Dynamics of dust particle motion in tokamak edge plasmas

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Introduction. The presence of substantial amounts of dust has been observed on the first walls of fusion devices (see Refs. 1 and the references therein). Although, the impact of dust on plasma parameters in current fusion devices is not clear [2], dust in burning plasma experiment may cause a significant safety threat. Here we consider some aspects of dust dynamics in tokamak edge plasmas and discuss an impact of dust on core plasma contamination with impurity. Following [3] we assume that dust density is rather small and ignore collective phenomena associated with the dust [4].

Forces and dust dynamics in the vicinity of smooth surface. In tokamak edge plasma the dust particle is usually negatively charged, $-eZ_d$, where Z_d can be estimated as $Z_d = \Lambda T r_d / e^2$ where r_d is the radius of dust particle and Λ is the numerical coefficient which in our estimates we will take $\Lambda \approx 3$. We assume that dust particle has no internal structure and its motion is determined by the momentum balance equation

$$d\mathbf{V}_d / dt = \mathbf{F} / M_d, \qquad (1)$$

where $\mathbf{V}_d$ and M_d are the velocity and mass of dust particle, and $\mathbf{F}$ is the total force acting on it. The analysis of the forces acting on a micron scale dust particle in fusion plasmas shows [3] that the dominant ones are: the electric force $\mathbf{F}_E = -eZ_d\mathbf{E}$ (where $\mathbf{E}$ is the electric field strength) and the plasma-dust particle friction force $\mathbf{F}_{fric} = \varsigma_F \pi r_d^2 M_i n V_i \mathbf{V}_p$, where n and $\mathbf{V}_p$ are the plasma density and hydrodynamic velocity (we assume that $V_p > V_d$), M_i and $V_i = \sqrt{T/M_i}$ are the ion mass and thermal speed, and $\varsigma_F \sim 10$ is a numerical factor. For a typical case of a small inclination angle, α, of magnetic field $\mathbf{B}$ to the wall surface (see Fig. 1) and $\mathbf{V}_p \sim (\mathbf{B}/B)V_i$, normal (y) component of friction force in the vicinity of the wall can be estimated as $\alpha F_{sh} \approx$ const. where $F_{sh} = \varsigma_F n T \pi r_d^2$. Therefore, net impact of friction and electric forces on y- component of dust velocity can be described with effective potential

$$U_d(y) = \alpha F_{sh} y + \Lambda r_d T \varphi(y)/e. \qquad (2)$$

Where $\varphi(y)$ is the electrostatic potential, which is of the order of T/e at the wall and falls quickly outside the sheath, $y > \rho_i$, where ρ_i is the ion gyro-radius. As a result $U_d(y)$ has a minimum at $y = y_{min}(r_d)$, where electric force balances normal component of friction force. The motion of dust particle, confined by the potential $U_d(y)$, can exhibit damping oscillations characterized by the frequency

$$\Omega_d^2 = M_d^{-1}(d^2U_d/dy^2)\Big|_{y=y_{min}} \sim (\alpha F_{sh}/M_d\rho_i)^{1/2}. \qquad (3)$$

For $n = 3\times10^{13}\text{cm}^{-3}$, $T = 10\,\text{eV}$, $\alpha \sim 0.1$, and the mass density of dust particle $\tilde{\rho}_d \sim 2\,\text{g/cm}^3$, we find that Ω_d is much larger than dumping rate and much smaller than the charging frequency, which are respectively of the order of $\sim 1\,\mathrm{s}^{-1}$ and $\sim 10^{11}\,\mathrm{s}^{-1}$ for the above dust and plasma parameters.

