Relativistic Electron Dynamics in Strongly-Nonlinear Regime of Laser-Plasma Interaction

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The laser-plasma interaction is the basic phenomenon for the new generation of the compact accelerators [1] and radiation sources [2], advanced fusion concepts [3]. One of the promising accelerator schemes is the electron acceleration in the plasma wave generated by the laser pulse [4]. An external electron bunch trapped in the plasma wave can be accelerated up to high energy. Background plasma electrons can also trapped and accelerated due to the wavebreaking [5]. However the energetic spectrum of self-generated is very broad that is not good for accelerator applications. Diffraction of the laser pulse is also limited the interaction length and the energy of the accelerated electrons. The plasma wave generation in the plasma channel is suggested in order to increase the interaction length the plasma channel [6].

Recently a new, highly-nonlinear regime of laser-plasma interaction has been observed in fully three-dimensional (3D) particle-in-cell (PIC) simulation [7]. The main features of the bubble regime are the following: (i) a cavity free from cold plasma electrons is formed behind the laser pulse instead of a periodic plasma wave; (ii) a dense bunch of relativistic electrons with a monoenergetic spectrum is self-generated; (iii) the laser pulse propagates many Rayleigh lengths in the homogeneous plasma without a significant spreading. These features are absent in the common regime of laser wake field acceleration [6].

We perform a numerical 3D PIC simulation of laser-plasma interaction in this regime. For the simulations, we use the fully electromagnetic 3D PIC code Virtual Laser-Plasma Laboratory [8]. The incident laser pulse is circularly polarized, has the Gaussian envelope $a(t, r) = A_0 \exp\left(-r^2/L^2 - t^2/T^2\right)$, and the wavelength $\lambda = 0.82 \mu$m. The parameters of the laser pulse are $r_L = 10\lambda$, $cT_L = 4\lambda$, $a_0 = 10mc^2$. The pulse propagates in a plasma with the density $n_0 = 6.1 \times 10^{-3}n_c$, where $n_c = (m\omega^2/4\pi e^2)^{1/2}$ is the critical density.

The plasma density distribution observed in the simulation is shown in Fig. 1 at two instants of time: (a) when the laser pulse has passed $l_{int} = 25c/\omega_p \simeq 50\lambda$ and (b) $l_{int} = 442c/\omega_p \simeq 900\lambda$ in plasma. These density distributions are very typical for the
bubble regime. It is seen from Fig. 1 that the wake behind the laser pulse takes the form of a solitary cavity, which is free from plasma electrons. The cavity is surrounded by a high density sheath of the compressed electron fluid. At later times, Fig. 1(b), a beam of accelerated electrons grows from the bubble base. Simultaneously, the bubble size increases.

First we are interested in the question what are the fields inside a spherical electron cavity moving in plasma. This cavity is similar to the hole in semiconductor physics. The ions are immobile in the cavity while the cavity runs with the relativistic velocity \( v_0 \simeq 1 \) along \( x \)-axis. Making of use the following convenient gauge

\[
A_x = -\varphi. \tag{1}
\]

and quasistatic approximation assuming that all quantities depend on \( \zeta = x - v_0 t \) instead of \( x \) and \( t \) we find from Maxwell’s equations the electromagnetic fields inside the spherical cavity

\[
E_x = \frac{\xi}{2}, \quad E_y = -B_z = \frac{y}{4}, \\
B_x = 0, \quad E_z = B_y = \frac{z}{4}. \tag{2}
\]

The calculated distribution of electromagnetic fields is close to the one observed in the 3D PIC simulation.

We assume that the laser field is circularly polarized and the azimuthal motion of plasma electrons is neglected. For simplicity we assume that the electron trajectory
lies in the plane $z = 0$. The group velocity of the laser pulse, $v_0$, is assumed to be close to the speed of light so that $\gamma_0^{-2} = 1 - v_0^2 \ll 1$. The laser pulse propagates along the $x$-axis. The averaged motion of an electron in the laser field and in the slowly varying electromagnetic fields of the bubble is defined by the averaged Hamiltonian

$$H = \sqrt{1 + (P + A)^2 + a^2 + \varphi}, \quad (3)$$

where $P$ is the canonical momentum of the electron, $a$ is the vector potential of the laser field, $A$ and $\varphi$ are the slowly varying vector and scalar potentials, respectively. In this description the fast electron oscillations in the laser field are averaged out and only the ponderomotive force remains.

We change variables in the Hamiltonian (3) from $x$ and $P_x$ to $\xi = x - v_0 t$ and $P_\xi = P_x$ by a canonical transformation with the generating function $S = (x - v_0 t) P_\xi$. The Hamiltonian in the new variables takes the form

$$H = \gamma - v_0 P_x - \varphi = \sqrt{1 + (P + A)^2 + a^2} - v_0 P_x - \varphi. \quad (4)$$

The Hamiltonian (4) is the integral of motion if we neglect the difference in velocity between the bunch and the cavity, where $v_b \approx 1 - 1/2 \gamma_b^2$ is the bunch velocity.

The necessary condition for the electron trapping in the cavity is the existence of the point of return where $d\xi/dt = 0$. It follows from Hamilton’s equations that at this point

$$p_x = v_0 \gamma. \quad (5)$$

The integral of motion (4) can be rewritten as follows

$$H = \gamma - v_0 p_x - \Phi = 0, \quad (6)$$

where the initial conditions $p = A_\perp = a = 0$ and $\Phi = 1$ are applied. Here we use the wake field potential $\Phi = A_x - \varphi$ instead of the scalar one.

The relations (5) and (6) can be expressed at the return point in the form

$$p_x = v_0 \gamma_0 \gamma_\perp = v_0 \gamma_0^2 \Phi, \quad (7)$$

where $\gamma_\perp^2 = 1 + p_y^2 + a^2$. The domain in the phase space, where the electron is trapped, can be defined as

$$p_x \geq v_0 \gamma_0 \gamma_\perp = v_0 \gamma_0^2 \Phi. \quad (8)$$

Eq. (7) gives the boundary of the domain. Integrating numerically Hamilton’s equations for spherical plasma cavity we find that the cavity can trap electrons, which have been initially at rest, if $R > \gamma_0$ (see Fig. 2a).
Figure 2: Electron trajectories in the plane $z = 0$ calculated by numerical integration of Hamilton’s equations for $\gamma_0 = 9$ and for spherical plasma cavity with $R = 10$. The coordinates are given in $c/\omega_p$.

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References