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Drift-Free Equilibrium Reconstruction Using Magnetic Probes

J.B. Lister, F. Saint-Laurent[2], Ph. Moreau[2], V. Dokouka[3], R.R. Khayrutdinov[3], V.E. Lukash[4]

Centre de Recherches en Physique des Plasmas, EPFL,
Association EURATOM-Confédération Suisse, 1015 Lausanne, Switzerland
[2]DRFC, Association Euratom-CEA, 13108 Saint-Paul-lez-Durance, France
[3]TRINITI, Moscow Region, Russia
[4]RRC Kurchatov Institute, Moscow, Russia

Introduction: Two sources of magnetic diagnostic drift have been well documented, namely electronic input offset of the integrators, since the fields and fluxes are time-integrals of the voltages from the physical probes, and radiation induced or thermoelectric emf due to neutron bombardment of the mineral insulated cables between the probes and the integrators. Tore Supra has recently demonstrated an additional effect, not previously taken into consideration, which is slight rotation of the magnetic probes by thermo-mechanical distortion during 300 second pulses absorbing more than 1GJ into the plasma, resulting in time-dependent pick-up of the toroidal magnetic field. The Tore Supra control system tracks these offsets and displaces the last closed flux surface from its reference location. This new effect of long high-power pulses has the properties of a drift, generating a slowly evolving unmeasured offset to the integrated magnetic signals. For this reason, we have decided to take another look at methods of reconstructing an offset-free equilibrium from magnetic diagnostics, whereas most proposed approaches to drift compensation rely on non-magnetic, non-integrated measurements of plasma emission or reflectometry. We note that a similar approach was tested on HT-7 [1] using the induction in the Poloidal Field (PF) coils as the sole measurement of the radial position, modulating in a similar way. Although this present work uses magnetic diagnostics, we are extending this idea to include the vertical position and the deformation of highly shaped plasmas.

Auto-calibration principle: An equilibrium flux surface plot shows that if we marginally displace an equilibrium, the change to the flux-loop signals depends on the curvature of the local flux surface at the loop. A loop which is placed at the tangency point for a given displacement direction will see no change, whereas either side will see a change of different sign. Modulating the position of the equilibrium in both directions, and the plasma current itself, therefore adds new information to that available from the probes for a static equilibrium. Modulating the equilibrium position is different from sweeping the probe position and does not directly give the local flux gradients. The changes of the PF coil currents, shell currents and deformation of the equilibrium itself must be taken into consideration. The changes to the signals are well modelled by a variety of linear and nonlinear models already extensively validated as part of plasma equilibrium control research. However, we can more easily use the tokamak itself to auto-calibrate the sensitivity, which is the first approach made in this paper. It is necessary to explicitly modulate the plasma current, since modulating the plasma radius modulates the current (and vice versa) and it is vital to separate these two effects.

The auto-calibration method proceeds step-wise, briefly summarised as follows for the particular case of TCV control and TCV diagnostics:

a) programme the reference waveforms so the plasma “wanders” over a range of interest;

b) modulate the geometry triplet [RI_p, ZI_p, I_p] (*RZI*) references at three frequencies;
c) run a full tokamak plasma discharge, or a full plasma simulation;
d) demodulate *RZI* using the three known frequencies, an offset and a drift, for short time-segments;
e) similarly demodulate all 76 magnetic diagnostics S_j (38 poloidal flux and 38 tangential poloidal field probes), to provide the 76*3 sensitivity matrix $Q_{jk}=dS_j/dRZI_k$;
f) identify the contributions of each element of *RZI* to the diagnostic modulation via an inversion of the *RZI* modulation triplet at the 3 frequencies in the same time-segments;
g) take all the time-segments and regress all 228 elements Q_{jk} to the mean values of [R,Z] in the time-segments to obtain an "auto-calibration" mapping *M* so that [R,Z] = *M(Q)*.