While in y- direction the friction force can be balanced by the electric force, in the directions along the wall the friction force is unbalanced and remains strong due to plasma flow along oblique magnetic field lines and ion diamagnetic and $\mathbf{E}\times\mathbf{B}$ flows in x-direction ($F_{fric}^{(z)} \sim F_{fric}^{(x)} \sim F_{sh}$). Assuming that the acceleration acts along a distance of length L, we can estimate the magnitude of radial, $V_d^{(x)}$, and toroidal, $V_d^{(z)}$, velocity components that dust particle gains during the time it moves along that distance

$$V_d^{(x)} \sim V_d^{(z)} \sim V_d^{(L)} \equiv V_i\left(\varsigma_F M_i n L/\tilde{\rho}_d r_d\right)^{1/2}. \qquad (4)$$

For $\tilde{\rho}_d \sim 2\,\mathrm{g/cm}^3$, $n = 3\times 10^{13}\mathrm{cm}^{-3}$, $T = 10\,\mathrm{eV}$, $L \sim 1\,\mathrm{cm}$, and $r_d \sim 3\times 10^{-4}$ cm from Eq. (4) we find $V_d^{(L)} \sim 3\times 10^3\ \mathrm{cm/s}$.

Dynamics of dust particle in the vicinity of corrugated surface. Thus, dust particle confined in the sheath region and accelerated by plasma flows can move along the surface with a very high speed. However, even rather small corrugation of the surface can cause an increase of the amplitude of particle oscillations in the potential well and even to fly out from the sheath region. In Ref. 3 an example of sinusoidal long wavelength corrugation of wall surface in x-direction was analyzed for the case of a small amplitude, h_s, of corrugation, $h_s << \rho_i$. In this case particle dynamics in the vicinity of the minimum of effective potential $U_d(y)$ can be described by the equations

$$\frac{d^2 y_d}{dt^2} = -\Omega_d^2\left(y_d - y_{min}(x)\right), \quad \frac{d^2 x_d}{dt^2} = \frac{F_{fric}^{(x)}}{M_d}. \qquad (5)$$

where $y_{min}(x) = \bar{y}_{min} + h_s \sin(k_s x)$ ($k_s \rho_i << 1$, k_s is the corrugation wave number) describes the variation of the location of minimum of potential $U_d(y)$ due to wall surface corrugation, $y_s(x) = \bar{y}_s + h_s \sin(k_s x)$. In [3] it was shown that due to acceleration of dust particle along x-direction the resonance $k_s V_d^{(x)} = \Omega_d$ occurs at time $t_{res} \approx \Omega_d M_d \big/ k_s F_{fric}^{(x)}$. For the case where $S_{res} \equiv t_{res}\Omega_d >> 1$, resonance coupling of x- and y- components of particle motion causes a strong increase of the amplitude, $\tilde{y}_d$, of particle oscillation in the potential well, which scales as $\tilde{y}_d(t) \approx h_s\left(\pi S_{res}\right)^{1/2}$.

Here we assume that $y_{min}(x) = \bar{y}_{min} + H_s(x)$, where small amplitude long wavelength function $H_s(x)$ ($|H_s(x)| < \rho_i$) determines the location of wall surface. We consider particle motion in the vicinity of $\bar{y}_{min}$ and solve equation Eq. (6) neglecting a small dependence of $F_{fric}^{(x)}(x, y \approx \bar{y}_{min})$ on both x and y, so that $M_d v_d^2(x) = 2xF_{fric}^{(x)}(y = \bar{y}_{min})$. We introduce the distribution function of dust particle, $f(\varepsilon,\varphi,x)$, using energy/phase, $\varepsilon = (v_y^2 + \Omega_d^2\tilde{y}^2)/2$ and $\cos(\varphi) = \Omega_d \tilde{y}/(2\varepsilon)^{1/2}$, variables, where $\tilde{y} = y_d - \bar{y}_{min}$ and v_y is the particle velocity along y coordinate. After some algebra one can show that the dust particle distribution function $f(\varepsilon,\varphi,x)$ obeys the following kinetic equation

$$v_d(x)\frac{\partial f}{\partial x} + \Omega_d \frac{\partial f}{\partial \varphi} = \frac{\Omega_d^2 H_s(x)}{\sqrt{2}}\left(\sin(\varphi)\sqrt{\varepsilon}\frac{\partial f}{\partial \varepsilon} - \frac{\cos(\varphi)}{\sqrt{\varepsilon}}\frac{\partial f}{\partial \varphi}\right). \qquad (6)$$

