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Error-field Penetration Thresholds in a Two-fluid Model

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I. INTRODUCTION

Error-field penetration (i.e., the sudden arresting of bulk plasma rotation and subsequent driven magnetic reconnection due to an error field) is fairly well understood in the context of resistive-viscous magnetohydrodynamics (MHD). The scaling of the critical error-field needed to drive reconnection depends strongly on the equilibrium flow velocity. In light of this, the validity of MHD in high temperature tokamaks is questionable, as it neglects the diamagnetic drift velocity, which can be comparable to, or greater than, the $E \times B$ velocity. With this in mind, we wish to "update" the current theoretical predictions of error-field penetration thresholds using a reduced four-field model derived from the well-known drift-MHD equations[1,2]

II. ANALYSIS

We begin by adopting standard right-handed Cartesian coordinates (x, y, z) and assume that there is no variation in the z-direction. Within this framework, we consider a two-species (singly-charged ion and electron) plasma with a uniform constant density profile and cold ions. Our reduced equations are as follows:[3]

$$\partial_t \psi = [\phi, \psi] + d_\beta [\psi, Z] + \eta \nabla^2 \psi \tag{1}$$

$$\partial_t Z = [\phi, Z] + c_\beta^2\, \eta \nabla^2 Z + c_\beta [V_z, \psi] + d_\beta \left[\nabla^2 \psi, \psi\right] \tag{2}$$

$$\partial_t U = [\phi, U] + \left[\nabla^2 \psi, \psi\right] + \mu \nabla^2 U \tag{3}$$

$$\partial_t V_z = [\phi, V_z] + c_\beta [Z, \psi] + \mu \nabla^2 V_z \tag{4}$$

$$U = \nabla^2 \phi \tag{5}$$

where ψ is the magnetic flux function, ϕ the ion stream function, $Z = b_z/c_\beta$ the normalized z component of the magnetic field less the guide field, and V_z the z component of the ion velocity. Here $\beta = \Gamma P_0/B_0^2$ (the ratio of specific heats is $\Gamma = 5/3$), B_0 is the guide field, P_0 the background pressure, $c_\beta = \sqrt{\beta/(1+\beta)}$, and $d_\beta = d_i c_\beta$, where $d_i = c/\omega_{pi}$ is the collisionless ion skin-depth. The resistivity η, and viscosity μ, are assumed constant.

The simplest model plasma equilibrium containing all the salient plasma features of error-field penetration is shown in Fig. 1. The plasma is periodic in the y direction, (with wavenumber k) and bounded by perfectly conducting walls at $x = \pm 1$. The y-directed magnetic field is sheared about the resonant surface located at $x = 0$. The parabola represents the background flow driven by a uniform y-directed force. The error-field is simulated by applying an anti-symmetric x-directed wall displacement (thus rippling the field lines in the outer region where ideal MHD is valid).

Balancing the y-averaged electromagnetic and viscous forces acting on the non-ideal resonant layer (centered at $x = 0$) gives

$$\frac{k}{2}(E_{sw}\Xi)^2 \frac{\mathrm{Im}[\Delta]}{|(-E_{ss}) + \Delta|^2} = 2\mu(V_0 - V) \qquad (6)$$

where $E_{sw} = 2k/\sinh k$ and $E_{ss} = -2k/\tanh k$ come from solving (1) - (5) outside the resonant layer. Note $E_{ss} < 0$ is the conventional tearing stability index. The wall displacement amplitude is given by Ξ, while V_0 (V) is the ion velocity at $x = 0$ in the absence (presence) of the error-field. Finally, Δ is found by solving the linearized versions of (1) - (5) in the resonant layer.[4] In general, equation (6) gives a cubic polynomial in $V(\Xi)$ whose roots determine the critical error-field amplitude needed to arrest the plasma motion at $x = 0$.

