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Dynamical Study of $q = 1$ Triple Kink-Tearing Modes

Andreas Bierwage[1], Satoshi Hamaguchi[1], Masahiro Wakatani[1†],
Xavier Leoncini[2], Sadruddin Benkadda[2]
[1] *Dept. Fundamental Energy Science, Kyoto University, Gokasho, Uji, 611-0011 Kyoto, Japan*
[2] *PIIM-UMR 6633 CNRS-Université de Provence, Centre Univ St Jérôme, case 321, 13397 Marseilles Cedex 20, France*
† *deceased*

Abstract

Configurations with three $q = 1$ resonant surfaces are studied with the aim of understanding partial sawtooth collapses. Preliminary results from nonlinear simulations show the formation of three magnetic islands which then lead to a flattening of the q-profile in an annular region between the resonant surfaces. It is likely that magnetic turbulence will develop in this region and there is evidence that this may occur even in the framework of the resistive reduced magnetohydrodynamic model, provided two or more $q = 1$ resonant surfaces are present.

Triple tearing modes arise in configurations where the safety factor profile has three resonant magnetic surfaces, in our case $q = 1$. They are more or less hypothetic, as, to our knowledge, there is no experimental evidence for such profiles so far. Our interest in their instability properties and the associated relaxation dynamics is motivated by the still incompletely understood phenomenon of sawtooth oscillations (see, e.g., [1], and also the review papers by Kuvshinov and Savrukhin [2] and Migliuolo [3]). The main problems are the sudden onset of the sawtooth crash, the rapidity of the relaxation and the possibility of partial sawtooth crashes.

The onset of the crash is often associated with the emergence of an internal $m = 1$ kink-tearing mode when the safety factor drops below unity in some region of the plasma (typically the center). This is the motivation for us to choose $q = 1$ resonances as the object of our study. The rapid-crash problem is generally believed to be associated with fast reconnection. Since we will use a resistive model, this problem is beyond our scope and we shall not discuss it further.

Our main focus will be on the problem of partial relaxation where the central value of the safety factor is found to be below unity after the crash, unlike the full-relaxation scenario as described by the Kadomtsev model [4]. While many studies were dedicated to configurations with monotonic q-profiles (one $q = 1$ resonant surface), we are interested in the effect of the presence of multiple $q = 1$ resonances. For $q_0 = q(r = 0) < 1$ this means that there must be at least three resonances, hence our focus on triple tearing modes. Triple-tearing configurations have, so far, attracted only relatively little attention, mainly in the context of trying to explain compound/double sawtooth oscillations (e.g., [5] [6]).

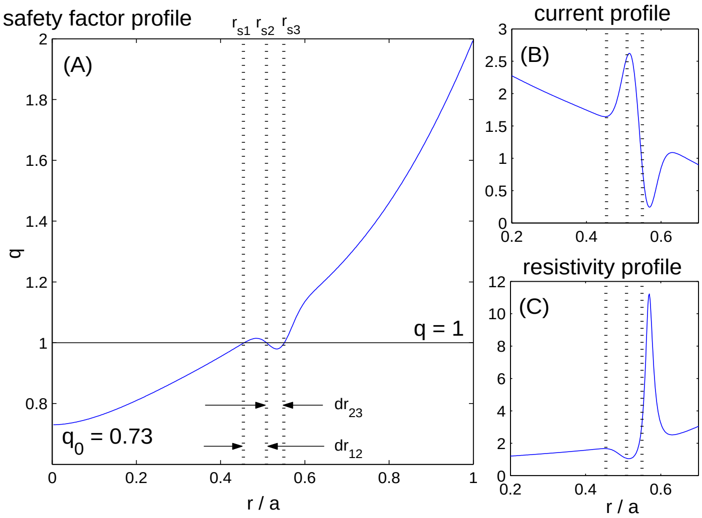

Figure 1: Triple-tearing configuration (A), with the corresponding current-density (B), and resistivity profile (C).

To study the characteristics of triple tearing modes, we use the reduced set of magnetohydrodynamic equations (RMHD) in the zero-beta limit. With the time normalized by the poloidal Alfvén time $\tau_{\mathrm{H_p}}$ and the radial coordinate by the minor radius a these well-known equations can be written as

$$\partial_t \psi = [\psi, \phi] - \partial_\zeta \phi - \frac{1}{S_{\mathrm{H_p}}} (\eta j_z - E_z) \tag{1}$$

$$\partial_t u = [u, \phi] + [j_z, \psi] + \partial_\zeta j_z + \frac{1}{Re_{\mathrm{H_p}}} \nabla_\perp^2 u. \tag{2}$$

$$\mathbf{B}/B_0 = \hat{\zeta} - \hat{\zeta} \times \nabla \psi, \quad j_z = \nabla \times \mathbf{B}/B_0 \approx -\nabla_\perp^2 \psi \quad \mathbf{v} \approx \mathbf{v}_{\mathrm{E\times B}} = \hat{\zeta} \times \nabla \phi$$
$$u = \nabla \times \mathbf{v} \approx \nabla_\perp^2 \phi \quad [f, g] = \frac{1}{r}\partial_r f \partial_\vartheta g - \frac{1}{r}\partial_r g \partial_\vartheta f, \quad \nabla_\perp^2 = \frac{1}{r}\partial_r r \partial_r + \frac{1}{r^2}\partial_\vartheta^2$$