Expanding the function $f(\varepsilon,\varphi,x)$ as a series of powers of H_s, from Eq. (6) in a quasi-linear approximation we find

$$\frac{\partial f_0}{\partial x} = \frac{\pi}{4}\frac{\Omega_d^4}{v_d^2(x)}\left|\hat{H}_s(k)\right|^2_{k=\frac{\Omega_d}{v_d(x)}}\sqrt{\varepsilon}\frac{\partial}{\partial \varepsilon}\left(\sqrt{\varepsilon}\frac{\partial f_0}{\partial \varepsilon}\right), \qquad (7)$$

where $\left\langle H_s^2(x)\right\rangle = \int\left|\hat{H}_s(k)\right|^2 dk$. Taking into account that $v_d^2(x) \propto x$, for $\hat{H}_s(k \to 0) = \hat{H}_s(0)$, from Eq. (7) we find asymptotic expression for the averaged energy ε of dust particle $\bar{\varepsilon}(x) \equiv \langle\varepsilon\rangle$:

$$\bar{\varepsilon}(x \to \infty) \approx \frac{3\pi}{16}\frac{M_d\Omega_d^4}{F_{fric}^{(x)}(\bar{y}_{min})}\left|\hat{H}_s(0)\right|^2 \ln\left(\frac{x}{\rho_i}\right). \qquad (8)$$

As a result, from Eq. (8) one finds that averaged amplitude $\left\langle \tilde{y}_d^2\right\rangle$ of dust particle oscillations in the potential well of $U_d(y)$ increases with x like

$$\left\langle \tilde{y}_d^2\right\rangle \sim \frac{\alpha}{\rho_i}\left|\hat{H}_s(0)\right|^2 \ln\left(\frac{x}{\rho_i}\right). \qquad (10)$$

In case of large amplitude of surface wave $H_s > \rho_i$, due to a strong effect of centrifugal force dust particle looses confinement within the sheath region even before it reaches resonance condition $V_d k_s = \Omega_d$ [3]. To study dust particle dynamics in this regime we use numerical modeling.

We solve Eq. (1) assuming that the wall is corrugated along either x or z directions and it affects the force **F**. In the case where the wall is corrugated along x direction we describe the force acting on the dust particle as $\mathbf{F} = -\nabla\Phi_\perp - \nabla\Phi_{||}\times\mathbf{e}_z$, where $\Phi_\perp(x,y) = \hat{\Phi}_\perp(y - y_s(x))$ and $\Phi_{||}(x,y) = \hat{\Phi}_{||}(y - y_s(x))$ describe respectively the effects of the corrugation on effective potential $U_d(y)$ and x-component of the friction force caused by diamagnetic and **E**×**B** plasma flows in the sheath region. As a reasonable approximation we take $\hat{\Phi}_{||}(y) = F_{sh}\rho_i\exp(-y/\rho_i)$ and $\hat{\Phi}_\perp(y) = \alpha F_{sh}\left\{y + \rho_i\exp\left((y_{min} - y)/\rho_i\right)\right\}$. We also assume specula reflection of dust particle from the surface. With this model we were able to recover the main results we obtained analytically for the case of a small corrugation of the surface. For the case of large corrugation, $|H_s(x)| > \rho_i$, we found that dust particles, being accelerated to large speed, fly at large distance from the surface (even toward the core) and hit the wall when they are coming back due to friction force effects. Such flights can have stochastic character (see Fig. 2 for $y_{min} = \rho_i/2$, $H_s(x) = h_s\sin(k_s x)$, $h_s/\rho_i = 3$, $k_s\rho_i = 0.3$, and $\alpha = 0.1$) with some intermittent features. More detail report on the result of numerical modeling of dust particle motion will be published elsewhere.

