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Interactive dynamics of two copropagating laser beams in underdense plasmas

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In this paper, the interaction of two co-propagating laser beams with crossed polarization in the underdense plasmas has been investigated analytically with the variational approach and numerically. The coupled envelope equations of the two beams include both the relativistic mass correction and the ponderomotive force effect. It is found that the relativistic effect always plays the role of beam attraction, while the ponderomotive force can play both the beam attraction and beam repulsion, depending upon the beam diameters and their transverse separation. In certain conditions, the two beam centers oscillate transversely around a propagation axis. In this case, the ponderomotive effect can lead to a higher oscillation frequency than that accounting for the relativistic effect only. The interaction of two beams decreases the threshold power for self-focusing of the single beam. A strong self-trapping beam can channel a weak one.

Recently, we have investigated the interaction between two co-propagating laser beams with the same polarization directions in underdense plasmas, and found that two beams can merge into one beam or split into three beams [1]. Ren et al. have observed spiraling of the two beams with crossed polarization directions in particle-in-cell (PIC) simulations [2]. They also have studied this phenomenon analytically, where their coupled beam equations only include the effect of the relativistic mass correction. However, the ponderomotive force effect, which has a significant influence on the self-trapping of a laser beam [3], has been ignored.

The coupled evolution equations for two laser beams with crossed polarization directions in underdense plasmas can be written as [3]

$$2i\frac{\partial a_{1,2}}{\partial \tau} + \nabla_\perp^2 a_{1,2} = \frac{n}{\gamma} a_{1,2}\,, \tag{1}$$

where a_1 and a_2 are the slowly varying vector potentials of the two beams normalized by mc^2/e, respectively, the relativistic factor $\gamma = \sqrt{1+(|a_1|^2+|a_2|^2)/2}$, the electron density $\mathrm{n} = \mathrm{Max}(0, 1+\nabla_\perp^2\gamma)$ is normalized by the unperturbed plasma density n_0. Here $\tau = \omega_p^2 t/\omega$ and $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ with transverse coordinate x and y normalized by c/ω_p, ω is light frequency and $\omega_p = (4\pi n e^2/m_e)$ is the electron plasma frequency. At the weakly relativistic approximation, $n/\gamma \approx 1-(1-\nabla_\perp^2)(|a_1|^2+|a_2|^2)/4$. Substituting n/γ into Eq. (1), we obtain the following coupled nonlinear equations

$$2i\frac{\partial a_{1,2}}{\partial \tau} + \nabla_\perp^2 a_{1,2} - a_{1,2} + \frac{1}{4}a_{1,2}\left(1-\nabla_\perp^2\right)\left(|a_1|^2+|a_2|^2\right) = 0\,. \tag{2}$$

Approximate solutions of Eq. (2) can be obtained by using the variation method [2,4]. Firstly, we need to find a Lagrangian density L_d, where the Euler-Lagrange equations can reproduce Eq. (2) by minimizing the action $\int_{-\infty}^{\infty} L_d\, dxdy$. Such a Lagrangian density is

$$L_d = \sum_{j=1,2}\left[i\left(a_j \frac{\partial a_j^*}{\partial \tau} - a_j^* \frac{\partial a_j}{\partial \tau}\right) + |a_j|^2 - \frac{1}{8}|a_j|^4 + (1+\frac{1}{4}|a_j|^2)\nabla_\perp a_j \cdot \nabla_\perp a_j^*\right.$$

$$+\frac{1}{8}|a_j|^2\,(a_j\nabla_\perp^2 a_j^* + a_j^*\nabla_\perp^2 a_j)\Bigg] + L_{12} \qquad (3)$$

where a_j^* is the complex conjugate, $L_{12} = -\frac{1}{4}\left(|a_1|^2|a_2|^2 + \nabla_\perp|a_1|^2\cdot\nabla_\perp|a_2|^2\right)$. We use the trial functions

