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A model for collisionless reconnection with arbitrary magnetic guide fields

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Introduction. A new set of reduced equations governing 2D, two-fluid, collisionless magnetic reconnection has been presented in Ref. [1]. These equations are valid for arbitrary values of the magnetic guide field. This represents a significant advance in magnetic reconnection theory, as it allows to bridge the limiting regimes where collisionless reconnection is mediated by either whistler waves at low guide fields (where the Hall term in the generalized Ohm law plays an important role) or by kinetic Alfven waves (where electron compressibility along the field lines becomes important). Indeed, the model exhibits a single scale length, which we denote by $d_\beta = c_\beta d_i$, where $c_\beta = (\beta/1+\beta)^{1/2}$, β is the plasma beta parameter (kinetic pressure/magnetic pressure) based on the magnetic guide field (i.e., β is large when the guide field is weak) and $d_i = c/\omega_{pi}$ is the ion skin depth. In the strong guide field limit, $c_\beta \approx \beta^{1/2}$ and $d_\beta \approx \rho_s$, the ion sound Larmor radius. In the opposite limit of weak guide field, $d_\beta \approx d_i$.

Using this new set of equations, we have computed the linear growth rates of tearing modes, both in the small and in the large Δ' regimes, where Δ' is the standard tearing stability parameter. In this short paper, a detailed mathematical derivation is presented only for the case where the guide field is absent. Results for arbitrary guide fields are briefly mentioned at the end.

Equilibrium. The standard Harris pinch equilibrium is assumed, with magnetic field $\vec{B}_{eq} = B_0 \tanh(x/L)\vec{e}_y$ and current density along the z-direction, $J_{eq}(x) = cB_0/4L\pi\cosh^2(x/L)$.Pressure balance with constant electron temperature T and negligible ion temperature yields the equilibrium density, $n_{eq}(x) = B_0 L J_{eq}(x)/2cT$. We introduce $n_0 = n_{eq}(0)$, $J_0 = J_{eq}(0)$, $v_A = B_0/(4m_i n_0 \pi)^{1/2}$. Furthermore, we normalized the magnetic field over B_0, the plasma density over n_0, the current density over J_0, distances over the equilibrium scale distance L and time over the Alfven time $\tau_A = L/v_A$. In dimensionless units, $n_{eq}(x) = J_{eq}(x)$.

Perturbation and linearized system. We represent the magnetic field as $\vec{B} = \vec{B}_{eq} + B_z\vec{e}_z + \nabla\psi\times\vec{e}_z$, where B_z and ψ are small amplitude perturbations, which are taken to depend on *x, y* and *t* as $\psi(x,y,t) = \psi(x)\exp(\gamma t + iky)$. The perturbed ion flow is $\vec{v}_i = \vec{e}_z \times \nabla\varphi + \nabla\chi + v_z\vec{e}_z$, the perturbed electron flow is $\vec{v}_e = \vec{v}_i - \vec{J}/en_{eq}$, the perturbed electric field is $\vec{E} = -\nabla\phi - \partial_t \vec{A}/c$, and the perturbed density is *n.* An isothermal equation of state is assumed for the electrons, while the ions are cold.

The starting equations are the ion continuity equation and the electron and ion momentum balance equations, where collisional friction terms are neglected. These equations are coupled with Maxwell's equations, where Poisson's law is replaced by quasineutrality. After relatively straightforward algebra, we obtain a closed set of three equations in dimensionless form for the variables ψ, n and B_z:

$$n_{eq}\gamma^2\psi - \left(\nabla^2\psi_{eq}\nabla\psi + \nabla^2\psi\nabla\psi_{eq} + \nabla n/2\right)\cdot\nabla\psi_{eq} = d_e^2\gamma^2\nabla^2\psi + d_i\gamma\left[\psi_{eq}, B_z\right] \quad (1)$$

$$\gamma^2 n - \nabla\cdot\left(\nabla^2\psi_{eq}\nabla\psi + \nabla^2\psi\nabla\psi_{eq} + \nabla n/2\right) = 0 \quad (2)$$

$$\gamma^2\left(n_{eq}B_z - d_e^2\nabla^2 B_z\right) + \left[\psi_{eq},\left[B_z,\psi_{eq}\right]\right] = d_i\gamma\left\{\left[\nabla^2\psi,\psi_{eq}\right] - \left[n_{eq},n\right]/2n_{eq}\right\} \quad (3)$$

