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Modulational instability and envelope excitations of dust–acoustic waves in a non–thermal background

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Abstract

The nonlinear amplitude modulation of dust acoustic waves is studied, in the presence of a non-thermal background population of ions and electrons. A nonlinear Schrödinger equation is derived for the wave's amplitude. The influence of non-thermality on the conditions for modulational instability is discussed.

1. Introduction. The *dust-acoustic* (DA) wave [1] is a fundamental wave mode in dust-contaminated (dusty) plasmas (DP), which has no analogue in ordinary e-i plasmas. It represents electrostatic oscillations of mesoscopic size, massive, charged dust grains against a background of electrons and ions which, given the low frequency of interest, are practically in a thermalized (i.e. Boltzmann) equilibrium state.

The amplitude (self-)modulation of these waves was recently considered [2], by means of the reductive perturbation formalism [3]. A study of the modulational (in)stability profile has shown that long/short wavelength DA waves are stable/unstable against external perturbations. The respective regions were associated with the occurrence of localized envelope DA excitations of the dark/bright type (i.e. voids/pulses) [3]. Obliqueness in perturbation was shown to modify this dynamic behaviour [2].

A departure from Boltzmann's distribution of the electrostatic background has been shown to bear a considerable effect on electrostatic (e.g. ion-acoustic) plasma modes. Inspired by those earlier works, the presence of non-thermal ions was recently shown to modify the nonlinear behaviour of DA waves, affecting both the form and the conditions for the occurrence of DA solitons. This brief paper is devoted to a study of the effect of a non-thermal background on the modulation properties of DA waves [4].

2. The model. Let us consider the (one-dimensional) propagation of dust-acoustic waves. The mass and charge of dust grains (both assumed constant for simplicity) will be denoted by m_d and $q_d = s\, Z_d e$, where $s = \text{sgn} q_d \equiv q_d/|q_d|$ denotes the sign of the dust charge ($= \pm 1$). Similarly, we have: m_i and $q_i = +Z_i e$ for ions, and m_e and $q_e = -e$ for electrons. The (moment) evolution equations for the dust fluid read:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(n\, u) = 0\,, \qquad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -s\,\frac{\partial \phi}{\partial x}\,, \qquad (2)$$

where the particle density n_d, mean fluid velocity u_d, and electric potential Φ are scaled as: $n = n_d/n_{d,0}$, $u = u_d/c_d$, and $\phi = |q_d|\Phi/(k_B T_{eff})$, where $n_{d,0}$ is the equilibrium dust density; the effective temperature T_{eff} is related to the characteristic dust speed $c_d \equiv (k_B T_{eff}/m_d)^{1/2} = \omega_{p,d}\lambda_{D,eff}$, defined below ($k_B$ is Boltzmann's constant). Time and space are scaled over $\omega_{p,d}^{-1} = (4\pi n_{d,0} q_d^2/m_d)^{-1/2}$ and $\lambda_{D,eff} = [(1-b_e)\lambda_{D,e}^{-2} + (1-b_i)\lambda_{D,i}^{-2}]^{-1/2}$ [where $\lambda_{D,e/i} = (k_B T_{e/i}/4\pi n_{e/i,0} q_{e/i}^2)^{1/2}$ is the Debye length for species e/i; $b_{e/i}$ are defined below]. Temperature and magnetization effects related to the massive dust grains were omitted. The system is closed by Poisson's equation $\nabla^2\Phi = -4\pi\, e(s\, n_d\, Z_d + n_i\, Z_i - n_e)$. Overall neutrality at equilibrium (viz. $\sum_{\Sigma=\alpha,\{\alpha'\}} n_{\Sigma,0}\, q_\Sigma = 0$) is assumed. Following the model of Cairns *et al.* [5], the non-thermal velocity distribution for both e and i reads

$$f_{s'}(v; a_{s'}) = \frac{n_{s',0}}{\sqrt{2\pi v_{th,s'}^2}} \frac{1 + a_{s'} v^4/v_{th,s'}^4}{1 + 3a_{s'}} \exp(-v^2/2v_{th,s'}^2)\,, \tag{3}$$

where $n_{s',0}$ and $v_{th,s'} = (k_B T_{s'}/m_{s'})^{1/2}$ denote the equilibrium density and thermal velocity of the species $s' \in \{1, 2\} \equiv \{i, e\}$ and the real parameter $a_{s'}$ expresses the deviation from the Maxwellian state (which is recovered for $a_{s'} = 0$). Eq. (3) leads to

$$n_{s'} = n_{s',0}(1 + \sigma_{s'} b_{s'} \Phi' + b_{s'} \Phi'^2)\, \exp(-\sigma_{s'}\Phi') \tag{4}$$

[4, 5], where $\sigma_{1/2} = \sigma_{i/e} = +1/-1$; $b_{s'} = 4a_{s'}/(1+3a_{s'})$; $\Phi'_{s'} = Z_{s'} e\Phi/k_B T_{s'}$; $Z_{1/2} = Z_i/1$.