$a_j = a_{0j}\exp(-i\phi_j)\times\exp\{-i[k_{xj}(x-X_{cj}) + k_{yj}(y-Y_{cj})]\}\exp\{[(x-X_{cj})^2 + (y-Y_{cj})^2](i/2R_j - 1/W_j^2)\}$

as in Ref. [2], where the amplitude a_{0j}, phase ϕ_{0j}, beam center (X_{cj}, Y_{cj}), perpendicular momentum (k_{xj}, k_{yj}), radius of curvature R_j, and the spot size W_j are all real and are functions of τ only. Substituting the trial functions into Eq. (3) and integrating the Lagrangian density L_d in the xy plane, we obtain the reduced Lagrangian density $L \equiv \frac{2}{\pi}\int_{-\infty}^{\infty} L_d\,dxdy$. One can find the evolution equations of the beam parameters by Euler-Lagrange equations for the reduced Lagrangian density L : $\partial L/\partial\beta - (d/d\tau)\partial L/\partial\dot{\beta} + (d^2/d\tau^2)\partial L/\partial\ddot{\beta} = 0$, where β is any variational parameter for the laser beams.

Varying ϕ_j, leads to the power conservation $d(a_{0j}^2 W_j^2)/d\tau = 0$. Varying R_j, one obtains $W_j/R_j = dW_j/d\tau$. By varying W_j, the equations for the evolution of each beam spot size are given as

$$\frac{d^2W_j}{d\tau^2} + \frac{4}{W_j^3}\left(\frac{P_j}{32} - 1\right) + \frac{P_j}{W_j^5}$$
$$= \frac{P_1P_2}{2(W_1^2+W_2^2)^2}\frac{W_j}{P_j}\exp\left(\frac{-2d^2}{W_1^2+W_2^2}\right)\left(-1 + \frac{2d^2}{W_1^2+W_2^2}\right) + \frac{8P_1P_2}{(W_1^2+W_2^2)^3} \qquad (4)$$
$$\frac{W_j}{P_j}\exp\left(\frac{-2d^2}{W_1^2+W_2^2}\right)\{(-1 + \frac{d^2}{W_1^2+W_2^2})[1 + 2(\alpha_X - X_{c1})(\alpha_X - X_{c2}) + 2(\alpha_y - Y_{c1})(\alpha_y - Y_{c2})]$$
$$+ [(2\alpha_X - X_{c1} - X_{c2})(\alpha_X + X_{c\bar{j}}) + (2\alpha_Y - Y_{c1} - Y_{c2})(\alpha_Y + Y_{c\bar{j}})]\},$$

where $\alpha_X = (X_{c1}W_1^2 + X_{c2}W_2^2)/(W_1^2 + W_2^2)$, $\alpha_Y = (Y_{c1}W_1^2 + Y_{c2}W_2^2)/(W_1^2 + W_2^2)$, the distance between the centers of the two beams is $d = \sqrt{(X_{c1} - X_{c2})^2 + (Y_{c1} - Y_{c2})^2}$, $\bar{j} = \begin{cases} 1, & j = 2 \\ 2, & j = 1 \end{cases}$. The right hand side of Eq. (4) shows the effect of the beam interaction through the relativistic effect and the ponderomotive force. From Eq. (4), neglecting the right hand side, one obtains that the normalized threshold power of relativistic self-focusing for individual beams is reached when $P_j \geqslant P_c \equiv 32$. On the other hand, let us consider the propagation of a weak beam two in the background of another powerful beam one with $P_2 \ll P_1$ and $P_2 \ll 32$. If the two beams propagate in co-axis, so that $X_{cj} = Y_{cj} = 0$, then one finds that beam two can be guided without diffraction through relativistic effect if the power of the beam one $P_1 \geqslant 8(1 + W_1^2/W_2^2)^2$, which can be less than 32 provided $W_1 < W_2$.

By varying (k_{xj}, k_{yj}), it gives $k_{xj} + dX_{cj}/d\tau = 0$, and $k_{yj} + dY_{cj}/d\tau = 0$. By varying X_{cj}, one obtains

$$P_1\frac{d^2X_{c1}}{d\tau^2} = \frac{-2P_1P_2(X_{c1} - X_{c2})}{(W_1^2+W_2^2)^2}\exp\left(\frac{-2d^2}{W_1^2+W_2^2}\right)\{\frac{1}{4} + \frac{2}{W_1^2+W_2^2}$$
$$+ \frac{4}{W_1^2+W_2^2}[(\alpha_X - X_{c1})(\alpha_X - X_{c2}) + (\alpha_y - Y_{c1})(\alpha_y - Y_{c2})]\}$$
$$+ \frac{2P_1P_2}{(W_1^2+W_2^2)^3}\exp\left(\frac{-2d^2}{W_1^2+W_2^2}\right)[\alpha_X(W_2^2 - W_1^2) + W_1^2X_{c2} - W_2^2X_{c1}],$$