In these equations, $d_e = c/\omega_{pe}L$ and d_i are the electron and ion skin depths normalized on the equilibrium scale length, respectively, $d_e = \left(m_e/m_i\right)^{1/2}d_i$ and we assume $d_i << 1$. The square brackets are defined as $[A,B] = \vec{e}_z\cdot\nabla A\times\nabla B$. Small terms of order m_e/m_i as compared to unity have been neglected.

Outer region. The system (1-3) is now solved analytically by standard asymptotic matching techniques. Let us define the vector $\vec{A} = \nabla^2\psi_{eq}\nabla\psi + \nabla^2\psi\nabla\psi_{eq} + \nabla n/2$. From Eq. (1) in the outer region, neglecting terms proportional to γ^2 and d_e^2, we find $A_x = \psi_{eq}''\partial_x\psi + \psi_{eq}'\nabla^2\psi + \partial_x n/2 = 0$. From Eq. (2), neglecting the term proportional to γ^2, we find $\partial_x A_x + ikA_y = 0$. These two equations taken together give $\nabla^2\psi + \left(dJ_{eq}/d\psi_{eq}\right)\psi = 0$, which is the standard equation for the perturbed magnetic flux in the outer ideal MHD region. Its solution for the Harris pinch equilibrium is well known. The normalized logarithmic jump of the outer flux across the singular layer at $x = 0$ yields the tearing stability index $\Delta' = 2\left(k^{-1} - k\right)$, which implies instability when $k < 1$. In the outer region, we also find $n \approx 2J_{eq}\psi$.

Inner region. For $x << 1$, Eqs. (1-3) reduce to

$$\gamma\left(1 - d_e^2\partial_x^2\right)\psi = ix\xi - id_i xB_z \quad (4)$$

$$\gamma\,\partial_x^2\xi = ix\partial_x^2\psi \quad (5)$$

$$\gamma\left(1 - d_e^2\partial_x^2 + x^2/\gamma^2\right)B_z = id_i x\partial_x^2\psi \quad (6)$$

where $\gamma \to \gamma/k$ and we have introduced the function $\xi = -\left(i/k^2\gamma\right)\left(\partial_x\psi + x\partial_x^2\psi - \partial_x n/2\right)$, which is proportional to the stream function for the ion flow. Also, we have neglected the term $\gamma^2 n$ in Eq. (2), the density term in Eq. (3), and we have used $\partial_x(\psi - n/2) \approx -x\partial_x^2\psi$.

The inner region has a nested layer structure. For $x << \gamma,\ d_i$, Eqs. (4-6) simplify further. Then, it is convenient to work in Fourier space. Let $\xi(p) = \int\xi(x)\exp(-ipx)dx$ the generalized Fourier transform. With the transformation $r = d_e p$, $W = \left[r^2/(1+r^2)\right]\xi$ and $Q = \gamma / d_i$, the following equation for $W(r)$ can be deduced for $p >> \gamma^{-1}$, d_i^{-1}:

$$\frac{d}{dr}\left(\frac{r^2}{1+r^2}\frac{dW}{dr}\right) - Q^2\left(1+r^2\right)W = 0 . \tag{7}$$

We anticipate that $Q << 1$ for the solutions of interest. For $r << Q^{-1/2}$, the term proportional to Q^2 can be neglected in Eq. (7), whose approximate solution takes the form

$$W = W_0\left[\left(r^{-1} - r\right) + \alpha + O\left(r^2\right)\right], \tag{8}$$

with α an integration constant. For $r >> 1$, Eq. (7) reduces to a Bessel equation. Choosing the solutions that decays to zero as $r \to \infty$ and matching this to the behaviour (8) in the asymptotic interval $1 << r << Q^{-1/2}$ gives $\alpha = [\Gamma(1/4)/2\Gamma(3/4)]Q^{-1/2}$.

