Electron viscosity in strong laser field

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The research in collisional dissipation processes play an important part in the problem of laser-plasma interaction. The collisional model of the absorption of laser radiation can work for ultrashort laser pulses only, when the duration of a pulse is shorter than the characteristic time of instabilities. These ultrashort pulses are of great importance in various practical problems (for example, recombination lasers [1]).

Originally the kinetic theory of transport phenomena was advanced for weak fields [2], when the electron velocity in electromagnetic wave \( V = eE/m\omega \) cannot exceed the thermal velocity \( u = \sqrt{T_e/m} \). The analytical treatment of the problem is available for almost equilibrium systems, in which the distribution function is represented as a local equilibrium part and small corrections caused by the factors, which disturb the equilibrium. It is also suggested that the action of these factors results in slow changing of the distribution function in comparison with the time between particle collisions. In the problem of laser-plasma interaction it means that laser frequency \( \omega \) is less than collision frequency \( \nu_{ei} \).

With the advent of high-power lasers the studies of transport phenomena in strong high-frequency fields have become urgent. In this regime electron-ion (e-i) collisions are shown to be suppressed by the factor \((V/u)^3\) [3]

\[
\nu_{ei} = \frac{16\Lambda Z e^4 n_e}{m^2 V^3},
\]

but e-e collisions remain frequent in dipole approximation, when the spatial dependance of the the field of electromagnetic wave is negligible

\[
\nu_{ee} = \frac{4\sqrt{\pi} \Lambda e^4 n_e}{3 \sqrt{m} T_e^{3/2}}.
\]

Due to relatively frequent electron collisions the maxwellian distribution function shifted by the quiver electron velocity may be considered as an equilibrium one. This assumption is valid for sufficiently strong fields \( V \gg Zu \) [4]. The contribution of e-i collisions can be found as a small correction to the equilibrium state, as it was done in the paper [3], which is concerned with the investigation of nonlinear plasma conductivity and inverse bremsstrahlung heating. It was shown that the power of the absorption caused by this mechanism decreases
with the increase of wave amplitude
\[ Q_{ei} = \frac{8\Lambda Z^2 e^4 n_e n_i}{m V}. \] (3)

With regard to other transport processes the kinetic theory of radiation-affected plasma is advanced in the paper [5], where the kinetic equation for the distribution function averaged over the laser period is obtained. In this averaged kinetic equation the harmonics of the distribution function are assumed to be negligibly small. It corresponds to the infinite wavelength approximation. Thus the question about finite wavelength effects remain to be answered. In spite of the smallness of the field inhomogeneity and the magnetic field of the wave, these effects cause the dissipation due to electron viscosity, which can compete with the inverse bremsstrahlung absorption. Contrary to the case of weak fields, in which the viscous dissipation is always relatively low, in strong fields we have to consider the question whether this absorption mechanism can become dominant.

In this case the equilibrium is disturbed by electrons coming from adjacent regions due to the thermal motion. This factor can also be taken into account as a small correction to the equilibrium state. This perturbation method is applicable if e-i collisions are suppressed and e-e collisions remain frequent. So the applicability area of the theory should be limited by the following conditions:
\[ \frac{u}{c} \ll \frac{\nu_{ee}}{\omega} \leq \min\left(\frac{\omega_p}{\omega}, \frac{V}{Zu}\right) > 1. \]
Thus we can develop the theory in case of arbitrary relation between \( \nu_{ee} \) and \( \omega \).

The kinetic equation in strong laser field contains e-e collisions only
\[ \frac{\partial f}{\partial t} + (\vec{v} \nabla) f - \left\{ \frac{e}{m} \left( \vec{E} + \frac{1}{c} [\vec{v} \times \vec{B}] \right) + \Omega [\vec{v} \times \vec{h}] \right\} \frac{\partial f}{\partial \vec{v}} = S_{ee}, \] (4)
where \( \vec{E} \) and \( \vec{B} \) are electric and magnetic fields of the wave, \( \vec{h} \) is a unit vector directed along the constant magnetic field \( \vec{B}_0 \), \( \Omega = eB_0/mc \) is a cyclotron frequency. We consider transversely polarized waves \( (\vec{k} \vec{E} = 0) \) propagating along \( \vec{h} \) direction
\[ \vec{E} = \vec{E}_1 \cos \phi + \vec{E}_2 \sin \phi, \quad \vec{B} = \frac{c}{\omega} [\vec{k} \times \vec{E}], \quad \phi = \omega t - \vec{k} \vec{r}. \]

Making the transform to the frame of reference, which moves with electron velocity \( \vec{V} \)
\[ f(\vec{v}, \vec{r}, t) = F(\vec{u}, \vec{r}, t), \quad \vec{u} = \vec{v} - \vec{V}(\vec{r}, t), \] (5)
and using the equilibrium distribution function in the standard form
\[ F_0 = n(\vec{r}, t) \frac{\theta^{3/2}}{\pi^{3/2}} \exp(-\beta u^2), \quad \beta = \frac{m}{2T(\vec{r}, t)} \]
(n – electron density, T – electron temperature), we obtain the following equation for the correction of the distribution function in the first approximation

\[
\frac{\partial F_1}{\partial t} - I_{ee}(F_1, F_0) - \Omega \left[ \vec{u} \times \vec{h} \right] \frac{\partial F_1}{\partial \vec{u}} = -2\beta F_0 \left( u_\alpha u_\beta - \frac{u^2}{3} \delta_{\alpha\beta} \right) \frac{\partial V_\alpha}{\partial x_\beta}.
\]