III. LAYER RESPONSE

The plasma flow levels in ohmically heated tokamaks are not generally large enough for the constant-ψ approximation to break down in the resonant layer. Furthermore, we expect the magnetic Prandtl number ($P = \mu/\eta$) to be greater than unity. Taking these facts into account, the relevant layer response regimes for the ohmically heated tokamak are the “visco-resistive”, “semi-collisional”, and “hall-resistive.” The layer parameter Δ

for each regime is:

$$\Delta_{VR} = 2.104 \left(\frac{k}{\eta}\right)^{2/3} [i\,(V - V_*)]^{5/6}\,(iV)^{1/6} P^{1/6}$$

$$\Delta_{SC} = 3.142 \left(\frac{k}{\eta}\right)^{1/2} [i\,(V - V_*)]\,(iV)^{1/2} d_\beta^{-1/2}$$

$$\Delta_{HR} = 2.124 \left(\frac{k}{\eta}\right)^{1/2} [i\,(V - V_*)]\, c_\beta^{1/2} d_\beta^{-1/2},$$

where V_* is the electron diamagnetic velocity at $x = 0$.

IV. TOKAMAK APPLICATIONS

Error-field penetration thresholds in slab geometry can be transformed into corresponding thresholds in a large aspect-ratio tokamak via a straight-forward and well-known process. Using a neo-Alcator confinement scaling model[5], the dependence of the critical vacuum radial error-field strength on the plasma major radius in an ohmic tokamak can be determined. The results are plotted in Fig. 2 for the $(2, 1)$ tearing mode in a deuterium plasma with $q(a) = 3.5$ (where q is the safety factor and a the minor radius). Note that small tokamaks (major radius $R_0 < 1$ m, shown as square points in Fig. 2) are in the "visco-resistive" response regime, while larger devices ($R_0 > 1$ m, displayed as triangular points in Fig. 2) fall in the semi-collisional response regime.

Unfortunately, scaling laws derived in this fashion are not consistent with Conner-Taylor dimensionless scaling arguments[6] applied to Eqns. (1) - (5). Proceeding instead using a dimensionless approach to derive scaling laws, we find in that if the density scaling is linear (as is found experimentally) then there must be a very strong inverse scaling with the toroidal field-strength, and a moderately strong inverse scaling with the major radius. In fact, both neo-Alcator and Connor-Taylor scaling models predict that the critical error-field necessary to trigger flow-damping and magnetic reconnection for an ITER sized ($R_0 \sim 6$ m) tokamak is

$$\left[\frac{b_r(r_s)}{B_\phi}\right]_{crit} \sim 2 \times 10^{-5}$$

where $b_r(r_s)$ is the vacuum radial magnetic field at the rational surface (calculated here for the (2,1) tearing mode), and B_ϕ the toroidal (guide) field-strength.

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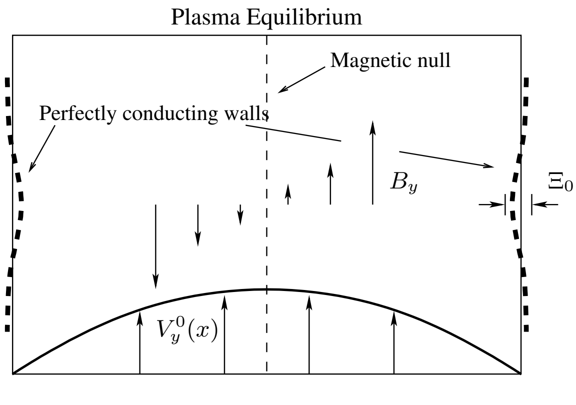

Figure 1.

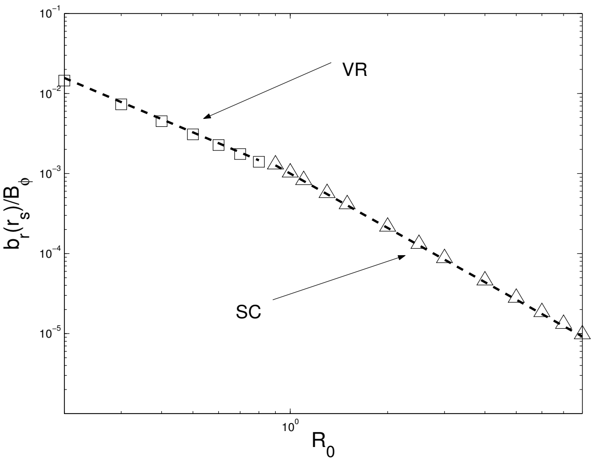

Figure 2.