Auto-calibration test in TCV geometry: In a first test, we took the extreme case of the TCV tokamak, in which the PF coils are close to the probes and in which the vacuum field is spatially highly structured for strong plasma equilibrium shaping. The self-calibration method was tested on simulation data generated by the 1.5D simulation code DINA-CH [2] and the results were encouraging, demonstrating that the magnetic probe modulation response when the triplet *RZI* is modulated can be roughly linearised as a function of the plasma position with a potentially useful noise sensitivity.
A number of important details were identified this first attempt which led to a second simulation with slightly different modulation frequencies and amplitudes to obtain a more diagonal and better conditioned 3x3 *RZI* matrix:

a) We are extracting the modulation amplitudes of RZI at three frequencies and then inverting the modulation of the three geometrical variables, which is inconsistent, since the modulation response is itself a function of frequency; out of precaution, we use frequencies as close as possible for modulating R and Ip, which provide the main cross-coupling; if the response is in the 1/f regime, the integrated quantities are roughly constant as a function of frequency; the problem will lie in the lower frequencies;
b) Estimating the modulation of [R,Z] from probes with unknown offsets is justified by their corresponding Shafranov integrals being known to be linear in the magnetic diagnostic measurements;
c) A crucial point is the propagation of noise or errors onto the raw signals thereby to the modulation amplitudes and to the estimate of the position, which we discuss in more detail; in order to guarantee a reliable mapping *M* between the modulation sensitivity Q_{jk} and the plasma position, we found it necessary to add noise to the simulated raw diagnostic data, and also to generate a large number of samples. In this way, the structure of *M* was adapted automatically to realistic noise on all diagnostics.

Linear mapping *M:* The first step was to generate a linear mapping function *M*. The noise addition was preferred to relying on an SVD truncation to improve the conditioning. Figure 1 shows the quality of the mapping for 3 different noise levels used for generating the mapping and for polluting the data to be analysed. The noise level of the raw diagnostic signal was estimated and used to quantify the added diagnostic noise level The mapping *M* generated with no added noise is incapable of approximating the plasma position from the Q_{jk} estimated from noisy signals (top right). With 10% of the assumed diagnostic noise level, the mapping *M* was reliable when reconstructing the noisier and noise-free data. With 100% of the assumed diagnostic noise level, the mapping becomes sensitive to noise.

The quality of reconstruction of R was inferior to the quality of reconstruction of Z. We attempted different fits to the position information, including cross-terms RZ and quadratic terms R^2. We have been unable to satisfactorily explain this difference. Potential causes include:

a) stronger coupling due to approximate poloidal flux conservation (but we tried several other combinations);
b) non-linearities during the inside-outside limiting of the plasma, although we found no increase in the residuals for the extrema in R;
c) small range of equilibria from which to extract the linearised dependence; this is inevitable due to the proximity of the separatrix to the wall.

Non-linear mapping to the R and Z positions: In view of this initially encouraging but ultimately disappointing result for finding a suitable linear mapping, we generated a simple non-linear mapping using a one hidden layer Multi-Layer Perceptron mapping which provides a universal mapping between two spaces, in this case the 228 modulation amplitudes to the single variable R or Z, a method previously used and described for the case of equilibrium mapping [3]. Using noise-free data led to a less noise sensitive map than the pseudo-inverse, due to the slow convergence to a solution with negligible improvement to the fit, seen in Fig.2. However, learning with itermediate noisy data still led to a better reconstruction of the very noisy data, and we assume to a good reconstruction of the noise-free data. The results are expressed in Table 1. The reconstruction is never performed on the learned data sample. Each condition is shown for a mapping generated with 2 and 4 neurons in the hidden layer.

The most realistic assumption is shown in bold face in the table, corresponding to 10% noise added to the learned data and 0% noise added to the measured data to be reconstructed. The resulting residuals are 1.0 to 1.3mm for R and Z respectively, obtained with 2 neurons in the hidden layer. The cases with 4 neurons suffer from over-learning with a number of examples too close to the number of free parameters in the mapping, already 559 with 2 neurons.

