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Impurity variable charge effect on the ponderomotive force based contamination control in plasmas

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Abstract

The impurity decontamination method in dusty plasmas proposed by Amroun and Annou [Phys. Plasmas 8, 1 (2001)] is revisited. The charging process of the grain is dealt with through appropriate sink terms in the governing equations. An additional term arises in the ponderomotive force that is at the root of the acceleration of the plasma components separation

Introduction

Solid sub-micron sized grains acquire an electric charge by attracting electrons and ions if immersed in a plasma [1]. Due to its high value as well as its dynamical character the grain charge greatly affects the modes of oscillations allowed to propagate in the plasma. This influence may be spurious in many industrial situations, such as, semi-conductors manufacturing industry, fusion reactors etc. [2]. In order to separate dust grains from electrons and ions of the plasma, it has been suggested to impose a magnetic field $\vec{B}_0$ and to launch a circularly polarized wave of a frequency that is chosen so to ensure an energy resonant feeding of the grain [3]. The gyro-radius of the grain grows whereas the electron and ion gyro-radii remain almost static, up to complete spatial separation. Indeed, it has been analytically shown through a compact solution away from the source, the feasibility of the proposed technique. Although the solution proves, in principle the possibility of the contamination control, it remains approximate since on one hand the ponderomotive forces generated by the wave and experienced by electrons and ions are neglected, and on the other the grain charge has been considered constant. The electrons and ions ponderomotive forces are reintroduced in Ref. (4) and the system of the governing equations is numerically solved. The results confirm the separation of the plasma constituents beyond a critical distance.
In this note we reconsider the charge of the grain that's self-consistently introduced. The grain charge equation is taken into account along with the loss effect experienced by electrons and ions through appropriate sink terms in the continuity and momentum equations.
It turns out, that an additional term that depends on the attachment frequency, appears in the ponderomotive force .This term accelerates the separation of the plasma constituents.

Modeling

Let's consider a left-handed circularly polarized wave, $\vec{E} = E(z)(\hat{x} + i\hat{y})\exp(i\omega t) + c.c$, where $E(z) = |E(z)|\exp(i\phi z)$, propagating in a magnetized dusty plasma.
The governing equations are given by,

$$m_d\left(\frac{\partial}{\partial t} + \vec{V}_d.\vec{\nabla}\right)\vec{V}_d = q\left(\vec{E} + \vec{V}_d \times \left(\vec{B} + \vec{B}_0\right)\right) - \frac{\nabla P_i}{n_d} + \frac{m_e n_e}{n_d}\nu_e\left(\vec{V}_e - \vec{V}_d\right) + \frac{m_i n_i}{n_d}\nu_i\left(\vec{V}_i - \vec{V}_d\right) \qquad (1)$$

$$m_e\left(\frac{\partial}{\partial t} + \vec{V}_e.\nabla\right)\vec{V}_e = -e\left(\vec{E} + \vec{V}_e \times \left(\vec{B} + \vec{B}_0\right)\right) - \frac{\nabla p_e}{n_e} - m_e\nu_e\left(\vec{V}_e - \vec{V}_d\right) \qquad (2)$$

$$m_i\left(\frac{\partial}{\partial t}+\vec{V}_i.\nabla\right)\vec{V}_i = e\left(\vec{E}+\vec{V}_i\times\left(\vec{B}+\vec{B}_0\right)\right)-\frac{\nabla p_i}{n_i}-m_i\nu_i\left(\vec{V}_i-\vec{V}_d\right) \tag{3}$$

$$\frac{\partial n_d}{\partial t}+\nabla.\left(n_d\vec{V}_d\right)=0 \tag{4}$$

$$\frac{\partial n_e}{\partial t}+\nabla.\left(n_e\vec{V}_e\right)=-\nu_e n_e \tag{5}$$

$$\frac{\partial n_i}{\partial t}+\nabla.\left(n_i\vec{V}_i\right)=-\nu_i n_i \tag{6}$$

$$\nu_e=\pi r_0^2\sqrt{\frac{8T_e}{\pi m_e}}\exp\left(\frac{eq}{CT_e}\right)n_d \tag{7}$$

$$\nu_i=\pi r_0^2\sqrt{\frac{8T_i}{\pi m_i}}\left(1-\frac{eq}{CT_i}\right)n_d \tag{8}$$

$$\frac{\partial q}{\partial t}+\vec{V}_d.\nabla q=\frac{e}{n_d}\left(\nu_i n_i-\nu_e n_e\right) \tag{9}$$

