Collisional electron heating by an ultraintense laser pulse

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In this paper, we consider strong high-frequency laser fields such that the laser frequency is higher than the plasma frequency and the electron velocity in a laser field is much higher than the electron thermal velocity. These conditions correspond to experiments with plasmas generated by ultraintense ultrashort laser pulses focused in a gas. The nature of electron–ion (e–i) collisions can be affected by such factors as extremely rapid photoionization of the gas, the periodicity of electron motion in a strong laser field, and small focal spot sizes. Our purpose here is to investigate how these factors influence the collisional electron heating rate.

We consider a fully ionized plasma with the ion density \( N_i \) and electron density \( N_e = zN_i \), where \( ze \) is the ion charge and \( -e \) is the charge of an electron. We assume that, at the initial time, the ion temperature is equal to the temperature of the atoms and that the electrons obey a Maxwellian initial energy distribution with a certain temperature \( T_e \) governed by the amount by which the absorbed photon energy exceeds the ionization energy. Let the plasma be affected by a linearly polarized laser wave field \( E(t) = (E_x, 0, 0) \),

\[
E_x(t) = E_0 \cos \omega t. \tag{1}
\]

In a collisionless plasma, the velocity of an electron and its coordinates are equal to

\[
v(t) = v(0) + v_E \sin \omega t, \quad r(t) = r(0) + v(0) t + r_E \cos \omega t,
\]

where the vectors \( v_E = -eE_0/m_0 \) and \( r_E = eE_0/m_0^2 \) determine the oscillatory velocity of the electrons and the amplitude of their oscillations, respectively, \( E_0 = E(0) \). Due to collisions, the directed electron motion in the external field becomes stochastic. If the electron oscillatory velocity \( V_E \) is much higher than the electron thermal velocity \( V_T = \sqrt{T/m} \) then the e–i collision frequency is equal to [1]

\[
\nu = \frac{16ze^4N_e}{m^2V_E^3} (1 + \ln \frac{V_E}{2V_T}) \ln \frac{k_{\text{max}}}{k_{\text{min}}}. \tag{2}
\]

In this case, the collisional electron heating power is equal to [2]

\[
W = \frac{8ze^4N_eN_i}{mV_E} (1 + \ln \frac{V_E}{2V_T}) \ln \frac{k_{\text{max}}}{k_{\text{min}}}. \tag{3}
\]
According to [1], the values $k_{\text{min}}$ and $k_{\text{max}}$ are determined by the Debye radius and the minimum impact parameter, which is found from the condition under which an electron with the velocity $V_E$ can be described either by classical mechanics or by perturbation theory. The Coulomb logarithm satisfies the equality

$$\Lambda = \ln \frac{k_{\text{max}}}{k_{\text{min}}} = \ln \frac{r_D}{\rho_{\text{min}}}. \quad (4)$$

In expression (2), the logarithm of the ratio of the oscillatory velocity to the thermal velocity stems from the fact that the collision frequency tends to infinity as the directed electron velocity decreases. As a result, the integral over velocities should be truncated at velocities below the thermal velocity. For a circularly polarized laser field, the absolute value of the electron velocity is constant and the logarithm drops out of expression (2) [1].

An analogous formula for the collisional electron heating power in the strong linearly polarized laser wave field was derived by Jones and Lee [2]:

$$W = \frac{8z^2 e^4 N_e N_i}{m V_E} \ln \frac{V_E}{V_T} \ln \frac{k_{\text{max}} V_T}{\omega}, \quad (5)$$

where the value of $k_{\text{max}}$ is determined from the applicability condition of classical mechanics, $k_{\text{max}} = m V_T/\hbar$. In the approximate expression derived by Shlessinger and Write [3] for the collisional electron heating power in a strong linearly polarized laser wave field, the collision frequency is multiplied by the factor that incorporates the difference between the oscillatory and thermal velocities of the electrons:

$$W = \frac{e^2 E_0^2}{2m \omega^2} v, \quad v = v_{ei}(1 + \frac{V_E^2}{3V_T^2})^{-3/2}, \quad v_{ei} = \frac{4\sqrt{2\pi} e^4 N_e}{3\sqrt{mT_{3/2}} \Lambda}, \quad (6)$$

where $\Lambda$ is the conventional Coulomb logarithm [1-5].

