \xi}+\varepsilon\frac{\partial v}{\partial \tau}-Z\frac{\partial \phi}{\partial \xi}+\frac{\gamma T}{n}\frac{\partial n}{\partial \xi}=0, \quad (12.b)$$

$$(v-M)\frac{\partial P}{\partial \xi}+\varepsilon\frac{\partial P}{\partial \tau}+\gamma P\frac{\partial v}{\partial \xi}=0, \quad (13.a)$$

$$(v-M)\frac{\partial T}{\partial \xi}+\varepsilon\frac{\partial T}{\partial \tau}+(\gamma-1)T\frac{\partial v}{\partial \xi}=0, \quad (13.b)$$

$$Z\varepsilon\frac{\partial^2 \phi}{\partial \xi^2}=n_e+n-n_i. \quad (14)$$

Let's expand in terms of ε, the variables n, P, v, ϕ and T to get,

$$n=1+\varepsilon n_1+\varepsilon^2 n_2..., \quad (15)$$

$$P=P_0+\varepsilon P_1+\varepsilon^2 P_2..., \quad (16)$$

$$v=\varepsilon v_1+\varepsilon^2 v_2..., \quad (17)$$

$$\phi=\varepsilon \phi_1+\varepsilon^2 \phi_2..., \quad (18)$$

$$T=T_0+\varepsilon T_1+\varepsilon^2 T_2... \quad (19)$$

Introducing Eqs.(15)-(19) in Eqs. (11)-(14) along with Eqs.(1) and (2) would yield at the lowest order,

$$n_1=-(n_{e0}+\sigma n_{i0})\phi_1=-l\phi_1, \quad (20)$$

$$M\frac{\partial n_1}{\partial \xi}=\frac{\partial v_1}{\partial \xi}, \quad (21)$$

$$\frac{\partial P_1}{\partial \xi}=M\frac{\partial v_1}{\partial \xi}+Z\frac{\partial \phi_1}{\partial \xi}, \quad (22)$$

$$\gamma T_0\frac{\partial n_1}{\partial \xi}=M\frac{\partial v_1}{\partial \xi}+Z\frac{\partial \phi_1}{\partial \xi}, \quad (23)$$

$$\lambda\frac{\partial P_1}{\partial \xi}=\gamma P_0\frac{\partial v_1}{\partial \xi}, \quad (24)$$

$$\lambda\frac{\partial T_1}{\partial \xi}=(\gamma-1)T_0\frac{\partial v_1}{\partial \xi}. \quad (25)$$

From Eqs.(20)-(25) we obtain,

$$P_1=-\gamma P_0 l\phi_1, \quad (26)$$

$$v_1=-Ml\phi_1, \quad (27)$$

$$T_1=(1-\gamma)T_0 l\phi_1, \quad (28)$$

and

$$M^2=Z/l+\gamma P_0, \quad (29.a)$$

$$M^2=Z/l+\gamma T_0. \quad (29.b)$$

At a higher order we get,

$$\frac{\partial n_1}{\partial \tau}+\frac{\partial n_1 v_1}{\partial \xi}-M\frac{\partial n_2}{\partial \xi}+\frac{\partial v_2}{\partial \xi}=0, \quad (30)$$

$$\frac{\partial v_1}{\partial \tau}+v_1\frac{\partial v_1}{\partial \xi}-M\frac{\partial v_2}{\partial \xi}-Z\frac{\partial \phi_2}{\partial \xi}+\frac{\partial P_2}{\partial \xi}=n_1\frac{\partial P_1}{\partial \xi}, \quad (31.a)$$

$$\frac{\partial v_1}{\partial \tau}+v_1\frac{\partial v_1}{\partial \xi}-M\frac{\partial v_2}{\partial \xi}-Z\frac{\partial \phi_2}{\partial \xi}+\gamma T_1\frac{\partial n_1}{\partial \xi}=\gamma T_0 n_1\frac{\partial n_1}{\partial \xi}, \quad (31.b)$$

$$\frac{\partial P_1}{\partial \tau}+v_1\frac{\partial P_1}{\partial \xi}-M\frac{\partial P_2}{\partial \xi}+\gamma P_1\frac{\partial v_1}{\partial \xi}+\gamma P_0\frac{\partial v_2}{\partial \xi}=0, \quad (32.a)$$

$$\frac{\partial T_1}{\partial \tau}+v_1\frac{\partial T_1}{\partial \xi}-M\frac{\partial T_2}{\partial \xi}+(\gamma-1)T_1\frac{\partial v_1}{\partial \xi}+(\gamma-1)T_0\frac{\partial v_2}{\partial \xi}=0, \quad (32.b)$$

$$Z\frac{\partial^2 \phi_1}{\partial \xi^2}+\frac{\sigma^2 n_{i0}-n_{e0}}{2}\phi_1^2=l\phi_2+n_2. \quad (33)$$

Using Eqs.(26)-(29.b) in Eqs.(30)-(33), the Korteweg-de Vries (KdV) equation governing the evolution of ϕ_1 is deduced for both formulations,

$$\alpha_{1,2}\frac{\partial \phi_1}{\partial \tau}+\beta_{1,2}\phi_1\frac{\partial \phi_1}{\partial \xi}+\frac{\partial^3 \phi_1}{\partial \xi^3}=0, \quad (34)$$

where, $\alpha_1=\dfrac{2M}{(M^2-\gamma P_0)^2}$,

$$\alpha_2=\frac{2M}{(M^2-\gamma T_0)^2},$$

$$\beta_1=\left(\sigma^2 n_{i0}-n_{e0}-Z^2\frac{3M^2+\gamma P_0(\gamma-2)}{(M^2-\gamma P_0)^3}\right)/Z,$$

$$\beta_2=\left(\sigma^2 n_{i0}-n_{e0}-Z^2\frac{3M^2+\gamma T_0(\gamma-2)}{(M^2-\gamma T_0)^3}\right)/Z.$$

It is clear that due to $P_0=T_0$, one has $\alpha_1=\alpha_2$ and $\beta_1=\beta_2$. Moreover, it appears that our results differ from those in Ref.(1) by the factor γ-2 (instead of γ-1).
The solution of Eq.(34) leads to,

$$\phi_1=\phi_0 \sec h^2\left(\frac{\xi-u_0\tau}{\delta}\right), \quad (35)$$

where u_0 is the soliton velocity, $\phi_0=\dfrac{3\alpha}{\beta}u_0$ and $\delta=\dfrac{2}{\sqrt{\alpha u_0}}$ being the amplitude and the width of the soliton respectively.

Conclusion

To conclude, we have shown that the conditions of the existence of adiabatic dust acoustic solitons as well as their characteristics such as amplitude and width, remain unchanged whether we consider the pressure term in dust momentum equation as ∇P or $\gamma T \nabla n$, if care is taken of the fact that T is variable, hence the closure of the fluid equations system by the equation of state in terms of temperature. In the large amplitude regime, both formulations agree well on Sagdeev potential. For the γ-model, unless we consider temperature constant whereas pressure is variable, the predictions are valid for all values of the temperature. We stress the fact that the present note does not deal with the existence of the dust acoustic solitons as such, but is a comparative study between the approaches cited above. A more detailed investigation of the dust acoustic solitons including charge fluctuation and particle trapping is found in Ref.(4).

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Reference

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