Generation of even harmonics in plasma skin layer
induced by a relativistic femtosecond laser pulse

V.S. Rastunkov, V.P. Krainov

Moscow Institute of Physics and Technology,
141700 Dolgoprudny, Moscow Region, Russia

The goal of this work is to consider generation of even and odd harmonics in skin layer produced at the interaction of a super-intense linearly polarized femtosecond laser pulse with a solid matter. Previously the yield of harmonics had been derived for underdense plasmas of multicharged atomic ions produced by a relativistic linearly polarized laser pulse [1]. Harmonics of the laser field are excited at the elastic electron-ion collisions in plasmas. In the non-relativistic case only odd harmonics can be excited. The harmonic conductivity is a tensor with odd components along the laser polarization and with even components along the direction of the laser pulse propagation. Even harmonics produce the alternating electric currents which are directed along the laser propagation vector. However, these harmonics cannot be observed directly, since it follows from Maxwell equations that the corresponding transverse harmonic electromagnetic field vanishes in the wave region (outside of plasma region). In the case of the overdense relativistic plasma both even and odd harmonics of laser radiation are excited due to complex motion of an electron inside the skin layer induced by both the electric and magnetic parts of the Lorentz force [2]. The intensity of harmonics is moderate because of the small electric field inside the skin layer. However, the electromagnetic field of the even harmonics can really propagate along the laser beam direction, unlike the case of underdense plasma. Excitation of harmonics occurs only at the leading edge of the super-intense femtosecond laser pulse since the electron beam leaves quickly the skin layer. Relativistic electron drift along the propagation of laser radiation is produced by a magnetic part of the laser field. It remains after the end of the laser pulse, unlike the relativistic drift of a free electron in underdense plasma. As a result, the penetration depth is much larger than the classical skin depth. The yields of even and odd harmonics are derived for typical values of laser parameters.

High efficiency of transformation of laser energy into radiation of high harmonics can be reached when the laser pulse propagates in plasma channel. Generation of even and odd
harmonics occurs at the reflection of radiation from walls of the channel. This two-dimensional problem was investigated in [3]. One-dimensional problem has been solved in [4].

We suggested solution of electron motion in overdense plasma perturbed by the relativistic laser pulse [2]. Here we consider the possibility of generation of even and odd harmonics inside the overdense plasma taking into account results of our work [2]. The skin layer prevents penetration of the electric field inside the overdense plasma. Here we do not consider the mechanism for the atomic ionization (tunneling ionization, above-barrier ionization or collision ionization). Though the electric field inside the overdense plasma is small, it is important for generation of even harmonics. Just the nonlinear electron collisions with atomic ions in the presence of a laser field are the reason for harmonic generation.

We investigated analytically and numerically equation of classical electron motion in overdense plasma. Further the electric current along the polarization direction is derived as a function of time. Then we find the electric field components on the harmonic frequencies and obtain the harmonic efficiency. An electron leaves skin layer with nonzero drift momentum along the propagation of laser pulse. The reason is that electric and magnetic fields in skin layer are shifted each to other by $\pi/2$.

The main results are:
1. even and odd harmonics are irradiated during the electron motion inside the skin layer produced by relativistic laser pulse;
2. the efficiency of generation of the even harmonics is less than that of the odd harmonics;
3. the generation of all harmonics occurs at the leading edge of the laser pulse, since electron leave skin layer region quickly;
4. efficiency of the harmonic generation decreases with the increasing of the harmonic number.

Dynamics of electrons inside the skin layer can be considered also using the equation of classical motion averaged over the laser period [5]. Then ponderomotive forces appear in right side of the equations which are due to dependence of the Hamiltonian function on time. In this case the relativistic electron kinetic energy depends on the coordinate $x$ along the
propagation of laser pulse, since the electric field strength decreases inside the skin layer. This dependence is just the physical reason for electron drift along the laser pulse propagation. However, in order to derive harmonics, we should use the equation of electron motion without averaging over the laser period.

In our approach we do not take into account the decreasing of the skin layer because of relativistic effects predicted in [5]. The relativistic \( \gamma \)-factor decreases the plasma frequency, while the relativistic ponderomotive force increases it. As a result, the depth of skin layer becomes less due to relativistic effects [5]. However, this effect is determined by the electric field \( F^m \) inside the skin layer which is small compared to the electric field in the incoming laser wave. Though \( F/\omega c \gg 1 \) for relativistic laser pulses, but \( F^m/\omega c < 1 \).

The boundary conditions on the surface plasma – vacuum are of the well known form

\[
F^m = \frac{2}{1 + \sqrt{\varepsilon}} F(t), \quad B^m = \frac{2\sqrt{\varepsilon}}{1 + \sqrt{\varepsilon}} F(t).
\]

Here \( F(t) \) is the amplitude of the electric field strength of the electromagnetic wave in the vacuum, \( \omega \) is its frequency, \( B^m \) is the amplitude of the magnetic field strength inside the plasma, and dielectric constant produced by free electrons is determined by equation

\[
\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}.
\]

(\( |\varepsilon| \gg 1 \)). The laser pulse is turned on and off adiabatically and is approximated by Gaussian envelope \( F(t) = F \exp(-t^2/\tau^2) \), where \( \tau \) is the duration of laser pulse; the condition \( \omega \tau \gg 1 \) is assumed.

Classical equations for the electron relativistic momentum \( p_x \) and \( p_y \) inside the skin layer (\( x > 0 \)) are of the form (the axis \( X \) is directed normal to the target surface, the axis \( Y \) is directed along the field strength vector of the linearly polarized laser pulse):

\[
\frac{dp_x}{dt} = 2v_x F(t) \exp(-x/\delta) \sin(\omega t), \tag{1}
\]

\[
\frac{dp_y}{dt} = \frac{2}{\omega_p} \frac{\omega}{\omega_p} F(t) \exp(-x/\delta) \cos(\omega t) - 2v_y F(t) \exp(-x/\delta) \sin(\omega t), \tag{2}
\]
units $m = c = 1$ are used here, $\delta = 1/\omega_p$ is the depth of the skin layer; $v_x$ and $v_y$ are the component of the relativistic electron velocity, i.e.

$$v_x = \frac{p_x}{\sqrt{1 + p_x^2 + p_y^2}} \quad v_y = \frac{p_y}{\sqrt{1 + p_x^2 + p_y^2}}$$

(3)

The value of $x = 0$ corresponds to the target surface. We do not consider the electron motion along the magnetic field strength. Results do not depend practically on the initial phase of the laser wave, since the laser pulse is turned on adiabatically.

We have found that an electron has relatively high drift velocity along the propagation of the laser pulse and high oscillating velocity along the electric field vector. Therefore electrons do not collide with atomic ions. Equations (1-3) are solved with the initial conditions: an electron is at rest when $t = -\infty$ and has coordinates $x = y = 0$.

In conclusion, we should make the important comments with respect to the observation of even harmonics in experiments. Even harmonics produce the alternating electric currents which are directed along the laser propagation vector. However, these harmonics cannot be observed directly, since it follows from Maxwell equations that the corresponding transverse harmonic electromagnetic field vanishes in the wave region. In order to observe even harmonics experimentally we can take into account also some additional fields in laser plasma. First of all, this is the quasi-static magnetic Weibel field [6]. This field is produced spontaneously by the plasma instability due to inhomogeneous velocity distribution of free electrons ejected in result of field ionization.