In the outer layer defined by the inequality $x >> d_e$, a solution can be found in x-space. Let $Y = \partial_x \xi / \xi_0$, where $\xi_0 = const$ is the asymptotic value of ξ at large x. Matching to the outer MHD solution requires that $Y \approx -2\gamma / \Delta' x^2$ at large x. Eqs. (4-6) reduce to

$$x^2 \frac{d}{dx}\left(\frac{\gamma^2 d_i^2}{\gamma^2 + x^2}\frac{dY}{dx}\right) - \left(\gamma^2 + x^2\right)Y = \frac{2\gamma}{\Delta'} . \tag{9}$$

Let us consider in detail the limit $\Delta' \to \infty$. With the transformation $s = x/\gamma$ and $V = \left(1+s^2\right)^{-1} dY/ds$, Eq. (9) becomes

$$\frac{d}{ds}\left(\frac{s^2}{1+s^2}\frac{dY}{ds}\right) - Q^2\left(1+s^2\right)V = 0 . \tag{10}$$

The small - s asymptotic solution of (10) behaves as

$$V = V_0\left[s^{-1-Q^2} + \alpha_1 s^{Q^2} + O\left(s^{1-Q^2}\right)\right] \tag{12}$$

where α_1 is determined by the condition that V not blow-up as $s \to \infty$. However, since Q is small and Eq. (10) has exactly the same form as the inner layer Eq. (7), we immediately deduce that $\alpha_1 = \alpha$. Fourier transforming (12) and relating $W(r)$ to $V(r)$, we obtain the asymptotic solution

$$W \approx W_2\left[\alpha\, \Gamma\left(1+\mathrm{Q}^2\right)\cos\left(Q^2\pi/2\right)r^{-1-Q^2} - \left(\gamma/d_e\right)^{1+2Q^2} r^{Q^2}\,\Gamma\left(-Q^2\right)\sin\left(Q^2\pi/2\right) + O\left(r^{1-Q^2}\right)\right] \tag{13}$$

Taking the limit $Q^2 << 1$ and comparing with Eq. (8) gives $\alpha^2 = (\pi/2)(\gamma/d_e)$ and the dispersion relation

$$\gamma = c_0 (d_e d_i)^{1/2} \tag{14}$$

where $c_0 = \left[\Gamma(1/4)/(2\pi)^{1/2}\Gamma(3/4)\right]$.

Inspection of Eq. (9) reveals that the dispersion relation (14) is valid for large values of the tearing stability index, $\Delta' d_e > 1$. For smaller values of Δ', the inhomogeneous Eq. (14) must be tackled. The algebraic details can be found in Ref. [1]. The dispersion relation applicable when $\Delta' d_e < 1$ is

$$\gamma \approx (c_0 \Delta')^2 d_e^2 d_i > 1 . \tag{15}$$

Discussion and conclusions. We have derived novel linear growth rates for tearing modes in collisionless plasmas, in the limits of small and large values of the tearing stability index Δ', applicable to equilibria where a magnetic guide field is absent. Under these circumstances, the Hall term in the generalized Ohm law cannot be neglected. This term introduces the ion skin depth, d_i, as a new scale length of the problem, upon which the growth rates now depend. The novel growth rates we have found also exhibit a dependence on the electron mass through the electron skin depth, d_e. The electron mass is related to the physical mechanism that breaks the frozen-in law in the collisionless two-fluid model we have considered. We point out that similar results have also been obtained recently by Mirnov *et al* [2].

A more general discussion, applicable to arbitrary magnetic guide fields, has been presented in Ref. [1]. According to this more general work, the results presented in this short contribution are recovered when the plasma beta based on the guide field exceeds the rather small critical value, $\beta_{crit} = (m_e/m_i)^{1/4}$ (= 0.16 for hydrogen). For these values of beta, the guide field ceases to have any significant effect on the linear growth rates. For smaller values of β below β_{crit}, the results of Ref. [3] are recovered.

References

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