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Potential and Frictional Drag on a Floating Sphere in a Flowing Plasma

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The interaction of an ion-collecting sphere at floating potential with a flowing collisionless plasma is investigated using the “Specialized Coordinate Electrostatic Particle and Thermals In Cell” particle-in-cell code SCEPTIC[1, 2].

Code calculations are given of potential and the total force exerted on the sphere by the flowing plasma. This force is of crucial importance to the problem of dusty plasmas, and the present results are the first for a collisionless plasma to take account of the full self-consistent potential. They reveal discrepancies amounting to as large as 20% with the standard analytic expressions, in parameter regimes where the analytic approximations might have been expected to be more accurate. They also provide definitive values in regimes where no analytic approximation is justified.

SCEPTIC calculates the collisionless ion orbits in 3 dimensions and the self-consistent potential on a spherical mesh having rotational symmetry about the external plasma flow direction, using a Boltzmann factor for the electron density and solving the resulting Poisson equation. The ions are injected on an outer computational boundary in a manner that quite accurately represents a drifting Maxwellian distribution at infinity, and are perfectly absorbed by the spherical surface at the inner computational boundary. Most calculations reported here are made on a 100×100 $(r \times \theta)$ grid, with 7 million particles.

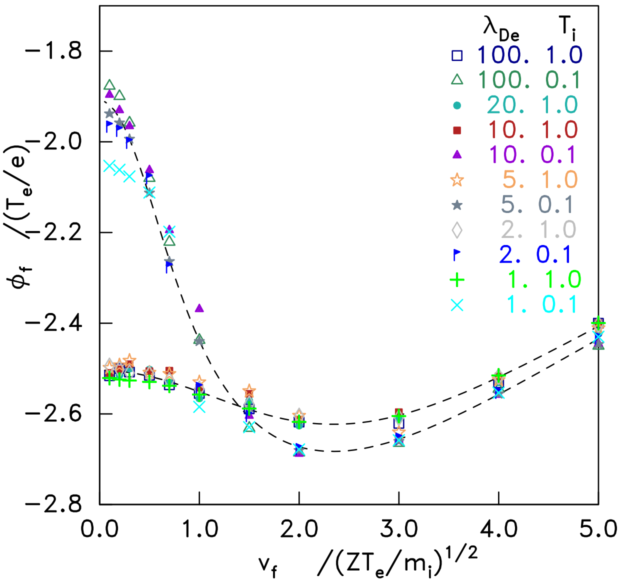

Figure 1: Floating potential calculated by SCEPTIC as a function of normalized drift velocity, for a range of λ_{De} measured in units of the sphere radius, and for $T_i = 1$ and 0.1 times ZT_e. The dashed line shows the OML theory.

In the case with finite flow, a distinction arises between a conducting isolated sphere, which we here call “floating”, and a non-conducting, or “insulating” sphere. The insulating sphere acquires a surface potential that varies with position on the surface so as to make the local current-density zero, whereas the floating sphere is an equipotential, whose value makes the total current zero. SCEPTIC tracks the ion flux (density) to the sphere surface and from it determines the self-consistent floating potential.

The floating potential of a surface which has no charged particle emission is that potential at which the electron collection current density, is equal to the ion collection current density:

$$\frac{1}{4} n_{e\infty} \left(\frac{8T_e}{\pi m_e}\right) \exp(e\phi/T_e) = f\ n_{e\infty} \sqrt{ZT_e/m_i}\ , \tag{1}$$

In the case of a Maxwellian ion distribution drifting with velocity $v_f = U\sqrt{2T_i/m_i}$, and a negatively charged sphere of potential $\phi = -\chi/ZT_i$, an OML value for average ion flux density can be obtained if one approximates the potential as spherically symmetric, yielding [3, 2]

$$f = \sqrt{\frac{2T_i}{ZT_e}} \frac{U}{4} \left\{ \left(1 + \frac{1}{2U^2} + \frac{\chi}{U^2}\right) \mathrm{erf}(U) + \frac{1}{U\sqrt{\pi}} \exp(-U^2) \right\} . \tag{2}$$

The floating potential is then the solution of

$$\phi_f = \frac{T_e}{e} \left(\tfrac{1}{2} \ln |2\pi Z m_e/m_i| + \ln |f| \right) . \tag{3}$$

The first term in the bracket is -2.84 for hydrogen and -4.68 for singly-charged argon; leading to typical floating potentials roughly $2 - 5T_e/e$.

In fig 1 are shown examples of the floating potential for an equipotential sphere in hydrogenic plasma ($m_i = 1837m_e$, $Z = 1$) compared with the values derived from the OML approximation, eqs (2), (3). The agreement is remarkably good, within the code uncertainty of perhaps 2% judged by the scatter, for all but a couple of points near $v_f = 1$, except that at low velocity and temperature, when $\lambda_{De} \sim 1$ the potential is dropping, indicating a gradual breakdown of the OML assumptions there.