In a collisionless plasma irradiated by a circularly polarized laser pulse $E(t) = (E_x, E_y, 0)$, where $E_x(t) = \frac{\sqrt{2}}{2} E_0 \cos \omega t$, $E_y(t) = -\frac{\sqrt{2}}{2} E_0 \sin \omega t$, electrons move at constant speed $V_E/\sqrt{2}$ along circles of radii $r_E/\sqrt{2}$. In the approximation of instantaneous binary collisions, the interaction of electrons with an immobile ion is taken into account, while the electron–electron interaction is neglected. In this case the mean “friction” force exerted by the ions with an impact parameter smaller than $\rho_{\text{max}}$ has the form [4]:

$$F = \frac{4\pi z^2 e^4 N_i}{m V_e^2} \Lambda,$$
where the Coulomb logarithm is equal to
\[ \ln \frac{\sqrt{\rho^2 + \rho_{\text{max}}^2}}{\rho_{\perp}} \approx \ln \frac{\rho_{\text{max}}}{\rho_{\perp}} \]
\[ \rho_{\perp} = \frac{ze^2}{mV^2} \]
is the impact parameter for the right angle. Note that the Coulomb logarithm is not truncated at \( \rho_{\perp} \) because it takes into account all of the e–i collisions with impact parameters \( \rho < \rho_{\text{max}} \).

The heating power is a product of the force, the velocity \( V_E/\sqrt{2} \) and electron density:
\[ W = \frac{4\sqrt{2\pi}z^2 e^4 N_e N_i}{mV_E^2} \Lambda. \] (7)

There exist systems with the Coulomb potential of interaction among the particles but without screening (e.g., systems of gravitating bodies and systems of immobile ions in semiconductors). For a system without screening, the problem of determining the collision frequency was studied by Kogan [5] in the straight-line motion approximation. He found that the Coulomb collision frequency for the particles moving along straight trajectories should be determined by taking the particle mean free path (the length of the straight portions of particle trajectories) as the maximum impact parameter. Accordingly, for a test particle moving at a constant speed along a straight trajectory among immobile charged particles, the dynamic friction force depends logarithmically on the time \( \Delta t \) that has passed after the test particle starts moving [5]:
\[ F = \frac{4\pi z^2 e^4 N_e}{mV^2} \ln \frac{\Delta t}{\tau_{\text{min}}} \]
Here, the time \( \tau_{\text{min}} \) is as usual determined from the applicability condition of perturbation theory. While the particle experiences straight-line motion under the action of an external force, the collision frequency is governed precisely by the straight portions of the particle trajectory, in which case Debye screening has no impact on the Coulomb forces, so that the Debye radius cannot serve as the maximum impact parameter. Consequently, with the particle motion in the strong field (1) taken into account, we determine the range of possible values of the impact parameter in terms of the straight portion of a particle trajectory and the squared electron oscillatory velocity averaged over the period of electron oscillations:
\[ \rho_{\text{max}} = 2r_e = 2eE_0/m\omega^2, \quad \rho_{\text{min}} = 2ze^2/mV_E^2 \] (10)

In this case, the \( \Lambda_E \) depends only on the frequency of field (1) and its strength:
\[ \Lambda_E = \ln \frac{\rho_{\text{max}}}{\rho_{\text{min}}} = \ln \left( \frac{eE_0^3}{z m^2 \omega^4} \right) \] (11)
The collisional electron heating power can be estimated as a product of the dynamic friction force with the electron velocity and density:

\[ W = \frac{16z^2 e^4 N_e N_i}{m V_f} \ln \frac{eE_0^3}{zm^2 \omega^4}. \]  
(12)

Here, the friction force is calculated from the electron oscillatory velocity averaged over the half-period of the laser field: \(<V> = V_f/\sqrt{2} \).

The approximation formula (12) for the collisional electron heating rate in the straight–line motion approximation agrees well with the results from molecular dynamic simulations. The results of computer simulation are compared also with other theoretical models. The new upper limit at which the Coulomb logarithm is proposed to be truncated – the amplitude of electron oscillations in a strong laser field – leads to a new functional dependence of the collisional heating rate on the laser field parameters. The proposed approximate formula applies to laser pulses with arbitrary polarization and does not contain the double logarithm that enters the corresponding formula derived by V.P. Silin for linearly polarized impulse.

We have shown [6, 7] that the gas density fluctuations and the periodic (nondiffusive) nature of collisions of an electron moving in a strong laser field with the ions change the collisional-related parameters of a plasma. This phenomenon is important for case of interaction of laser pulse with gas and case of cluster plasma.

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REFERENCES