Here, ψ is the magnetic flux function, ϕ the electric potential, j_z the toroidal current and u the vorticity. E_z is a constant electric field compensating the resistive dissipation of the current profile. $\eta = \eta(r)$ is the resistivity profile, normalized by $\eta_0 = \eta(r = 0)$. The Lundquist number $S_{\mathrm{H_p}}$ and the kinetic Reynolds number $Re_{\mathrm{H_p}}$ are chosen to be $S_{\mathrm{H_p}} = Re_{\mathrm{H_p}} = 10^6$. Our simulation code uses the quasi-spectral method and a two-time-step predictor-corrector scheme. In Fourier space each mode is labelled by a poloidal mode number m and a toroidal mode number n, e.g. $\psi(r, \vartheta, \zeta, t) = \sum_{m,n} \psi_{(m,n)}(r, t) e^{i(m\vartheta - n\zeta)}$.

Consider a flowless equilibrium defined by a hypothetic q-profile as the one shown in Fig. 1. There are three $q = 1$ resonant surfaces located at the radii r_{s1}, r_{s2} and r_{s3}. The central q-value is chosen to be 0.73. We are dealing with a collisional plasma with finite resistivity, so the system is indeed unstable to triple kink-tearing modes. In the following we shall limit ourselves to one single helicity given by $m/n = 1$.

In our study, we adopt a procedure commonly used in computational tearing mode studies: an unstable equilibrium is taken and a small flux perturbation applied. Perturbing the mode $(m, n) = (1, 1)$ in a triple-tearing configuration can lead to a situation

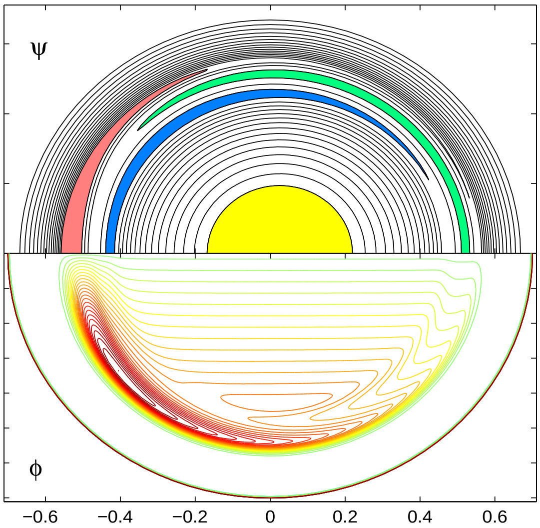

Figure 2: Magnetic island structures in a triple-tearing configuration.

as that shown in Fig. 2. This snapshot was taken during the saturation phase (shortly after the end of the linear growth phase). We observe three magnetic islands, one at each resonant surface, emerging with alternating X- and O-points. During the further evolution we observe that the q-profile is flattened in the region between the inner and outer resonant surface. Such a state is thought to occur after a partial sawtooth crash (e.g., [1]). Porcelli [1] proposed that magnetic turbulence may emerge in the region where the q-profile is flat. Suggested mechanisms for driving the turbulence are "the coupling of the $m = 1$ mode with the poloidal modulation of the equilibrium profiles" and "the secondary instability of modes, such as tearing-parity resistive g-modes" ([1] and references therein).

We do not include these effects, however, preliminary results from our simulations suggest that the triple-tearing mode as such may give a basis for the development of magnetic turbulence even in the framework of the resistive RMHD model. The same seems to be true for $q = 1$ double tearing modes. With further investigations, we expect to obtain insights into the interaction between magnetic reconnection and turbulence as well as multi-scale dynamics in general. The simplicity of the chosen model will aid a better understanding of fundamental processes at work.

In summary, during the nonlinear evolution of triple kink-tearing modes at $q = 1$ we have observed that the region between the resonant surfaces can be flattened before the Kadomtsev-type sawtooth collapse occurs. Magnetic turbulence can develop in that region and may significantly influence the further evolution of the global mode, possibly leading to a behavior phenomenologically similar to the partial sawtooth collapse.

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