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \tag{10}$$

$$\nabla\times\vec{B}=\mu_0\,\vec{j}+\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \tag{11}$$

The different quantities have their usual meaning while r_0, C are the grain radius and capacitance. The ponderomotive force responsible for particles separation is defined as, $\vec{F}=q(R_e\vec{V}\times R_e\vec{B})$ where $\mathrm{Re}(x)$ stands for real part of x. After some lengthy algebra, the perpendicular component of the velocities of each particle is given by,

$$\vec{V}_{d\perp}=\frac{1}{\alpha^2+\tilde{\Omega}_d^2}\left\{\alpha\,\vec{A}-\vec{\Omega}\times\vec{A}\right\}=\frac{\alpha+i\beta}{\alpha^2+\beta^2}\gamma\,\vec{E} \tag{12}$$

where, $\bar{\nu}_i=\frac{m_i n_i}{m_d n_d}\nu_i;\bar{\nu}_e=\frac{m_e n_e}{m_d n_d}\nu_e\ ;\Omega_\sigma=q_\sigma B_0/m_\sigma$ for $\sigma=e,i,d$

$$\alpha=i\omega+\bar{\nu}_e+\bar{\nu}_i-\frac{\nu_e\bar{\nu}_e\left(i\omega+\nu_e\right)}{\Omega_e^2+\left(i\omega+\nu_i\right)^2}-\frac{\nu_i\bar{\nu}_i\left(i\omega+\nu_i\right)}{\Omega_i^2+\left(i\omega+\nu_i\right)^2},$$

$$\tilde{\vec{\Omega}} = \vec{\Omega}_d + \frac{\nu_e \overline{\nu}_e}{\Omega_e^2 + (i\omega + \nu_e)^2}\vec{\Omega}_e + \frac{\nu_i \overline{\nu}_i}{\Omega_i^2 + (i\omega + \nu_i)^2}\vec{\Omega}_i = \beta\,\hat{z},$$

$$\vec{A} = \frac{q}{m_d}\vec{\mathrm{E}} - \frac{\frac{e\overline{\nu}_e}{m_e}(i\omega+\nu_e)}{\Omega_e^2 + (i\omega+\nu_e)^2}\left\{\vec{\mathrm{E}} + \frac{\vec{\mathrm{E}}\times\vec{\Omega}_e}{i\omega+\nu_e}\right\} + \frac{\frac{e\overline{\nu}_i}{m_i}(i\omega+\nu_i)}{\Omega_i^2 + (i\omega+\nu_i)^2}\left\{\vec{\mathrm{E}} + \frac{\vec{E}\times\vec{\Omega}_I}{i\omega+\nu_i}\right\} = \gamma\,\mathrm{E}.$$

$$\vec{\mathrm{V}}_{e\perp} = -\frac{\nu_e + i(\omega+\Omega_e)}{\Omega_e^2 + (i\omega+\nu_e)^2}\left\{\frac{e}{m_e} - \frac{\gamma\nu_e}{\alpha - i\beta}\right\}\vec{\mathrm{E}} \tag{13}$$

$$\vec{\mathrm{V}}_{i\perp} = \frac{\nu_i + i(\omega+\Omega_i)}{\Omega_i^2 + (i\omega+\nu_i)^2}\left\{\frac{e}{m_i} + \frac{\gamma\nu_i}{\alpha - i\beta}\right\}\vec{\mathrm{E}} \tag{14}$$