Learned noise level	Neurons	Reconstructed with 0% noise		Reconstructed with 10% noise		Reconstructed with 100% noise	
		R [mm]	Z [mm]	R [mm]	Z [mm]	R [mm]	Z [mm]
0%	2	0.88	1.07	2.3	2.4	16.7	20.5
	4	0.84	1.04	2.2	3.1	17.2	27.8
10%	2	**1.0**	**1.3**	1.3	1.8	7.4	11.0
	4	0.9	1.2	1.3	1.7	7.6	13.3
100%	2	2.4	5.2	2.4	5.1	5.1	8.4
	4	3.1	5.5	3.2	5.8	6.5	11.7

Table 1. Residuals using a 1-hidden layer neural network [mm]. Columns are the noise level in the reconstructed data and the rows refer to the noise level in the learned data.

Effect of changing the equilibrium: These results provide the basis for confidence that the position of a given equilibrium can be reliably estimated from the modulated components. The question of sensitivity of the mapping *M* to changes in beta and li has to be addressed and the simulations have already been run to be analysed in the near future.

Noise level in the measured signal and required modulation amplitude: We selected 0% noise on the data for testing the quality of the reconstruction. This is justified by the reduction of the noise level for a given amplitude modulation by the square root of the modulation duration. The signal level is proportional to the modulation amplitude. We can therefore always increase the signal-to-noise level for a given tolerable amplitude by reducing the bandwidth of the new measurement. Since we are only trying to compensate drift, in principle extremely low frequency, this bandwidth will be extremely low. The modulation should be continuous stimulation, rather than pulsed stimulation, for the same reason, since the signal-to-noise of the modulated component varies as $\sigma \sim A\ \mathrm{sqrt}(\Delta t)$ where A is the amplitude of the modulation and Δt is the duration of the stimulation. Initial experiments have already been carried out on Tore Supra in a simplified form of the auto-calibration method, but the results illustrate the noise sensitivity and show clean demodulated strengths, but with fairly large modulation amplitude.

Application to ITER: Assuming superposition of the noise, the error on the modulation will diminish with the square-root of the demodulation window duration. The demodulation itself also improves by increasing the duration since the conditioning of the fit to the known frequencies improves as the number of complete cycles increases.

AC losses will increase linearly with the duration of the modulation and the coupling losses will increase with the square of the amplitude and the square of the frequency, $P_{AC} \sim A^2\ \omega^2\, \Delta t \sim \sigma^2\, \omega^2$. We assume that the modulation is sufficiently small that the hysteresis losses will be negligible. This relationship illustrates that the AC losses are minimised for a given signal-to-noise ratio by low modulation frequencies, but are independent of the mark to space ratio of the modulation, unlike the signal-to-noise ratio itself.

Model based calibration: An alternative approach, for the moment a gedanken approach, might be to take a validated plasma equilibrium response model for a given equilibrium, to calculate the closed loop responses of the diagnostics and position to any imposed voltage modulation $S_j(j\omega) = H(j\omega)\ U(j\omega)$ and to derive the same modulation sensitivities Q_{jk} for the nominal position. The response is then recalculated over a grid in [R,Z] and the same procedure followed by analogy. The fundamental approach can then be restated as finding combinations of the [R,Z] dependence of some of the transfer functions which are sensitive to [R,Z]. Considering the problem in this way illustrates some of the weaknesses and strengths. The first assumption is linearity of the diagnostic response, well established experimentally. The next assumption is that this linearity varies with position over short ranges, roughly a local linearisability of the Green's functions. On the negative side, when we divide by the position modulation, we are assuming rough independence of $H(j\omega)$ on $j\omega$, which applies carefully choosing the frequencies for a region in which the responses are flat or implying close frequencies.

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