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Impact of field line label and ballooning parameter on infinite-*n* ballooning stability in compact stellarators

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Abstract. The impact of field line label and ballooning parameter on the infinite-*n* ballooning stability of compact, quasi-poloidal symmetric stellarators is investigated. Previously, the ballooning stability of quasi-poloidal stellarators has been examined for free-boundary, moderate β (β >4%) plasmas in the Quasi-Poloidal Stellarator (QPS) [1]. These previous calculations were performed with ballooning parameter, $\zeta_k = 0$ and field line label $\alpha = 0$. Here, these results are extended to include other possible values of ζ_k and α. First ballooning instability β-limits for these devices are well described by the $\alpha, \zeta_k = 0$ results. Changing either ζ_k or α increases the β required for first instability. The β values required to enter second ballooning stability are higher when ζ_k, $\alpha \neq 0$.

I. Introduction

Access to regions of second stability in axisymmetric devices makes possible advanced tokamak operation. Exploration of regions of second stability in three–dimensional configurations could allow for high-β operation in a compact stellarator. Here, β is the ratio of the plasma pressure to the magnetic pressure, $\beta = p/B^2$, and $\langle\beta\rangle$ indicates the volume average β. A number of previous studies of ballooning modes in stellarators [1,2] have focused on a narrow range in parameter space, namely $\alpha, \zeta_k = 0$. The ballooning parameter is given by $\zeta_k = k_\iota / k_\alpha$ and the field line label is $\alpha = \iota\zeta - \theta$. Here, k_ι and k_α are the components of the wavenumber perpendicular to the magnetic field and ι is the rotational transform. The limitation in the range of ζ_k, α in previous studies was motivated by both computational limitations and the result of limited testing that indicated these values (i.e., $\alpha, \zeta_k = 0$) had the lowest β-limits for stability.

The focus of this work is the impact of ballooning parameter and field line on ideal MHD ballooning stability of free-boundary equilibria in the Quasi-Poloidal Stellarator

(QPS), a quasi-poloidal, compact stellarator. For more details on the QPS configuration, see [3]. Previously, it was shown that a region of second ballooning stability (i.e., a region of interchange stability for large pressure gradients) does exist for QPS with $\alpha,\zeta_k = 0$ [1]. Here, the impact of field line and ballooning parameter on both the first stability boundary and second-stability access in QPS is explored.

II. Ideal MHD Ballooning Stability

Ballooning modes are short perpendicular wavelength ($k_\perp >> k_{||}$), pressure-driven modes, localized to regions of bad curvature along a field line. The plasma displacement is taken to be of the form $\xi = \hat{\xi}(\mathbf{r},\varepsilon)\exp\left[iS(\iota,\alpha)/\varepsilon + \gamma t\right]$ where $\hat{\xi}(\mathbf{r},\varepsilon)$ is slowly-varying, $S(\iota,\alpha)$ is the eikonal, and $\alpha = \iota\zeta - \theta$ is the field line label. The perpendicular wavenumber is then

$$\mathbf{k} \equiv \nabla S(\iota,\alpha) = k_\alpha \nabla\alpha + k_\iota \nabla\iota = k_\alpha\left(\nabla\alpha + \zeta_k \nabla\iota\right) \tag{1}$$

where $k_\alpha = \partial S(\iota,\alpha)/\partial\alpha$, $k_\iota = \partial S(\iota,\alpha)/\partial\iota$, and $\zeta_k = k_\iota / k_\alpha$ is the ballooning parameter. Note that the ballooning parameter is often written as $\theta_k = k_q / k_\alpha$ where $k_q = \partial S(q,\alpha)/\partial q$ and $q = 1/\iota$ is the safety factor. Applying the ballooning formalism to the MHD energy equation leads to the ballooning equation [4], an eigenvalue equation along a field line coordinate, ζ:

$$\frac{\partial}{\partial\zeta}\left[c_1(\zeta)\frac{\partial\hat{\xi}}{\partial\zeta}\right] + c_2(\zeta)\hat{\xi} + \gamma^2 c_3(\zeta)\hat{\xi} = 0 \tag{2}$$

The coefficients c_1, c_2, and c_3 depend on the field line as determined by the choice of α. The perpendicular wavenumber only appears in the coefficients c_1, c_2, and c_3 in the form of the ballooning parameter as: $\zeta - \zeta_k$. Previously [1,2] it was shown that quasi-poloidal stellarators exhibit a region of second ballooning stability for $\alpha,\zeta_k = 0$. In the following, these results are expanded to a range of values for α and ζ_k.

