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Complex plasma as a left-handed medium

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In 1968, Veselago [1] studied theoretically the electrodynamics properties of a medium having both negative dielectric permittivity ε(ω) and magnetic permeability μ(ω) simultaneously and concluded such media must have surprising propagation characteristics in comparison with the conventional ones. These media were called the "left-handed materials" (LHM). In particular, they posses the reversal effects of the Doppler shift and the Cherenkov radiation. Such a medium was manufactured in form of a long line of the three-dimensional array of intersecting thin straight wires. The propagating modes of this line have the dispersion relation analogous to the usual plasma [2]. The anomalous optical properties of the LHM were discussed in [3].

In this report we consider the low frequency dispersion properties of the dusty plasma with ferromagnetic negatively charged granules. We assume that typical sizes of the granules are small enough to treat them as one-domain ferromagnetic particles. All granules possess the same constant magnetic dipole moment d_m and the moment of inertia J. A notation about an experimental study of this type of dusty plasma is given in [4]. The collective properties of the electron-ion plasma with the neutral component consisting of paramagnetic atoms were considered in [5].

In the low frequency domain, the magnetic permeability is a constant and given by the Langevin formula

$$\mu(\omega) = 1 + \frac{4\pi N_g d_m^2}{3T}, \qquad (1)$$

where N_g is a density number of the granules, d_m is a magnetic moment of an individual granule.

In the high frequency domain $\mu(\omega)$ has a real and an imaginary parts;

$$\mathrm{Re}\,\mu(\omega) = 1 - \frac{4\pi N_g d_m^2}{3J\omega^2}, \qquad (2)$$

$$\mathrm{Im}\,\mu(\omega) \sim \exp(-\omega^2/\omega_T^2). \qquad (2a)$$

The imaginary part (2a) relates to a special type of the Landau damping [3] that appears due to the interaction of plasma waves with the so called resonant dipoles whose angular velocities close to the wave frequency. The main peculiarity of this damping is that it does not depend on a wave vector unlike the convention Landau damping [7]. This means that the plasma waves with the frequencies ω close to ω_T will be suppressed.

We consider the case when the energy of magnetic moment in the constant homogeneous magnetic field H_0 is much larger their thermal energy proportional to the system temperature T. Therefore, we may consider the magnetic moments "cold" and oriented practically along the magnetic field. In a field of the external electromagnetic wave with a magnetic component $\vec{H}(\vec{r},t) = \vec{H}\exp[i\vec{k}\vec{r} - i\omega t]$ the dipoles slightly change orientation with respect to z-axis. In a general case, the Lagrange function of an individual magnetic dipole may be written in the form

$$L = \frac{m_g v^2}{2} + \frac{J}{2}(\dot{\theta}^2 + \dot{\varphi}^2) + d_m H_0 \cos\theta + \vec{d}_m \cdot \vec{H}(\vec{r},t) \ , \qquad (3)$$

where m_g is the mass of granule, $\vec{v}$ is a velocity of the translation motion of the mass center of magnetic dipole, spherical polar θ and azimuth φ angels describe an orientation of the vector of magnetic moment, H_0 is the external magnetic field directed along z-axis, $\vec{H}(\vec{r},t)$ is the varying magnetic component of the self-consistent electromagnetic field that is the first order of smallness in comparison with H_0 The equations of motion that follows from (3), we present in the form

$$m_g \dot{\vec{v}} = -\frac{\partial U}{\partial \vec{r}}, \quad U = -\vec{d}_m \cdot \vec{H}(\vec{r},t);$$
$$J\ddot{\theta} + d_m H_0 \sin\theta = -\frac{\partial U}{\partial \theta}, \qquad (4)$$
$$\ddot{\varphi} = -\frac{\partial U}{\partial \varphi}.$$

Keeping in mind that in the spherical system of coordinate θ<<1, we rewrite the potential energy U of the dipole introducing the amplitudes of varying magnetic field

$$U \approx -d_m \{\theta \cos\varphi H_x + \theta \sin\varphi H_y + H_z\} \exp(-i\omega t), \qquad (5)$$

Here we ignore dependence of the magnetic field on r in accordance with the long wave approximation.