$$P_2 \frac{d^2 X_{c2}}{d\tau^2} = -P_1 \frac{d^2 X_{c1}}{d\tau^2}. \quad (5)$$

Similar equations hold in the y direction. Eq. (5) shows that the beam centroids move like two particles with the mass proportional to their powers. Momentum conservation $P_1 \dot{X}_{c1} + P_2 \dot{X}_{c2} =$ constant can be obtained from Eq.(5). In a simplified case: $P_1=P_2=P$ and $W_1=W_2=W$, the motion equations of the two beams become

$$\frac{d^2 \Delta X_c}{d\tau^2} = \frac{-P}{W^4} \Delta X_c \exp(-\frac{d^2}{W^2})(\frac{3}{4} + \frac{1}{W^2} - \frac{d^2}{2W^2}) \quad (6)$$

where $\triangle X_c = X_{c1} - X_{c2}$. Similar equations hold in the y direction. As compared with what obtained in Ref. [2], new terms at the right hand side of Eq. (6) appear owing to the introduction of the ponderomotive force effect in Eq.(2). In the factor $(3/4+1/W^2-d^2/2W^2)$, the relativistic effect contributes only 1/4 and the ponderomotive force contributes $(1/2+1/W^2-d^2/2W^2)$. This suggests that the relativistic effect always contributes to the beam attraction. While the ponderomotive force can play the role for beam attraction only for a transverse separation between beam centers $d<(2+W^2)^{1/2}$, beyond which it plays the role for beam repulsion. Physically, the ponderomotive repulsion is caused by the density increase in space between two beams, which leads to a low refractive index there. The beam repulsion can overcome the relativistic beam attraction when $d>(2+3W^2/2)^{1/2}$.

To account for more general cases, we solve the coupled equations (1) numerically. A rectangular simulation box is used in the x-y plane. The input beams are launched parallel to each other along the z-direction, and without initial perpendicular momentums. The transverse beam profiles are in Gaussian with $a_i=a_{0j}\exp\{-[(x-X_{0j})^2+y^2]/W_{0j}^2\}$. In the whole simulation processes, energy center $\langle x \rangle_j$ of the two beams are tracked, where $\langle x \rangle_j = I_{tot}^{-1} \int_{-\infty}^{\infty} x\,|a(x,y)|^2\, dxdy$, $I_{tot} = \int_{-\infty}^{\infty} |a(x,y)|^2\, dxdy$.

Figure 1 illustrates the evolution of the two beams when $a_{01}=a_{02}=0.15$, $W_{01}=W_{02}=20\sqrt{2}$, $X_{01}=-X_{02}=12$. Even though the corresponding power for single beam is P=18, much less than P_c, the two beams are still trapped while propagating. As Fig. 1 shows, the two beams attract, intersect, and separate, like a damped oscillation with an increasing oscillation frequency. Based on the assumption that both beams always have constant $W=20\sqrt{2}$, from Eq. (5), one obtains that the two Gaussian beams have a non-damping and sinusoidal oscillation, as shown with the dashed line in Fig. 1. The oscillatory period is T=2244. The dotted line in Fig. 1 show theory prediction without ponderomitive effect, and the corresponding oscillatory period is T=3098. The theory better agrees with the simulation when including ponderomitive effect. The two beam width decreases to 10 and beam amplitudes increase to 0.39 in τ=2000. Eq. (5) also predict that oscillatory frequency will increase when the beam amplitudes increase and their distance decreases. For $a_{1,2}=0.39$, $W_{1,2}=10$, d=4.5, the oscillatory

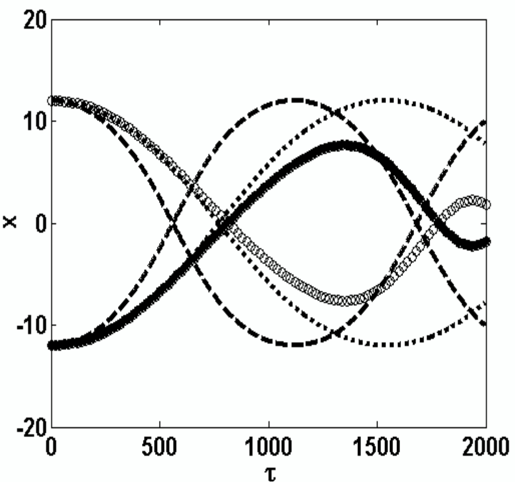

Fig. 1. Evolution of two beams with initial parameters: $a_{01}=a_{02}=0.15$, $W_{01}=W_{02}=20\sqrt{2}$, $X_{01}=-X_{02}=12$. circles: energy center of the beams from simulations, dashed line: theory including both the poderomotive and relativistic effects, dotted line: theory only including the relativistic effect.

