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Low Dimensional Model Study of Intermittent Thermal Transport in Toroidal Ion-Temperature-Gradient Turbulent Convection

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Abstract

Ion temperature gradient (ITG) driven turbulence is studied using a low degree-of-freedom model composed of 18 ordinary differential equations (ODEs). In the strongly turbulent state, intermittent bursts appear due to the competition of the ITG mode, self-generated sheared flows and viscosity. The time averaged Nusselt number N_u with the ion temperature gradient K_i scales as $N_u \propto K_i^3$ in the presence of intermittency. On the other hand, the full PDE and the 1 dimensional simulations show $N_u \propto K_i^{1/3}$ like the Rayleigh–Bénard convection in the strongly turbulent regime.

1 Introduction

One of the most important issues in recent fusion research is to control anomalous transport caused by micro-turbulence. The ITG mode is a sort of micro-instabilities, and induces the anomalous ion thermal transport.

In order to understand the mechanisms of interaction between ITG driven turbulence and self-generated sheared plasma flows, we present and analyse a low degree-of-freedom model composed of 18 ODEs, which is expected to capture the essential nonlinear physics of the system.

In Sec. 2, the toroidal ITG equations and low dimensional models treated here are briefly explained. Numerical results are discussed in Sec. 3.

2 Models

ITG modes have 2 branches, i.e., slab and toroidal modes. We now consider the toroidal ITG mode described by the following vorticity and ion pressure equations in the 2 dimensional slab geometry [1];

$$\frac{\partial}{\partial t}(\nabla_\perp^2\phi - \phi) + [\phi, \nabla_\perp^2\phi] = (1 - g + K_i\nabla_\perp^2)\frac{\partial\phi}{\partial y} - g\frac{\partial p}{\partial y} + \mu\nabla_\perp^2\nabla_\perp^2\phi \qquad (1)$$

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and

$$\frac{\partial p}{\partial t} + [\phi, p] = -K_i \frac{\partial \phi}{\partial y} + \kappa \nabla_\perp^2 p \, , \tag{2}$$

where

$$[a,b] = \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x} \tag{3}$$

are Poisson brackets. In the equations, ϕ is the electrostatic potential, p the ion pressure, g the effective gravity due to magnetic curvature, K_i the ion pressure (or temperature) gradient, μ the viscosity and κ the thermal conductivity. Here $K_i = (1+\eta_i)\, T_i/T_e$ and $\eta_i \equiv L_n/L_{Ti} = \mathrm{d}\ln T_i/\mathrm{d}\ln n_0$.

Here the linear growth rate is discussed to estimate the most unstable wave numbers for given parameters. Assuming fluctuation of the form $\exp(\mathrm{i}k_x x + \mathrm{i}k_y y - \mathrm{i}\omega t)$ and linearizing Eqs. (1) and (2), we obtain the dispersion relation as follows,

$$\left[-\mathrm{i}(1+k_\perp^2)\omega + \mathrm{i}k_y(1-g-K_i k_\perp^2) + \mu k_\perp^2\right]\left(-\mathrm{i}\omega + \kappa k_\perp^2\right) - gK_i k_y^2 = 0 \, . \tag{4}$$

This is a quadratic equation which can be solved with respect to ω as

$$\omega = \frac{1}{2(1+k_\perp^2)}\left\{k_y(1-g-K_i k_\perp^2) - \mathrm{i}\left[\kappa k_\perp^2(1+k_\perp^2) + \mu k_\perp^4\right] \pm \sqrt{D_k}\right\}, \tag{5}$$

where

$$D_k = \left\{k_y(1-g-K_i k_\perp^2) + \mathrm{i}\left[\kappa k_\perp^2(1+k_\perp^2) - \mu k_\perp^4\right]\right\}^2 - 4(1+k_\perp^2)gK_i k_y^2. \tag{6}$$

From the linear growth rate $\gamma = \mathrm{Im}(\omega)$, the most unstable wave numbers are estimated as

$$k_x^2 \ll k_y^2 \sim k_\perp^2 = \frac{1-g}{K_i} \, . \tag{7}$$

In this study, we assume $k_x = k_y/2$ for simplicity, and set

$$k_x = k_y/2 = [(1-g)/5K_i]^{1/2} \, , \tag{8}$$

so that this mode satisfies Eq. (7).