The second equation of (4) with account of the inequality θ<<1 is the equation of motion of driven harmonic oscillator with the natural frequency $\omega_0 = \sqrt{d_m H_0 / J}$. From the third equation of (4), one may see that in zero approximation when the varying field is zero the azimuth angle with accuracy to integration constant $\varphi = \Omega t$. The angular frequency Ω is obviously coincides with the thermal frequency $\omega_T = \sqrt{T / J}$. Comparing these frequency, we get $\omega_T / \omega_0 = \sqrt{d_m H_0 / T}$ >>1 that requires the model of the cold ferromagnetic granules.

$$\ddot{\theta} + \omega_0^2 \theta = \frac{d_m}{J}[H_x \cos\varphi + H_y \sin\varphi] \exp(-i\omega t). \qquad (6)$$

The partial solution of this equation that describes forced vibrations could be written as

$$\theta(t) = \frac{d_m}{J} \frac{H_x \cos\varphi + H_y \sin\varphi}{\omega_0^2 - \omega^2} \exp(-i\omega t). \qquad (7)$$

Now we can find the magnetization of a unit volume of the ferromagnetic component by comparing the expression

$$M_i = N_g d_{mi} \qquad (8)$$

Recalling that in our model

$$d_{mx} = d_m \theta \cos\varphi, \ d_{my} = d_m \theta \sin\varphi, \ d_{mz} \cong d_m, \qquad (9)$$

with the help (7), we get the tensor of magnetic susceptibility has the following nonzero components

$$\chi_{xx}(\omega) = \frac{d_m^2 N_g}{2J}\frac{1}{\omega_0^2 - \omega^2},\ \chi_{yy}(\omega) = \frac{d_m^2 N_g}{2J}\frac{1}{\omega_0^2 - \omega^2}. \quad (10)$$

The rest components of this tensor are zeros. A factor ½ in the denominators is a result of averaging $\cos^2\varphi$ and $\sin^2\varphi$. Now we could write down the permeability tensor that emerges in the dusty plasma due to availability of the granules with magnetic dipole moments. It has diagonal form with the components

$$\mu_{xx} = \mu_{yy} \equiv \mu = 1 + \frac{\Omega_m^2}{\omega_0^2 - \omega^2},\ \mu_{zz} = 1;\ \ \Omega_m = \sqrt{\frac{2\pi N_g d_m^2}{J}}. \quad (11)$$

The rest components are equal to zero.

The dispersion law for monochromatic waves propagating in a medium with arbitrary permittivity $\hat{\varepsilon}(\vec{k},\omega)$ and permeability $\hat{\mu}(\vec{\kappa},\omega)$ tensors can be obtained from Maxwell's equations. For electromagnetic fields depending on space-time coordinates according to exp $(i\vec{\kappa}\cdot\vec{r} - i\omega t)$, these give

$$\begin{aligned} &\vec{\kappa}\times\vec{E} = \frac{\omega}{c}\vec{B}, \quad \vec{\kappa}\times\vec{H} = \frac{\omega}{c}\vec{D}, \\ &\vec{k}\cdot\vec{B} = 0, \qquad \vec{k}\cdot D = 0, \\ &\vec{D} = \hat{\varepsilon}\left(\vec{k},\omega\right)\cdot\vec{E}, \ \vec{B} = \vec{H} + 4\pi\vec{M} = \hat{\mu}\left(\vec{k},\omega\right)\cdot\vec{H}. \end{aligned} \quad (12)$$

