Quark-Gluon Plasma and Transition to Hadrons at Nucleation in
Astrophysics Plasmas at Boltzmann Equilibrium

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One key problem in astrophysics is to explain how the elements above iron are created. Based on the key role in the transition of the Fermi statistics for hadrons changing from the relativistic branch into the subrelativistic branch the stable configuration for the nuclei just at the well-known density of nuclei was calculated. If the transition of higher than nuclear density at the big bang or at expansion of neutron stars e.g. in a supernova appears under Boltzmann equilibrium, the nucleation including the heavy elements leads to a relation of the magic numbers explaining the transition in the Bagge series. An understanding is derived why then only the elements up to about uranium appear and that the elements above curium are typical manmade non-equilibrium creations. This is based on the experimental facts on the standard abundance distribution (SAD) in the Universe, and the Boltzmann relation

1. Introduction

This paper uses the theory of classical plasmas, especially the importance of the Debye thickness of the surface or interface of plasmas and the subsequent surface tension and surface energy for a generalization to the theory of nuclei where the changed interpretation of the temperature as Fermi energy is essential. This interpretation was successful to generalize the plasma surface tension theory to the degenerate electrons in a metal reproducing the measured surface tension of metals [1]. Following this success, the use of the hadron Fermi energy of protons and neutrons is leading to the thickness of the surface range of nuclei as a quasi-Debye length and to the nuclear force as the result of this density decay where the surface energy, compensates the Fermi energy of the nucleons of the whole nucleus just arriving at the well known density of nuclei [2]. One consequence is the generation to the quark gluon plasma at higher densities without surface property effects, and as a consequence, the nucleation up to elements like uranium when the quark gluon plasma expands to the well known nuclear densities [3]. This explains the generation of the heavy nuclei in the universe due to a Boltzmann equilibrium [4].

The Boltzmann equilibrium uses the exponential increment form the observed standard abundance distribution of the elements in the universe [5] and derives ratios for the magic numbers of nuclei with a direct explanation of the jump between the two Bagge sequences [3,6] without needing the interpretation of spin and spin-orbit coupling of the theory of Jensen and Maria Goeppert-Mayer [7] leading to a quark property of the nuclear structure [3]. This in retrospect may be used for revealing the very extensively studied spin-orbit coupling.
2. Surface tension in laser ablated plasmas

The study of the expansion of a laser produced plasma into vacuum led to recognize the Debye length as the thickness of the double layer at the plasma surface (see Fig. 2.2 of Ref. 8). One of the first observed anomalies at laser interaction with plasmas produced by irradiation of targets was the fact of nonlinearity [8-10]. For laser powers below about one megawatt (MW), the plasma generated behaved classically when the temperature was about 5 eV (corresponding 50,000 K). The emitted ions had similar maximum energies of the expected thermalised plasma, and the electron emission followed fully the thermionic Richardson laws with emission current densities of few mA/cm$^2$. When the Q-switched laser provided smooth and reproducible pulses of about 10 ns duration and powers of 10 MW and more, Linlor (see [8,9]) measured up to 10 keV energy of ions $E_i$ from irradiated carbon. He found that the ions were non-thermally separated by their charge number $Z$ linearly increasing according $E_i = \text{const} \times Z$. The $Z$-separation was first assumed to be due to the ambipolar fields. But these fields could only explain a number of about $10^8$ ions per interaction from the double layer of the plasma surface, while the measured number of ions was $10^{13}$ and more. The non-thermal effects were soon recognized by applying the dielectric modified nonlinear force (of which a simplified case is the ponderomotive force) [8-10] to explain production of the 10 keV ion energies after the laser beam underwent ponderomotive self-focusing [11] at a threshold just around the observed MW threshold.

The double layer at the plasma surface of Debye length $\lambda_d$ thickness contained electrostatic field energy, which counted per surface area, resulted in a surface tension

$$\sigma_v = 0.27 \frac{T^2}{(8\pi \varepsilon^2 \lambda_d)}$$  \hspace{1cm} (1)

of the fully ionized plasma as seen also from the smooth plasma plums expanding form a laser-irradiated target. Wehne instead of the temperature $T$ in (1) and in the Debye length, the Fermi energy of the degenerate electrons in a metal was used [1], the double layer at a metal surface equal to the work function was derived leading to a straightforward quantum theory of the surface tension of metals in agreement with the observations.