III. Impact of Ballooning Parameter and Field Line on First Ballooning Stability

For the reference configuration, a quadratic pressure profile, $p(S) = p_0(1 - s)^2$, was chosen for simplicity. Infinite-n ballooning mode stability was analyzed using the COBRAVMEC stability code [5]. The following are parameter space plots of the ballooning growth rates as a function of flux coordinate s, field line label α (here mapped to θ), and ballooning parameter ζ_k (here mapped to ζ). Below $\langle\beta\rangle = 2\%$, the plasma is ballooning stable at all s, α, ζ_k. As $\langle\beta\rangle$ is raised above 2%, the plasma crosses the first instability threshold for infinite-n ballooning modes. Figure 1 shows a contour plot of ballooning growth rates for a QPS plasma just above the stability boundary ($\langle\beta\rangle = 2.1\%$).

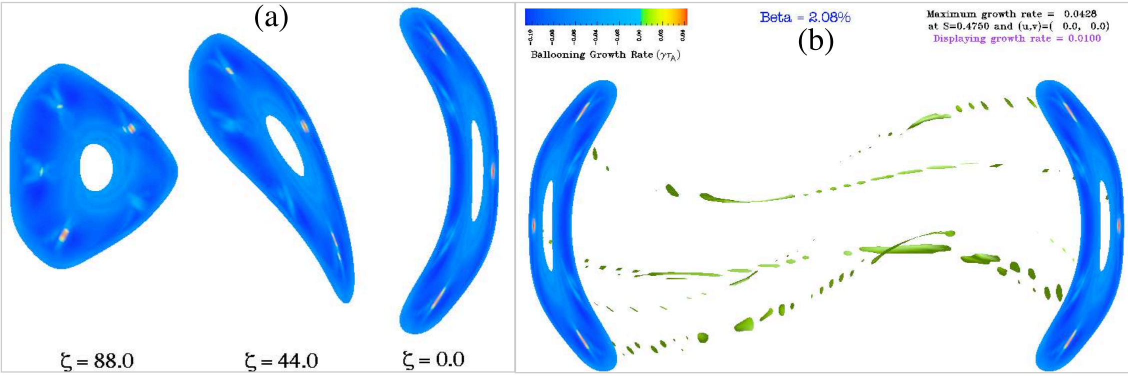

Figure 1. Ballooning growth rates for $\langle\beta\rangle = 2.1\%$ as a function of s, α, ζ_k, for (a) 3 toroidal cuts and (b) the $\gamma\tau_A = 0.01$ surface. Negative values (blue) indicate stability. (Note that here and in Figure 2, α is mapped to θ and ζ_k is mapped to ζ).

The results for $\alpha, \zeta_k = 0$ accurately describe the first stability boundary for infinite-n ballooning modes. It is at these values that the plasma first goes unstable. In addition, only a small region of field lines near $\alpha = 0$ are unstable for values of $\langle\beta\rangle$ near the stability boundary ($\langle\beta\rangle < 3\%$).

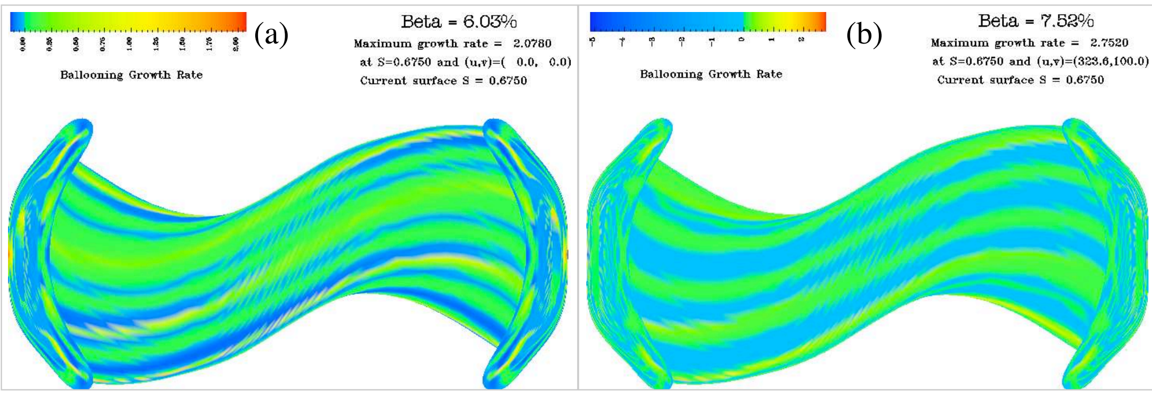

Figure 2. Contours of infinite-n ballooning growth rates for (a) $\langle\beta\rangle = 6.03\%$ and (b) $\langle\beta\rangle = 7.52\%$ on the $s = 0.675$ surface. Blue regions indicate ballooning stable regions.