Here $\vec{E}(\vec{k},\omega)$ and $\vec{H}(\vec{k},\omega)$ are the Fourier transforms of the electromagnetic field. The permeability tensor of our media is given by (11). The permittivity tensor of the dusty plasma in the coordinate system with z-axis along the constant magnetic field H_0 could be presented in the following form [7]

$$\begin{aligned} &\varepsilon_1 = \varepsilon_{xx}(\omega) = \varepsilon_{yy}(\omega) = 1 - \sum_\sigma \frac{\Omega_\sigma^2}{\omega^2 - \omega_{H\sigma}^2},\ \ \varepsilon_3 = \varepsilon_{zz}(\omega) = 1 - \sum_\sigma \frac{\Omega_\sigma^2}{\omega^2}, \\ &\varepsilon_{xy}(\omega) = \varepsilon^*{}_{yx}(\omega) = -i\varepsilon_2 = -i\sum_\sigma \frac{\omega_{H\sigma}}{\omega}\frac{\Omega_\sigma^2}{\omega^2 - \omega_{H\sigma}^2}, \end{aligned} \quad (13)$$

$\Omega_\sigma = \sqrt{\frac{4\pi e_\sigma^2 N_\sigma}{m_\sigma}}$ is the plasma frequency, $\omega_{H\sigma} = \frac{e_\sigma H_0}{m_\sigma c}$ is the cyclotron frequency, σ numerates kinds of charged particles: *e*(electrons), *i* (ions), and *g* (granules).

It will be convenient to introduce the inverse tensor permeability μ_{ij}^{-1} with the help of a relation $\mu_{ik}^{-1}\mu_{kj} = \delta_{ij}$. It follows from (11) that

$$\mu_{xx}^{-1} = \mu_{yy}^{-1} = \frac{1}{\mu},\ \mu_{zz}^{-1} = 1. \quad (14)$$

Eliminating the vector of magnetic induction *B* from the Maxwell equations (12), we obtain the system of linear algebraic equations with respect to components of the electric field vector

$$\begin{aligned} &[\mu\varepsilon_1 - \eta^2\cos^2\theta]E_x - i\varepsilon_2 E_y + \eta^2\sin\theta\cos\theta\, E_z = 0, \\ &i\varepsilon_2 E_x + [\mu\varepsilon_1 - \eta^2(\cos^2\theta + \mu\sin^2\theta]E_y = 0, \\ &\eta^2\sin\theta\cos\theta\, E_x + [\mu\varepsilon_3 - \eta^2\sin^2\theta]E_z = 0, \end{aligned} \quad (15)$$

$\eta = kc/\omega$ is the refraction index, θ is the angle between the wave vector $\vec{k}$ and the magnetic field $\vec{H}_0$. Equating to zero the determinant of the system (15), we get the equation

$$\mu(\omega)\{a\eta^4 + b\eta^2 + c\} = 0, \tag{16}$$
$$a = (\varepsilon_1 \sin^2\theta + \varepsilon_3 \cos^2\theta)[1 + (\mu - 1)\sin^2\theta],$$
$$b = \mu(\omega)(\varepsilon_2^2 - \varepsilon_1^2)\sin^2\theta - \varepsilon_1\varepsilon_3[1 + \cos^2\theta + (\mu - 1)\sin^2\theta],$$
$$c = \mu^2(\omega)[\varepsilon_1^2 - \varepsilon_2^2]\varepsilon_3 .$$

Equation (16) splits into two equations $\mu(\omega)$=0 and $a\eta^4+b\eta^2+c$=0. The first one relates to collective vibration of magnetic dipoles in the plane perpendicular to the constant magnetic field H_0. We will call them the waves magnetization. The dispersion law of these waves is given by the following expression

$$\omega = \sqrt{\frac{d_m H_0}{J}\left(1 + \frac{2\pi N_g d_m}{H_0}\right)} . \tag{17}$$

The second equation describes the dispersion of waves in the dusty plasma with participation of magnetic dipoles. In the case of ferromagnetic granules with anomalous magnitude of magnetic moments their contribution to the dispersion properties of the dusty plasma could be important. However, this problem requires a special study.

Here we would like to note that components of the permeability tensor (11) in a frequency range $\omega > \sqrt{\omega_0^2 + \Omega_m^2}$ are negative. We may hope that at a special set of parameters of the dusty plasma with ferromagnetic granules it is possible to meet conditions for realization of negative dielectric and magnetic permittivity (ε<0 and μ<0) simultaneously.

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