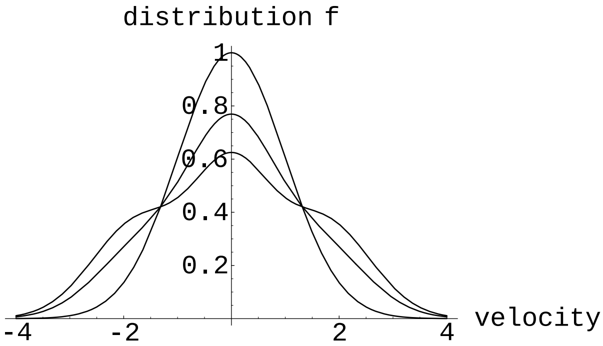

Figure 1: The non-thermal distribution $f(v; a)$ [scaled by $f(v; 0) = 1/\sqrt{2\pi}$] vs. the reduced velocity v/v_{th}, for $a = 0,\ 0.1,\ 0.2$ (from top to bottom).

3. Nonlinear oscillations. Near equilibrium ($\phi \ll 1$), Poisson's Eq. becomes:

$$\nabla^2\phi \approx \phi - \alpha\,\phi^2 + \alpha'\,\phi^3 - s\,(n-1)\,. \tag{5}$$

Note that the right-hand side in Eq. (5) cancels at equilibrium. Here, $\alpha = [Z_i/(\lambda_{D,i}^2 T_i) - 1/(\lambda_{D,e}^2 T_e)]\, T_{eff}\lambda_{D,eff}^2/(2Z_d)$ and $\alpha' = [(1+3b_i)\, Z_i^2/(\lambda_{D,i}^2 T_i^2) + (1+3b_e)/(\lambda_{D,e}^2 T_e^2)]\ T_{eff}^2\ \lambda_{D,eff}^2/(6Z_d^2)$. For $T_e \gg T_i$, one may retain the approximate expressions: $\alpha \approx -s(1-\mu)/[2(1-b_i^2)^2]$ and $\alpha' \approx (1+3b_i)(1-\mu)^2/[6(1-b_i^2)^3]$ [also: $\lambda_{D,eff} \approx \lambda_{D,i}(1-b_i)^{-1/2}$], defining the dust parameter $\mu = n_{e,0}/(Z_i n_{i,0}) = 1 + sZ_d n_{d,0}/(Z_i n_{i,0})$ (the dependence on the electron parameters is recovered if the assumption $T_e \gg T_i$ is *not* satisfied).

Following the reductive perturbation technique [3], we define the state vector $\mathbf{S} = \{n, u, \phi\}$ [governed by Eqs. (1), (2) and (5)] and then expand in the vicinity of $\mathbf{S} = \mathbf{S_0} \exp[i(kx - \omega t)] + \text{c.c.}$, where $\mathbf{S}^{(0)} = (1, 0, 0)^T$, viz. $\mathbf{S} = \mathbf{S}^{(0)} + \epsilon \mathbf{S}^{(1)} + \epsilon^2 \mathbf{S}^{(2)} + ...$, where $\epsilon \ll 1$ is a smallness parameter. We assume that $Sj^{(n)} = \sum_{l=-\infty}^{\infty} S_{j,l}^{(n)}(X, T)\, e^{il(kx-\omega t)}$ (for $j = 1, 2, ...;$ $S_{j,-l}^{(n)} = S_{j,l}^{(n)*}$, for reality), thus allowing the wave amplitude to depend on the stretched ($slow$) coordinates $X = \epsilon(x - v_g\, t)\,, \quad T = \epsilon^2\, t$, where $v_g = \omega'(k)$ is the *group velocity*.

