Plasma Self-induced Current in Reversed Field Pinch with Low Aspect Ratio

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Abstract

It is found that the plasma self-induced current becomes dominant (~100%) by taking account of the additional bootstrap current due to the finite banana-width effect of fusion-produced alpha particles in the low aspect ratio neoclassical RFPs with a relatively flat pressure profile, thereby minimizes non-inductive seed current to generate the steady state configuration, its hollow current profile makes the magnetic shear or the stability beta high and its parallel current makes the force-free field dominant, in the relaxed-equilibrium state.

1. Introduction

The previous report showed that the low aspect ratio neoclassical RFP equilibrium, which was solved self-consistently considering the bulk bootstrap current and the Ohmic current, gave an economical attractiveness of reducing the auxiliary current required to attain the equilibrium and enhancing the net electric power, hence lowering the cost of electricity for the fusion power plant [1]. In this report, the additional plasma self-induced current due to the finite banana-width effect of trapped particles and the non-inductive seed current by means of radiofrequency current drive (RFCD) to attain the steady state neoclassical RFP equilibrium with low aspect ratio are considered beside the bulk bootstrap current.

2. Finite banana-width effect on parallel current

In the neighborhood of magnetic axis, a particle orbit is quite different from the conventional banana orbit and a thin-banana approximation in the standard neoclassical transport theory is broken down. The finite banana-width effect is shown to modify the conventional bootstrap current around the magnetic axis satisfying the condition, \( \sqrt{\varepsilon} < \delta_{s}^{1/3} \), where \( \varepsilon = r / R_{0} \) and \( \delta_{s} = 2q_{0}\rho_{a} / \kappa R_{0} \) with the major radius \( R_{0} \), the elongation parameter \( \kappa \), the safety factor at the magnetic axis \( q_{0} \) and the Larmor radius \( \rho_{a} \) for species \( a \) [2]. This modification is significant for high energy particles, indicating that the fraction of trapped particles is increased owing to the finite banana-width effect.
3. Plasma self-induced currents

The total toroidal current in MHD equilibrium is given by

\[ I_T = (1/2\pi) \int (dp/\partial \psi) (\langle B_{\phi}^2 \rangle / \langle B^2 \rangle) \, dV + (1/2\pi) \int F (\langle j \cdot B \rangle / \langle B^2 \rangle) <1 / R^2 > \, dV, \]

where \( V \) is the volume of the region enclosed by a flux surface, \( F = R B_\phi (B; \text{toroidal magnetic field}) \) is toroidal field function, \( F \) and pressure \( p \) are function of only a poloidal flux function \( \psi \) and \( \langle X \rangle \) denotes the flux surface average of \( X \). The first term gives a substantial contribution to the confinement for usual RFPs being \( B_p \geq B_t \) although it too small to contribute confinement for usual tokamaks. Hereafter it is called “P-S current (\( I_{PS} \))”, which includes the diamagnetic current and the P-S current. The second term represents the contribution from the parallel current necessary to maintain the equilibrium parallel momentum balance along the magnetic field, which includes the bootstrap current and the Ohmic current. The bootstrap current is composed of the bulk current (\( I_{\text{BS}} \)) depending on plasma pressure / temperature profiles [3] and the additional bootstrap current (\( I_{\text{BS}}^\alpha \)) due to the finite banana-width effect of high energy particles around the magnetic axis satisfying the condition \( \epsilon^{1/2} < \delta_s^{1/3} \). The bulk bootstrap current increases with lowering aspect ratio since it arises owing to anisotropy in electron pressure tensor or viscosity, which is neoclassical (toroidal geometry) effect. The additional bootstrap current becomes relatively large in the case of small \( R_t \) and then large \( \delta_s \). Accordingly the plasma self-induced current is expected to be increased in low aspect ratio neoclassical RFPs.

