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Wake potential of a moving test charge in a dusty plasma with dynamical grain charging

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Abstract:

A test charge moving through a dusty plasma produces a wake field potential in addition to the usual Debye-Hückel potential. The wake-field excited by a test charge moving through a multicomponent dusty plasma is calculated by using the Vlasov-Poisson model. The potential response is given in the standard way by a multidimensional inverse Fourier transform involving the plasma dielectric function. The physics of the plasma is represented by the choice of the dielectric function. The dynamics of the charging of dust grains is taken into consideration. This leads to an enhanced shielding of a test charge, and since there is a finite charging rate for a dust grain the shielding is delayed. For low test charge velocities the grain charge can, to good approximation, be given using a delay operator and the charging term in the dielectric function is an exponential function of the frequency. At high velocities the charging term behaves as the inverse frequency. The potential response is then the sum of a damped dust acoustic mode and a shielded potential. In general both analytical and numerical methods are needed to analyse the form of the wake field potential.

Introduction:

The physics of dusty plasmas, whose constituents are electrons, ions and charged dust particles, has recently attracted considerable interest and is an active and growing field of plasma physics. In particular, there has been a great deal of interest in the study of wake fields in plasma due to their applications in many phenomena e.g., in the acceleration of particles [1,2] and in the formation of dust particles into regular crystalline structures in dusty plasmas [3,4,5,6]. Dusty plasmas consist of heavy highly charged dust particles immersed in a plasma environment. The particles attain negative charges due to the high flux of plasma electrons. The massive and highly charged dust species introduces a number of new and interesting phenomena into plasma physics.

The theory of the wake potential [7,8,9] has attracted much attention in recent years. For dust Coulomb crystals Takahashi et. al. [10] have experimentally demonstrated the role of the wake potential in a plasma with a finite ion flow. Vladimirov and Nambu [7] first showed that the collective interaction of the static dust particulate with low frequency oscillations in the ion flow in a dusty plasma can provide an attractive oscillatory wake potential along the ion flow direction. Vlamidimirov and Oshihara [11] and Oshihara and Vlamidimirov [12] extended this theory to consider periodic structures along and perpendicular to the ion flow direction.

Nambu [13] and Shukla and Rao [14] explained the wake potential formation in terms of the resonant interaction of the drifting grains and the extremely low frequency dust-acoustic waves in an unmagnetized dusty plasma. They argued that the resonant interaction of the slowly moving or static grains with the extremely low frequency dust-acoustic wave involving the dust grains would be more effective in forming the wake potential.

Recently, Nasim [15] studied wake-field excitations in a multi-component dusty plasma by using the fluid as well as the Vlasov-Poisson model. The form of the wake field was found

to critically depend upon the size of the test charge velocity relative to the dust acoustic speed. More recently, Ali [16] has studied attractive wake field formation due to an array of dipolar projectiles in a multicomponent dusty plasma for modified dust acoustic waves.
In this paper, we study the Debye screening and wake potentials for a fast moving test charge in a dusty plasma with dynamical charging.

Response to a moving test charge:

For a moving test charge q_t in a dusty plasma, the electrostatic response potential is given by

$$\phi = \frac{q_t}{(2\pi)^3 \varepsilon_0} \int \frac{\delta(\omega - KV_t \cos\theta)}{K^2 D(K,\omega)} \exp[i\mathbf{K}\cdot\mathbf{r} - i\omega t] d\mathbf{K} d\omega \qquad (1)$$

where θ is the angle between K and V_t, the test charge velocity, and $D(K,\omega)$ is the plasma dielectric including a dust component is given by the expression

$$D(K,\omega) = 1 + \frac{K_1^2}{K^2} + W\left(\frac{\omega}{KV_{td}}\right) + i\delta(K,\omega) \qquad (2)$$

where in terms of the electron and ion Debye numbers $K_1 = \sqrt{K_e^2 + K_i^2}$. The dynamical charging term $\delta(K,\omega)$ may be written as

$$\delta(K,\omega) = \left(K_{dch}^2 / K^2\right)\nu / (\omega + i\nu) \qquad (3)$$

The charging rate ν and K_{dch} may be found by comparing with the expression given by Melandsø [17]. For low frequencies (for slowly moving test charge) $\delta(K,\omega)$ may be approximated by an exponential function of ω which is the Fourier transform of the delay operator. The effect of the charging dynamics is then equivalent to a delayed shielding [18]. Here we instead assume large V_t so that $\delta(K,\omega)$ is proportional to the inverse of ω. Introducing cylindrical coordinates (ρ, φ,Z) with Z-axis parallel to V_t, equation (1) may be integrated with respect to the parallel component of K and the azimuthal angle to give (in a reference frame moving with the test charge)

$$\phi = \frac{q_t V_t}{(2\pi)^2 \varepsilon_0} \int_{-\infty}^{\infty}\int_0^{\infty} \frac{J_0(K_\perp \rho) \exp\left(i\omega Z / V_t\right)}{\left[\left((\omega / V_t)^2 + K_\perp^2\left(1 - \omega_{pd}^2 / \omega^2\right) + \left(K_1^2 - \omega_{pd}^2 / V_t^2\right) + i\beta\right)\right]} K_\perp dK_\perp d\omega \qquad (4)$$

with $\beta = \left[K_1^2 + \left(\omega / V_t\right)^2\right]\delta$ and where $J_0(K_\perp \rho)$ is the zero order Bessel function. Here we note that the denominator of the integrand is (apart from a division by ω^2) a quadratic in ω^2 giving four poles. The integration over ω can therefore be performed using the calculus of residues. Two of the poles are at the dust acoustic frequency $\omega = \omega_\pm$ including damping. Assuming a long wavelength approximation now gives