IV. Impact of Ballooning Parameter and Field Line on Second Ballooning Stability

For $\alpha, \zeta_k = 0$, a large region of the plasma is in the second ballooning stability regime for $\langle\beta\rangle > 6\%$ [1]. Figure 2a shows a contour plot of ballooning growth rates on the $s = 0.675$ surface for a QPS plasma near the onset of second stability ($\langle\beta\rangle = 6.03\%$). The plasma is still in the first instability regime for a wide range of ζ_k and α. At higher β more of the plasma is in the second stability regime as shown in Figure 2b for $\langle\beta\rangle =$

7.52%. Parameter space plots of the ballooning growth rate surfaces (a) $\gamma\tau_A = 0.5$, (b) $\gamma\tau_A = 1.0$,, and (c) $\gamma\tau_A = 1.5$ for a $\langle\beta\rangle = 7.52\%$ are shown in Figure 3. These surfaces are irregular torii and not cylinders [4].

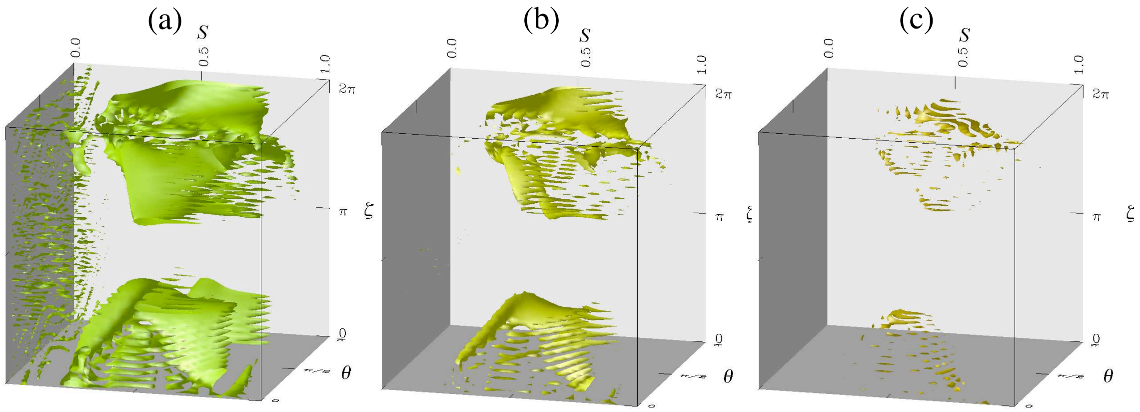

Figure 3. Contours of constant infinite-*n* ballooning growth rates as a function of s, α, ζ_k. for (a) $\gamma\tau_A = 0.5$, (b) $\gamma\tau_A = 1.0$,, and (c) $\gamma\tau_A = 1.5$ for a $\langle\beta\rangle = 7.52\%$ QPS case.

V. Conclusions

The first stability boundary for infinite-*n* ballooning modes in QPS has been tested over a wide range of ballooning parameter and field line labels. Near the first instability threshold, the plasma first goes unstable at $\alpha,\zeta_k = 0$, and only over a narrow range of field lines. At $\langle\beta\rangle$ values high enough to enter second stability for infinite-*n* ballooning modes at $\alpha,\zeta_k = 0$, more of the plasma remains first unstable at other values of α and ζ_k.

A primary open question is how do these local mode solutions over a range of field lines and ballooning parameter relate to global ballooning modes? Results from an analysis of finite-*n* ballooning modes (up to $n = 19$) using the Terpsichore code [4] indicate that QPS plasmas can be stable to $\langle\beta\rangle > 5\%$ [1]. Construction of global modes from the local mode solutions may increase the range of $\langle\beta\rangle$ for which ballooning modes are stable.

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