The agreement shows that the effects of asymmetry in the potential are virtually negligible in respect of the total ion flux. In itself this is a new and valuable result. Prior multidimensional PIC results [4] treating the electrons, as well as the ions, via particle dynamics (unlike SCEPTIC) had uncertainties too large to validate the OML model even with an artificially low mass ratio ($m_i = 100m_e$).

The charge on the sphere when it is floating is of course mostly a reflection of its floating potential and capacitance. In figure 2 are shown the total sphere charge determined from SCEPTIC for a range of Debye lengths. The charge

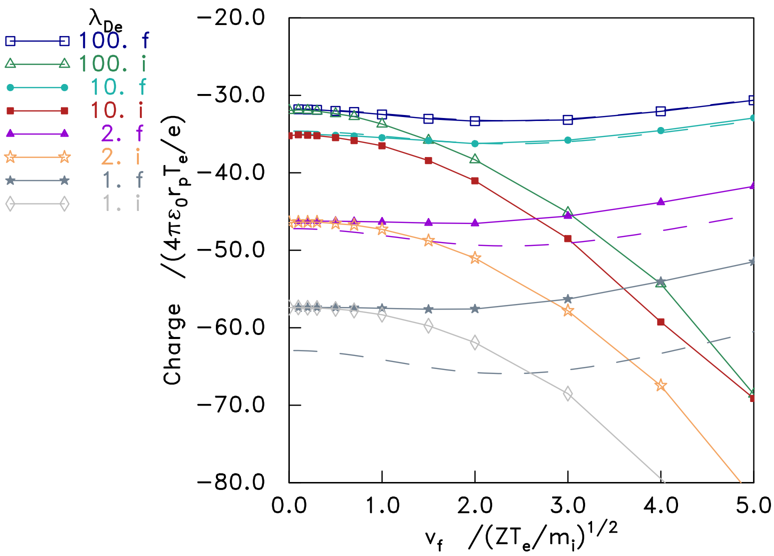

Figure 2: Charge as a function of drift velocity for an isolated sphere in a hydrogenic plasma ($m_i = 1837m_e$, $T_i = ZT_e$). A range of Debye lengths (λ_{De}) is plotted, and both floating (f) and insulating (i) spheres. Dashed lines show the linearized-capacitance analytic approximation.

Q is expressed in normalized units as $Q/(4\pi\epsilon_0 r_p T_e/e)$, where r_p is the sphere radius. The floating equipotential sphere shows little variation in the total charge with flow velocity. In contrast there is a substantial increase of total (negative) charge with flow velocity for an insulating probe. This effect is caused by the strong negative potential that develops on the down-stream side of the sphere for supersonic flow, because the flux on that side is much smaller.

This figure also shows the charge that would be predicted by using the OML potential (which fig 1 shows to be quite accurate) and for capacitance the expression appropriate to the linearized plasma shielding approximation $C = 4\pi\epsilon_0 r_p(1 + 1/\lambda_{De})$ (see e.g. [3]).