The calculation provides the first harmonic amplitudes (to order $\sim \epsilon^1$)

$$n_1^{(1)} = s(1 + k^2)\, \psi\,, \qquad u_{1,x}^{(1)} = (\omega/k) \cos\theta\, n_1^{(1)}\,, \qquad u_{1,y}^{(1)} = (\omega/k) \sin\theta\, n_1^{(1)}\,, \tag{6}$$

in terms of the potential correction $\phi_1^{(1)} \equiv \psi$, along with the DA wave (reduced) dispersion relation $\omega^2 = k^2/(k^2+1)$. The amplitudes of the 2nd and 0th (constant) harmonic corrections are obtained in $\sim \epsilon^2$ (the lengthy expressions are omitted for brevity). The potential correction ψ obeys the *nonlinear Schrödinger–type equation* (NLSE)

$$i\, \frac{\partial \psi}{\partial T} + P\, \frac{\partial^2 \psi}{\partial X^2} + Q\, |\psi|^2\, \psi = 0\,. \tag{7}$$

The *dispersion coefficient* P is related to the curvature of the dispersion curve as $P = \omega''(k)/2 = -3\omega^5/(2k^4) < 0$. The *nonlinearity coefficient* Q, due to carrier wave self-interaction, is given by a complex function of k, α and α':

$$Q = \frac{1}{12k(1+k^2)^{3/2}(3+3k^2+k^4)} \Big\{ -4\alpha^2(k^4+3k^2-3) + 3\Big[9 + 6(5+3\alpha')k^2 \\ +(35+18a')k^4 + 3(5+2\alpha')k^6 - k^{10}\Big] + 12s\alpha(1+k^2)^2(k^4+5k^2+3)\Big\} \tag{8}$$

which is essentially a function k, μ and $a_{i,e}$ (make use of the definitions above); for $k \ll 1$, $Q \approx (3 + 2s\alpha)^2/(12k) > 0$. For shorter wavelengths, i.e. for $k > k_c r$, Q changes sign (becomes negative). These exact expressions may now be used for a numerical investigation of the wave's modulational stability profile.

4. Stability profile – envelope excitations. The evolution of a wave whose amplitude obeys Eq. (7) essentially depends on the sign of the coefficient product PQ, which may be numerically investigated in terms of the physical parameters involved. For $PQ > 0$, the DA wave is modulationally *unstable* and may either *collapse*, when subject to external perturbations, or evolve into a series of *bright* localized envelope modulated wavepackets (*pulses*) [3]. For $PQ < 0$, on the other hand, the DA wave is *stable* and may propagate as a *dark/grey*-type envelope wavepackets, i.e. a propagating localized envelope *hole* (a *void*) amidst a uniform wave energy region (see e.g. in [3] for details). We note that, in either case, the localized excitation L and maximum amplitude ψ_0 satisfy $L\psi_0 = (2P/Q)^{1/2} =$ constant. The dynamics therefore essentially depends on

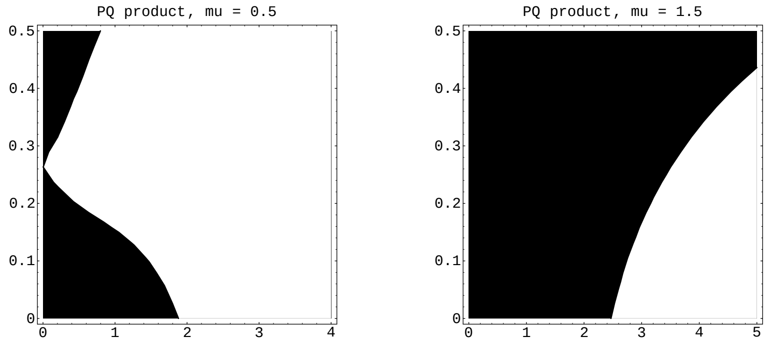

Figure 2: The sign of the product PQ vs. $k = 2\pi\lambda_{eff}/\lambda$ (horizontal axis) and a_i (vertical axis): black (white) regions, i.e. $PQ < 0$ (> 0), denote carrier wave stability (instability) and favor dark- (bright-) type localized excitations. Here $\mu =$: (a) 0.5 (negative dust); (b) 1.5 (positive dust);.

the quantity $\eta = P/Q$, whose sign (magnitude) determine the instability profile and the type (characteristics, respectively) of the localized envelope excitations. A numerical analysis [4] shows that low k (i.e. long wavelength λ) DA waves will be *stable*, while for shorter λ they will be *unstable*. An increase in the non-thermal population (i.e. in the value of a_i) – for *negative* dust charge (see e.g. Fig. 2a) – favors instability at lower k, as well the generation of higher λ *bright* envelope structures [3]. Large wavelengths always remain *stable* and may only propagate as *dark*-type excitations. The opposite qualitative behaviour is witnessed for *positive* dust (see e.g. Fig. 2b), where deviation from thermality appears to favor stability. Even though the approximate expressions for α, α' were used here (for $T_e \ll T_i$), a similar profile is obtained in the general case [4].

5. Conclusions. We have investigated the modulation of dust acoustic waves in the presence of a non-thermal (off-Maxwellian) and have pointed out the influence of the latter on the stability profile, and on the conditions for the occurrence of envelope excitations. An extended account is under way, to be reported soon [4].

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