For numerical simulations, we use the spectrum method in the x and y directions. In order to derive a low dimensional model considered here, ϕ and p are expanded into the Fourier series and truncated in 18 low order components as follows,

$$\begin{aligned}
\phi(x,y,t) &= \phi_1^0(t)\sin(k_x x) + \phi_2^0(t)\sin(2k_x x) + \phi_3^0(t)\sin(3k_x x) \\
&\quad + \phi_1^c(t)\sin(k_x x)\cos(k_y y) + \phi_1^s(t)\sin(k_x x)\sin(k_y y) \\
&\quad + \phi_2^c(t)\sin(2k_x x)\cos(k_y y) + \phi_2^s(t)\sin(2k_x x)\sin(k_y y) \\
&\quad + \phi_3^c(t)\sin(3k_x x)\cos(k_y y) + \phi_3^s(t)\sin(3k_x x)\sin(k_y y) \, , \qquad (9) \\
p(x,y,t) &= p_1^0(t)\sin(k_x x) + p_2^0(t)\sin(2k_x x) + p_3^0(t)\sin(3k_x x) \\
&\quad + p_1^c(t)\sin(k_x x)\cos(k_y y) + p_1^s(t)\sin(k_x x)\sin(k_y y)
\end{aligned}$$

$$+p_2^c(t)\sin(2k_xx)\cos(k_yy)+p_2^s(t)\sin(2k_xx)\sin(k_yy)$$
$$+p_3^c(t)\sin(3k_xx)\cos(k_yy)+p_3^s(t)\sin(3k_xx)\sin(k_yy)\ , \qquad (10)$$

where k_x (k_y) is the minimum wave number defined by Eq. (8). It is noted that ϕ_1^0, ϕ_2^0 and ϕ_3^0 represent the sheared flows. By substituting Eqs. (9) and (10) into Eqs. (1) and (2), 18-coupled ordinary differential equations are obtained for each Fourier coefficients [3, 4].

3 Results and Discussion

Equations (9) and (10) are solved numerically by the 5th order Runge–Kutta method. When the system is close to the threshold of ITG instability, an L-H-like transition occurs in the kinetic energy and Nusselt number fluctuations [4]. As K_i is slightly increased, the system is in the convection regime and convective heat transport exhibits periodic oscillations. As the ion temperature gradient is increased further, the system bifurcates to a turbulent regime. In the strongly turbulent state, edge localized modes (ELM)-like oscillations associated with intermittent bursts in the kinetic energy and the convective flux are observed [3, 4] unlike the previously presented low degree-of-freedom model composed of 11 ODEs [2]. Figure 1 and 2 show time evolutions of kinetic energy for each harmonics and the Nusselt number N_u for $K_i = 5$. Here the Nusselt number characterizes the convective heat flux and defined as

$$N_u(t) = \int \kappa K_i + pv_x\frac{\mathrm{d}V}{V}\Big/(\kappa K_i) = 1 + \frac{\langle pv_x\rangle_V}{\kappa K_i}\ . \qquad (11)$$

This behaviour is due to competition between 3 factors of the ITG mode, the sheared flows and the viscosity, and is classified into 3 phases: (a) generation of sheared flows and suppression of ITG turbulence, (b) gradual reduction of the sheared flows due to viscosity, and (c) rapid regrowth of ITG modes due to the reduction of stabilizing effect by the sheared flows.

Dependence of the Nusselt number N_u on the ion temperature gradient K_i indicates transition of transport regimes and is expressed as $N_u \propto K_i^3$ in the presence of the intermittency [4]. This scaling is quite defferent from the full PDE and the 1D simulations results which show $N_u \propto K_i^{1/3}$ in the strongly turbulent regime like the Rayleigh–Bénard convection or the resistive interchange mode [5]. Here R_a is the Rayleigh number.

Intermittent oscillations are the essential dynamics of the system in the sense that such phenomena are also obtained by the 1D model and the full PDE simulations, although these solutions show more complex behaviour. The 18 ODE model is the minimal and useful model to study the mechanism of intermittent oscillations of the system.

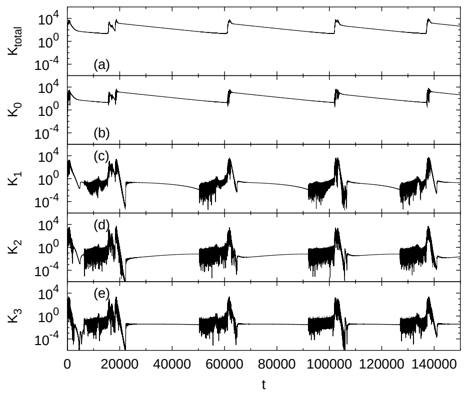

Figure 1: Time evolution of (a) total kinetic energy and kinetic energy of (b) sheared flows K_0, (c) the 1st harmonics K_1, (d) the 2nd harmonics K_2 and (e) the 3rd harmonics K_3 for $K_i = 5$.

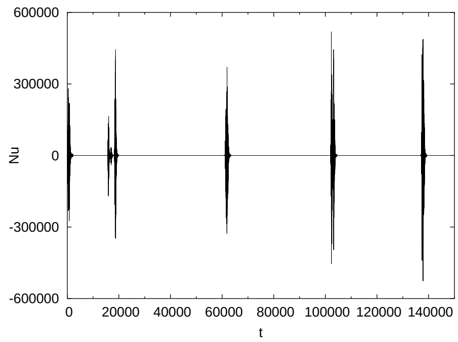

Figure 2: Time evolution of the Nusselt number N_u for $K_i = 5$.

As a conclusion, the 18 ODE model is useful to study the intermittent oscillations, however the 1D model is needed for transport analysis.

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