(6)

where \( I_{ee}(F_1, F_0) \) is a linearized Landau collision term. To derive the spatial and temporal dependance of the function \( \vec{V}(\vec{r}, t) \) we use the hydrodynamic equation

\[
\frac{\partial \vec{V}}{\partial t} + (\vec{V} \nabla) \vec{V} = -\frac{e}{m} \left( \vec{E} + \frac{1}{c} \left[ \vec{V} \times \vec{B} \right] \right) - \Omega \left[ \vec{V} \times \vec{h} \right].
\]

(7)

Following the standard procedure we seek for the solution in the form of the expansion of \( F_1 \) in terms of Sonin’s polynomials. The modification of this calculation method in our case is in need to solve differential equations for the expansion coefficients instead of usual algebraic equations in case of slowly varying fields. The final result of the calculation for the absorbed power

\[
Q_{ee} = -\left( \pi_{ij} \frac{\partial V_i}{\partial x_j} \right), \quad \pi_{ij} = m \int \left( u_i u_j - \frac{u^2}{3} \delta_{ij} \right) F_1 d^3 u
\]

(8)

is rather cumbersome, that is why further we consider different limiting cases.

In the low frequency limit (\( \omega \ll \Omega, \nu_{ee} \)) the expression of the dissipation power may be reduced to the form

\[
Q_{ee} = 0.36 \ nT \left( \frac{\omega}{\Omega} \right)^2 \frac{k^2 V_0^2}{\nu_{ee}} \propto T^{5/2}
\]

(9)

in the regime of weak magnetic field (\( \Omega \ll \nu_{ee} \)). In strong magnetic field viscosity is suppressed by the factor \( (\Omega/\nu_{ee})^2 \)

\[
Q_{ee} = 0.6 \ nT \left( \frac{\omega}{\Omega} \right)^2 \frac{k^2 V_0^2 \nu_{ee}}{\Omega^2} \propto \frac{1}{\sqrt{T}}
\]

(10)

It resembles the well-known result from the conventional theory of weak fields [2]. The difference between the dissipation of strong and weak electromagnetic waves becomes pronounced in high Z plasmas, since contrary to the case of weak waves the dissipation power of strong waves does not depend on Z due to negligible role of e-i collisions.

If the magnetic field is sufficiently small (\( \Omega \ll \omega, \nu_{ee} \)), one can obtain the result from the previous expressions (9,10) replacing \( \Omega \) to \( \omega \). Since the plasma should be ideal and transparent, we can consider high frequency limit (\( \omega \gg \nu_{ee} \)) only

\[
Q_{ee} = 0.6 \ nT \frac{k^2 V_0^2 \nu_{ee}}{\omega^2} \propto \frac{1}{\sqrt{T}}.
\]

(11)
which is analogous to the regime of viscosity suppression in strong magnetic field.

In the vicinity of the cyclotron resonance ($\omega = \Omega - \delta$) the temperature dependence of the absorbed power can also be approximated by the same regimes $T^{5/2}$ and $T^{-1/2}$. This case is characterized by the fast growth of $Q_{ee}$ even in high frequency limit ($\omega \gg \nu_{ee}$). For $\nu_{ee} \gg \delta$ the dissipation power is

$$Q_{ee} = 0.71 \, nT k^2 V_0^2 \left( \frac{\Omega}{2\delta} \right)^2 \frac{1}{\nu_{ee}} \propto T^{5/2}. \quad (12)$$

This regime is switched to the dependence $T^{-1/2}$, when e-e collision frequency becomes comparable with $\delta$:

$$Q_{ee} = 1.2 \, nT k^2 V_0^2 \left( \frac{\Omega}{2\delta} \right)^2 \frac{\nu_{ee}}{\delta^2} \propto \frac{1}{\sqrt{T}}. \quad (13)$$

The increase by the factor $(\Omega/2\delta)^2$ is connected with the increase of electron velocity near the cyclotron resonance, which causes the increase of the contribution of viscous dissipation and the decrease of the inverse bremsstrahlung mechanism.

Thus in the absence of constant magnetic field the dissipation due to electron viscosity becomes dominant if the following condition is fulfilled

$$\frac{Q_{ee}}{Q_{ei}} = \frac{\sqrt{\pi}}{10Z} \left( \frac{V}{c} \right) \left( \frac{V}{u} \right) \gtrsim 1. \quad (14)$$

In the vicinity of the cyclotron resonance in the limit $\nu_{ee} \ll \delta$ the corresponding condition is

$$\frac{Q_{ee}}{Q_{ei}} = \frac{2}{5 \sqrt{\pi} Z} \left( \frac{V}{c} \right) \left( \frac{V}{u} \right) \left( \frac{\Omega}{\delta} \right)^6 \left( \frac{\omega_p}{\Omega} \right)^2 \gtrsim 1. \quad (15)$$

References