The bootstrap current due to the fusion-produced alpha particles by taking account of the finite banana-width effect is written as

\[ j_\alpha = (B^2 / \langle B^2 \rangle) \left\{ (m_\alpha v_\alpha^2 \gamma / \partial \psi) / 2B_p \right\} \langle B_{\phi}^2 \rangle \]

where \( m_\alpha \) is the mass for the alpha particles; \( v_\alpha \) is the birth velocity; \( \partial \psi / \partial r \) is the birth rate of alpha particles; \( \gamma \), is the slowing-down time; \( v_e \), is the critical velocity defined as \( (3 \sqrt{\pi} / 4) \Sigma_j (m_i n_i e_j^2 / m_e n_e e^2) v_e \) with \( \Sigma_j \) meaning a summation over only ion species and \( v_e \) electron thermal velocity; \( L_{\alpha}^\alpha \) and \( L_{\alpha}^{\gamma} \) are the transport coefficients [2]. The alpha particle-induced bootstrap current is accompanied by an electron return current due to the parallel momentum transfer from alpha particles. The resulting net current density is written as \( j'' = j_\alpha [1 - (Z_e / Z) F] \), where \( 1 - (Z_e / Z) F \) represents the shielding factor due to the electron return current, therefore becomes zero on the axis at the critical effective charge number \( Z_e \). For the RFPs with \( q_t < 1 \), \( Z_e \) is expected to be smaller than 1.1, then the net bootstrap current on the axis becomes positive when \( Z > 1.1 \).
4. Steady state neoclassical RFP equilibrium

Especially, in the neoclassical MHD equilibrium with the aspect ratio of \( A = 2 \) and a relatively flat pressure profile, which is solved self-consistently considering the bulk bootstrap current and the Ohmic current, the current profile of a hollow type aligns well with the plasma self-induced current profile because \( I^\text{eq}_q \) (total toroidal current in the equilibrium) \( \sim I_p [1] \). The Ohmic current \( (I^\text{Oh}_q) \) should be replaced for the steady state configuration with the alpha particle-induced bootstrap current and the non-inductive seed current. The alpha particle-induced bootstrap current \( (I^\text{a}_q) \) flows in a wide region with the fraction of 0.174 to \( I^\text{eq}_q (=30.8 \text{MA}) \) for the D-T-fueled plasma with \( \delta_\alpha = 0.05 \) at \( Z = 2.0 \) and \( T_o = 35 \text{keV} \) as shown in Fig. 1. The RF-driven toroidal current \( (I^\text{RF}_q) \) is required to compensate for the Ohmic current in conjunction with the alpha particle-induced bootstrap current making a central safety factor \( q_0 \), smaller than unity. The required RF current is \( I^\text{RF}_q \sim 0.38 I^\text{eq}_q \) = 1.17MA in the normalized flux surface region of \( \psi = 0 \sim 0.11 \) (\( \psi = 0 \) at the magnetic axis). Using low frequency fast wave (LFFW; \( \omega \sim 2 \Omega_\alpha \)) providing the seed current in the core region with a high density / temperature, the requisite RF power \( (P_{\text{CD}}) \) is given by

\[
I^\text{RF}_q[A] / P_{\text{CD}}[\text{W}] = (0.122 <T_e>[\text{keV}] / R_p[\text{m}] <n_e>[\text{m}^{-3}]) \ln \Lambda \times \frac{f}{p^s}
\]

where \( \frac{f}{p^s} \) is the normalized current drive efficiency and the function of parallel wave phase velocity normalized to electron thermal velocity and \( \ln \) the Coulomb logarithm \( (\sim 15) \), which depends on the current driving system only. The RF power spectrum is selected in order to that RFCD should create a current profile \( G(\psi) = \frac{<j \cdot B>^\text{RF}}{<B^2>} \sim \frac{<j \cdot B>^\text{eq}}{<B^2>} - H(\psi) \), where \( H(\psi) \) is the bootstrap current density. The value of \( \frac{f}{p^s} \) has been known to be 0.0175 from the RFCD in the dominant bootstrap current-aided reverse shear (RS) tokamak with a similar \( G(\psi) \)-profile using the same RF (Low Frequency Fast Wave, \( \omega \sim 2 \Omega_\alpha \)) current driving system [4]. Then the requisite RF power for \( I^\text{RF}_q \sim 1.17[\text{MA}] \) is evaluated to be \( P_{\text{CD}} = 11.0[\text{MW}] \) for the D-T-fueled plasma and \( P_{\text{CD}} = 3.5[\text{MW}] \) for the D-\(^3\text{He}\)-fueled plasma. Each composition of toroidal currents in the steady state neoclassical RFP equilibrium with \( A = 2.0, \kappa / \delta = 1.4 / 0.4 \) is listed in Table I.