By considering the following conditions viz, $|\Omega_d/\Omega_e| << 1, \overline{\nu}_e/\nu_e << 1$, $\overline{\nu}_i/\nu_i << 1$ and $\Omega_d < \nu_e$, $\nu_i < \Omega_i$ and using Eqs.(10)-(12), we determine the ponderomotive force experienced by the dust grains,

$$\vec{\mathrm{F}} = -\frac{q}{\Omega_d}\left\{\frac{\delta_2}{2}\frac{\partial}{\partial z}|\mathrm{E}|^2 + \delta_1|\mathrm{E}|^2\frac{\partial\phi}{\partial z}\right\}\hat{z} \tag{15}$$

where, $\mathrm{Re}\,\vec{\mathrm{V}}_{d\perp} = \delta_1\,\mathrm{Re}\,\vec{\mathrm{E}} + \delta_2\,\mathrm{Im}\,\vec{E}$, $\delta_1 \approx \frac{q}{m_d}\frac{1 + \left(\frac{\overline{\nu}_e + \overline{\nu}_i}{\Omega_d}\right)^2}{2(\overline{\nu}_e + \overline{\nu}_i)\left[1 + \left(\frac{\overline{\nu}_e + \overline{\nu}_i}{2\Omega_d}\right)^2\right]}$, $\delta_2 \approx -\delta_1\frac{\overline{\nu}_e + \overline{\nu}_i}{\Omega_d}$

An additional term appears that depends on the phase derivative. It turns out that this term accelerates the separation process. The low frequency component of the governing equations, where the electron and ion ponderomotive forces are neglected, are given by the following system provided the kinetic energy is slowly varying, viz, $\left(\frac{\partial}{\partial z}\left(\frac{1}{2}m_\sigma V_{\sigma z}\right) \approx 0\right)$,

$$\frac{dn}{dz} \approx 0, \tag{16}$$

$$\frac{dn_i}{dz} = -n_i\left(a\,\frac{d\in^2}{dz} + b\,\in^2\frac{d\phi}{dz}\right), \tag{17}$$

$$\frac{d^2\tilde{\phi}}{dz^2} + \frac{2}{\in}\frac{d\in}{dz}\frac{d\phi}{dz} = 0, \tag{18}$$

$$\frac{d^2\in}{dz^2} - \in\left(\frac{d\phi}{dz}\right)^2 + \in\left(1 - \frac{n_d}{\alpha}\right) \approx 0, \tag{19}$$

where, $n = n/(\varepsilon_0 B_0^2 / m_d)$, $\in^2 = |E|^2 / 2T_e ZB_0^2 / m_d$, $Z = \frac{|q|}{e}$, and $\alpha = (\omega - \Omega_d)/\Omega_d$.

The ion density away from the source is given by,

$$n_i(z) \approx n_i(0)\exp\left\{-\frac{\in^2(0)}{4}\exp(\beta z)\right\}\exp\left(\frac{bk}{\in^2(0)}z\right), \tag{20}$$

where, $\in^2 \frac{d\phi}{dz} = k$. Away from the source the solution tends to $\frac{n_d - n_i}{n_i} \approx 1$.The additional term $\exp\left(\frac{bk}{\in^2(0)}z\right)$ that accelerates the separation process, is due to particles attachment by the grain.

Conclusion

In summary, when a circularly polarized wave propagates in a magnetized dusty plasma, electrons, ions and dust grains gyrate around the field lines. However, if one chooses a wave frequency close to the grain cyclotron frequency, the dust gyro-radius grows and a dust void develops. However, it is established that when a dust grain is immersed in the plasma, it acquires an electric charge by way of electrons and ions attachment. The grain charge is then a dynamical variable, which has been proved to be very influential on plasma properties. In this note, it is shown that an additional term arises in the expression of the ponderomotive force generated by the wave and experienced by the grains. In the range $(\bar{\nu}_e + \bar{\nu}_i) < 2\omega$ the correction to the ponderomotive force reduces to $\frac{\Omega_d}{\bar{\nu}_e + \bar{\nu}_i}|E|^2 \frac{\partial\phi}{\partial z}$ and happens to give rise to an acceleration of the void formation (separation process).

References

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