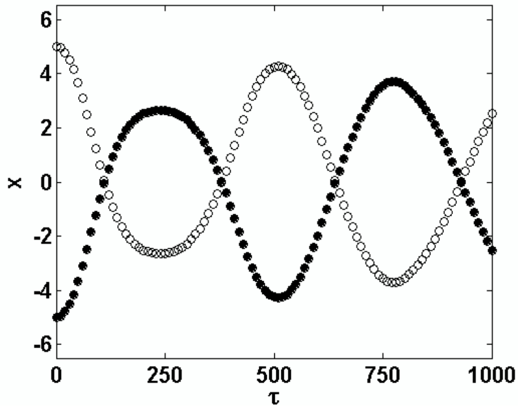

Fig. 2. Evolution of two beams with initial parameters: $a_{01}=a_{02}=0.5$, $W_{01}=W_{02}=6.35\sqrt{2}$, $X_{01}=-X_{02}=5$.

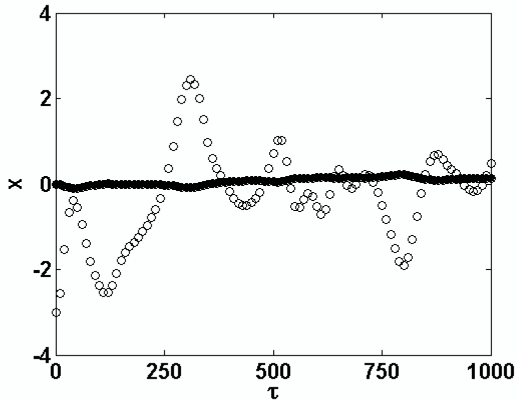

Fig. 3. Evolution of two beams with initial parameters: $a_{01}=1$, $a_{02}=0.2$, $W_{01}=W_{02}=3.9\sqrt{2}$, $X_{01}=0$, $X_{02}=-3$.

period T=207, which is consistent with the accelerated oscillation in $\in[1750,2000]$.

Figure 2 illustrates another example for beam evolution when $a_{01}=a_{02}=0.5$, $W_{01}=W_{02}=6.35\sqrt{2}$, and $X_{01}=-X_{02}=5$. The beam widths are almost constant in the whole process. The power of one beam is 20.2. It is clear to see that the two beams have a oscillatory motion and the oscillation period is about 514. Our analytical solution from Eq. (6) predicts a oscillation period of T=349. This large difference is probably due to the weakly relativistic approximation and the ideal Gaussian beam used in the above variational approach.

Figure 3 displays the interaction between the two beams when $a_{01}=1$, $a_{02}=0.2$, $W_{01}=W_{02}=3.9\sqrt{2}$, $X_{01}=0$, $X_{02}=-3$. Note that $P_2=1.2$ is much less than $P_1=30.4$. The large power means large mass when one takes an analogy between the laser beams and particles. It is expected that the beam with a small power will twist along the beam with a large power. As shown in Fig. 3, the beam one is almost transversely immobile, but the beam two oscillate around beam one, with a period of about 200. Meanwhile, the beam two remains to be trapped without significant spreading even though its power is much lower than the self-focusing threshold. This is also due to the focusing effect of the beam one as also discussed analytically following Eq. (4).

In summary, the interaction of two co-propagating laser beams with crossed polarization in the underdense plasma has been investigated analytically and numerically. It is found analytically that the relativistic effect always plays the role of beam attraction, while the ponderomotive force can play both the beam attraction and beam repulsion, depending upon the beam diameters and their center separation. In certain conditions, the two beam centers oscillate transversely around a propagation axis. In this case, the ponderomotive effect can lead to a higher oscillation frequency than that accounting for the relativistic effect only. The interaction between two beams decreases the threshold power for self-focusing of the single beam. A strong self-trapping beam can channel a weak one.

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