We consider the ensemble of a number of protons and neutrons and how to squeeze them into a spherical volume with a density with a nucleon density $n$. Obviously there are strong Coulomb forces trying to drive the protons apart. Another force against confinement of the nucleons is the quantum pressure expressed by the Fermi energy when locating a particle into a volume $V$ that from the quantum relation with the necessary momentum corresponding to an energy $E_F$ increases with another exponent for smaller and smaller $V$ than that for the Coulomb repulsion. The Fermi and the Coulomb energy are equal at a radius of 285 fm [4], such that for smaller radii the Fermi energy is the dominating part of the internal energy of the nuclei. Looking into cases of small radii, we can then consider the Coulomb forces and other components as small perturbations, which are discussed later.
3. **Surface energy of Nuclei**

The following success will justify us to compare the Fermi energy of the proton and neutron ensemble with the surface energy given from a surface tension using a Debye length given by the Fermi energy of the nucleons. The Fermi energy splits into the branches

\[
E_F = \begin{cases} 
\left[ \frac{3}{(\pi)^{2/3}/4}\right] \frac{h^2 n^{2/3}}{(2m)} & \text{(subrelativistic)} \\
\left[ \frac{3}{(\pi)^{2/3}/4}\right] \frac{hc n^{1/3}}{(2\pi)} & \text{(relativistic)} 
\end{cases}
\]

(2a)

(2b)

using \(\lambda_C\) the Compton wavelength \(\frac{h}{(2\pi mc)}\) with “2\(\pi\)” which option is just ascertained by the following treatment modifying the preceding work [2]. The surface energy of the nucleus [2] is then

\[
E_{\text{surf}} = 0.27 \left[ 3A(4\pi)^{1/2} \right]^{2/3} A^{1/3} E_F^{2/3} / (\pi^{1/2} 2^{5/2} n^{1/6} e) \quad (3)
\]

For comparison between the surface energy and the internal energy we have

\[
E_{\text{surf}}/(AE_F) = \begin{cases} 
0.27 \left( \frac{3^{3/2}/2^{10/3}}{(2^{7/3} \alpha^{1/2} A^{1/3})} \right) & \text{(subrelativistic)} \\
0.27 \left[ \frac{3^{8/3}}{(2^{7/3} \alpha^{1/2} A^{1/3})} \right] & \text{(relativistic)} 
\end{cases}
\]

(4a)

(4b)

using the fine structure constant \(\alpha = e^2/2\pi \hbar c\). From (4a) we see that the nucleus cannot be confined for too low a density. The nucleus is stable only when the density reaches a value of the density \(n_n\) where the ratio (4a) is equal to one. This is the case at the well-known value of the nuclear density as checked e.g. for bismuth [2]. The surface “Debye”-layer has a thickness of about 2 to 3 fm, just the measured Hofstadter decay of the surface charge of heavy nuclei.

At relativistic densities just above that of the subrelativistic case reproducing the well known density of nuclei, we see that the value

\[
E_{\text{surf}}/(AE_F) = 6.28/A^{1/3} \quad (5)
\]

This equation does not does not longer depend on the nucleon mass or on the density. We have then no nucleation by the surface energy and a soup of matter. Even the independence of the mass shows that we can either have hadrons (as assumed in neutron stars) or quark-gluon plasma. Only when this dense matter is expanding at the big bang or from a neutron star in a supernova, when reaching the nuclear density, the surface energy will produce the nucleation. The numerical factor in (5) may mean that higher values for \(A\) than 247 are not possible. This may just explain, why the nucleation by expansion of a quark-gluon plasma at higher than nuclear density from the relativistic branch of the nucleon Fermi energy to the lower nuclear density can produce elements only up to uranium of up to curium at the most within such equilibrium processes.
Fig.1. Measured standard abundance distribution of the elements (SAD) in the Universe where the line follows the exponential Boltzmann dependence of Eq. (6) with $Z' = 10$.

Higher trans-uranium nuclei by heavy ion collisions as an extremely non-equilibrium process are then really manmade, but following the rule of magic numbers with the Bagge sequences [3,4] as a Boltmann equilibrium derived from a fit of the astrophysics observations of Fig. 1. and discussed before [3]

$$N(Z) = N' \exp(Z/Z')$$  \hspace{1cm} (6)