$$\omega_\pm = \frac{\pm K_\perp V_{da} V_t}{\sqrt{V_t^2 - V_{da}^2}} - i\frac{K_{dch}^2}{K_1^2}\nu \qquad (5)$$

where $V_{da} = \omega_{pd} / K_1$ is the dust acoustic velocity and a small imaginary part due to charging dynamics is included. The remaining poles are given approximately by

$$\omega = \pm i \frac{\sqrt{V_t^2 - V_{da}{}^2}}{V_{da}} \omega_{pd} \tag{6}$$

neglecting a small contribution from the charging dynamics. They are strongly damped and lead to a Debye shielded potential. Since the poles at $\omega = \omega_{\pm}$ are damped (i.e. imaginary part less than zero) they give no contribution for Z>0. For Z<0 we find

$$\phi = \frac{\sqrt{V_t^2 - V_{da}{}^2}\, q_t}{(2\pi)^2 V_{da} \varepsilon_0} \int_0^{\infty} \frac{J_0(K_\perp \rho)\left(-\dfrac{\exp(i\,\omega_+ Z / V_t) - \exp(i\,\omega_- Z / V_t)}{2i} \right)}{\left[K_\perp^2 + \left(\dfrac{V_t}{V_{da}}\right)^2 \left\{ 1 - \left(\dfrac{V_t}{V_{da}}\right)^2 \right\}^2 K_1^2 \right]} K_\perp^2 dK_\perp \tag{7}$$

We now make an approximate integration with respect to $K_\perp$ for ρ = 0 (on the negative Z-axis) by truncating for large $K_\perp$. The wake potential response is then found to be

$$\phi = \frac{2q_t}{Z\varepsilon_0} \left\{ 1 - \left(V_{da} / V_t \right)^2 \right\} \exp\left(\frac{K_{dch}^2 \nu Z}{K_1^2 V_t} \right) \cos\left(\frac{V_{da}}{\sqrt{V_t^2 + V_{da}^2}} \frac{Z}{\lambda_L} \right) \tag{8}$$

where V_{da} is the dust acoustic velocity and λ_L is given by

$$\lambda_L = \frac{V_t V_{da}}{\left(V_t^2 - V_{da}^2\right)} \frac{1}{K_1} \tag{9}$$

The wake potential given here by equation (8) is similar to one proposed earlier [7,9]. Note that for negative Z the exponential factor corresponds to damping for large Z. The maxima in the oscillating wake potential can trap negatively charged grains.

Discussion:

The wake-field excited by a test charge moving through a multicomponent dusty plasma containing electrons, ions and negatively charged dust particles is calculated by using the Vlasov-Poisson model. The density perturbations of electrons and ions has been assumed as a Boltzmannian type while the dust grains satisfy the Vlasov equation. The dielectric constant incorporating the effect of charging dynamics is studied. By employing this dielectric constant, we have derived the most general expression for the wake potential.
By taking some typical parameters like $q_t = 1$, $V_{da} = 1\cdot 7$, $K_1 = 97$, ν=1.5 and $\lambda = 0\cdot 28$, we numerically solve equation (6) and plotted against Z for a range of different test charge velocities and a range of charging dynamics parameter K_{dch}. It was observed (Fig. 1) that when the test charge velocity is close to dust acoustic velocity (V_{da}), a wake potential behind the test charge in the negative Z-direction is formed. This wake potential increases as we increase the ratio between the test charge velocity and dust acoustic velocity. This wake potential is oscillatory and extends to several oscillations, though with decreasing amplitude. The numerical results for the wake potential with and without the effect of charging dynamics (K_{dch}) are also shown for different K_{dch} values(Figure 2). It was shown that the charging dynamics contributes much to the damping of the oscillatory wake field structure. With an increase in K_{dch} values damping becomes high and vice versa.

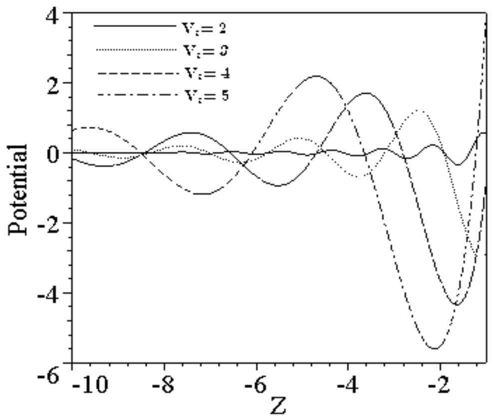

Figure 1: Electrostatic potential (ϕ) versus Z for different test charge velocities and fixed K_{dch}.

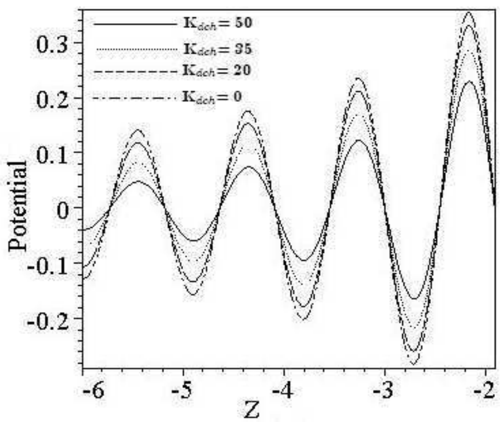

Figure 2: Electrostatic potential (ϕ) versus Z for different K_{dch} values and for fixed V_t.

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