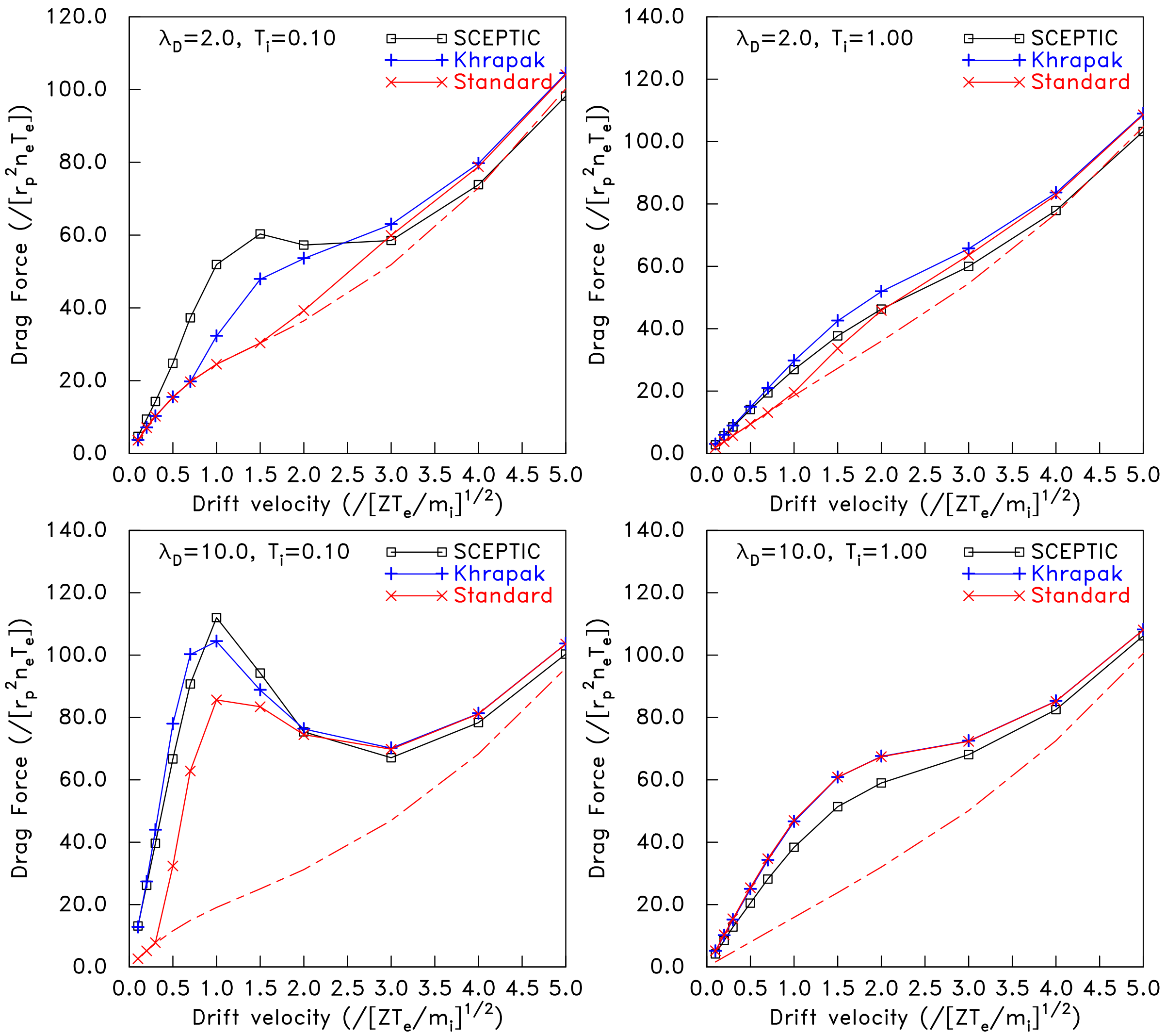

Figure 3: SCEPTIC calculations for $\lambda_{De} = 2$ and 10, on a computational domain of radius 10 and 20 respectively, times the probe radius; with $T_i = 0.1$ and 1.0 ZT_e.

The SCEPTIC code can directly evaluate the drag force on the sphere by considering the momentum flux in three components (1) Ion momentum flux. (2) Electric field forces. (3) Electron pressure. The ion momentum flux is obvious, and in the code is evaluated by summing the momentum of all ions crossing the surface. The electric field forces are expressed in terms of the Maxwell stress tensor which gives the net electric force on all particles inside the surface. This also can be evaluated in the code. The electron pressure is also significant and is evaluated by an appropriate integral of the electron pressure over a spherical surface.

These three contributions can be integrated over any surface surrounding the sphere. A good test of the accuracy and convergence of the code is whether the forces derived on the sphere surface and the outer domain boundary are the same.

In Fig 3 are shown a examples for ion temperatures of 1 and 0.1 (times ZT_e), for floating spheres. The code results are compared with the theoretical drag summing the direct and scattering contributions predicted by the analytic theory's standard [5] and Khrapak [6] forms for $\ln \Lambda$, using charge equal to whatever SCEPTIC determines. The dashed line shows the direct ion collection force.

The agreement is fairly satisfactory. The Khrapak form remains viable to a somewhat lower Debye length than the standard form (all relative to sphere radius). But for the $T_i = 1$ cases there is little difference between the two theory values for cases where the orbital scattering is significant. Both appear somewhat to over-estimate the drag force relative to the fully self-consistent results of SCEPTIC.

The computational results presented here are the first to take account of the full, non-linear, asymmetric, self-consistent problem of collisionless flowing plasma interacting with floating or insulating spheres, at an accuracy that is sufficient for critical comparisons with approximate analytic theory. The results show that the asymmetry in the plasma potential is rather small for most situations and does not have a strong effect on the results. Consequently, the OML approximation, when it is justified by a large value of λ_{De}/r_p, provides a reliable measure of the total ion flux to a floating sphere, and hence its potential. Of course, the OML expression for a drifting ion distribution must be used. The charge on the sphere, however, is not well represented by typical analytic approximations to the capacitance, except when it is close to the vacuum value, because of the plasma non-linearity. The asymmetry in ion flux to the sphere surface has been documented (but not presented here for lack of space) for a wide range of Debye lengths. When the flow is subsonic, it proves not to be greatly different for floating and for insulating spheres. However, for the insulating case, the potential is greatly depressed on the downstream side at high flow-velocities, which substantially increases the negative charge on the sphere.

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