Simultaneously the hollow current profile enhances the plasma stability beta to \( \beta_1 \) (toroidal beta) = 63% stable against both ideal kink and Mercier’s localized modes, making the magnetic shear increase locally and globally and suggesting the relaxed-equilibrium state predicted by Lyapunov functional [5]. The plasma parameters for the steady state configuration of neoclassical RFP equilibrium with low aspect ratio are listed in the Table II. The economical analysis on the steady state fusion power plant at the design point of the plasma parameters leads to the lowest cost of electricity in the used cost algorithm [6].
Conclusion

According to the conventional neoclassical transport theory, the bootstrap current density is well known to be zero at the magnetic axis. However, it is suggested that finite banana-width effect leads to the possibility of the completely bootstrapped RFP. Indeed, the low aspect ratio neoclassical RFP equilibrium has a good alignment with plasma self-induced current profile, which reduces the requisite power of non-inductive rf current drive to generate steady state configuration. The resulting configuration has a strong magnetic shear due to the hollow current profile giving the high stability beta of $\beta_t = 63\%$ and a dominant force-free field in the relaxed-equilibrium state.

References


Table I

The composition of toroidal currents in the steady state neoclassical RFP equilibrium without Ohmic current ($A = 2.0, \kappa = 1.4/0.4$).

<table>
<thead>
<tr>
<th>$L_{\psi}^{(\text{ii})}$ [MA]</th>
<th>$L_{\psi}^{(\text{BS})}$ [MA]</th>
<th>$L_{\psi}^{(\text{RF})}$ [MA]</th>
<th>$L_{\psi}^{(\text{a})}$ [MA]</th>
<th>$L_{\psi}^{(\text{SF})}$ [MA]</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.8</td>
<td>17.8</td>
<td>6.39</td>
<td>5.35</td>
<td>1.18</td>
</tr>
<tr>
<td>57.8%</td>
<td>20.7%</td>
<td>17.4%</td>
<td>3.8%</td>
<td></td>
</tr>
</tbody>
</table>

Table II

The plasma parameters for the steady state neoclassical RFP equilibrium.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\kappa$</th>
<th>$\beta_t$</th>
<th>$\rho_i$</th>
<th>$F$</th>
<th>$B_0$</th>
<th>$B_e$</th>
<th>$L_{\psi}^{(\text{ii})}$</th>
<th>$\langle n_e \rangle_1$</th>
<th>$\langle T_e \rangle_1$</th>
<th>$\langle n_e \rangle_2$</th>
<th>$\langle T_e \rangle_2$</th>
<th>$E_\perp$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-T</td>
<td>2.0</td>
<td>1.4</td>
<td>63</td>
<td>0.10</td>
<td>2.8</td>
<td>3.0</td>
<td>30.8</td>
<td>1.10</td>
<td>1.26</td>
<td>1.88</td>
<td>29.6</td>
<td>0.96</td>
<td>2.0</td>
</tr>
<tr>
<td>D-$^3$He</td>
<td>2.0</td>
<td>1.4</td>
<td>63</td>
<td>0.10</td>
<td>2.8</td>
<td>6.0</td>
<td>30.8</td>
<td>1.10</td>
<td>1.26</td>
<td>4.40</td>
<td>50.0</td>
<td>0.96</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Fig. 1 The steady state neoclassical RFP equilibrium replaced for the Ohmic current with both alpha particle-induced bootstrap current and RF (LFFW)-driven current to have the central safety factor of $q_0 \leq 1.0$; (a) magnetic flux surface; (b) safety factor as function of normalized poloidal flux; (c) toroidal current density in the midplane: solid curve is target current density, short dashed curve is from alpha particle-induced bootstrap current, chain dashed curve is from the bulk bootstrap current density, dotted curve is from “P-S” current density ad chain dotted curve is from RFCD.