Dust dynamics and core plasma contamination. As we saw, dust particles flights caused by wall surface corrugation can result in the motion of dust toward core and, therefore, contaminate core plasma with impurity. Assuming that the main dust particle material is carbon, we find that in each dust particle with $r_d \sim 3\times10^{-4}$ cm and $\tilde{\rho}_d \sim 2\ g/cm^3$ there are $N_C \sim 10^{13}$ carbon atoms. In order to maintain a constant impurity fraction, ξ_{imp}, in the core, the carbon influx through the separatrix, $\Gamma_{imp}^{(sep)}$, should be about $\xi_{imp}\Gamma_{H}^{(sep)}$, where $\Gamma_{H}^{(sep)}$ is the flux of hydrogenic species though the separatrix. For $\xi_{imp} \sim 10^{-2}$ and $\Gamma_{H}^{(sep)} \sim 10^{21}\ s^{-1}$, we find $\Gamma_{imp}^{(sep)} \sim 10^{19}\ s^{-1}$, and it would take a flux, $\Gamma_{d}^{(sep)} \sim 10^{6}\ s^{-1}$, of dust particle to establish the required impurity influx and corresponding density of dust particles near the separatrix is $n_d \sim 10^{-2}\ cm^{-3}$. Where this dust may come from? If it originates at the walls of the main chamber due to plasma flux to the wall then to maintain impurity balance we have to have $\Gamma_{imp}^{(sep)} \sim \xi_{imp}\Gamma_{H}^{(sep)} \sim Y_C \eta_{dust} \eta_{flight} \Gamma_{H}^{(sep)}$, where η_{dust} is the fraction of sputtered carbon converted into dust, η_{flight} is the dust fraction that flies to the core. Since $Y_C \sim \xi_{imp} \sim 1\%$ we have to assume that roughly ~100% of sputtered carbon is transformed into dust and then flies to the core, which is unlikely the case. Another possibility, dust formation in the vicinity of divertor striking points, looks much more probable. Indeed, due to the strong plasma recycling in the divertor, the plasma flux to divertor targets, $\Gamma_{H}^{(div)}$, can easily be as high as $10^{23}\ s^{-1} >> \Gamma_{H}^{(sep)}$. Then, the impurity influx into the core caused by dust, $\xi_{imp}\Gamma_{H}^{(sep)} \sim Y_C \eta_{dust} \eta_{flight} \Gamma_{H}^{(div)}$, can be established for Y_C~3%, η_{dust}~3%, and η_{flight}~10%, which looks very feasible [3].

Conclusions. Due to acceleration by the friction forces caused by plasma flows in the vicinity of material surface, dust particles can acquire very high speeds ($\sim 10^3$ cm/s and higher). Interactions of dust particles with corrugated surface can cause an escape of dust particle to from near wall region and flights toward tokamak core. It is likely that dust formation in and transport from divertor region can play an important role in core plasma contamination. However, even then, dust particle density around the separatrix is $\sim 10^{-2}\ cm^{-3}$, which makes it difficult to diagnose.

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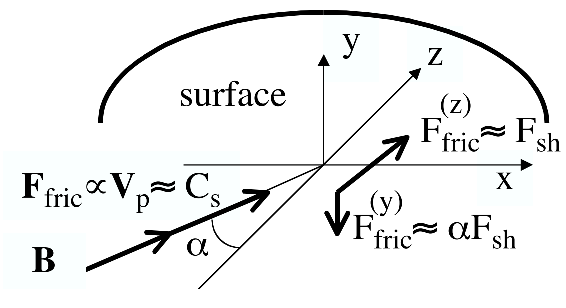

Fig. 1. Orientation of magnetic field, plasma flows, and friction forces

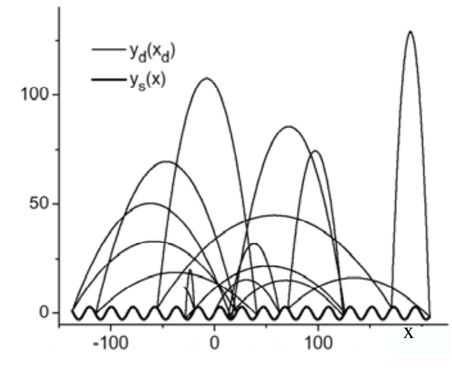

Fig. 2. Stochastic flights of dust particle over corrugated surface (x and y are in ρ_i)