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Analytic investigations on tearing mode

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The understanding of the saturation process of the tearing mode in a Tokamak plasma is an important issue in Magnetohydrodynamics. The aim of the two preliminary analytic investigations summarized here is to set a new and simple procedure for a more complete treatment of the nonlinear evolution of tearing instability in realistic geometry (i.e. cylindrical geometry).

In the first work (F. Militello, G. Huysmans, M. Ottaviani and F. Porcelli, *Phys. Plasmas* **11** (2004), 125), it is shown that local features of the equilibrium current density profile, i.e., its gradient and curvature evaluated at the resonant magnetic surface, can change considerably the linear behavior of the asymmetric tearing modes, i.e., modes for which the perturbed magnetic flux has a mixed parity in the radial direction across the resonant layer. The first analysis on this issue was developed by Bertin (*Phys. Rev. A.* **25**, 1786 (1982)) that modified the standard tearing mode dispersion relation (H. P. Furth, J. Killeen and M. N. Rosenbluth, *Phys. Fluids* **6** (1963), 459) introducing the effects of local and global asymmetries. Our work extends and improves that of Bertin, resolving the big discrepancy that is sometimes observed between Bertin's dispersion relation and numerical computations of tearing mode growth rates. In particular, we treat correctly a logarithmic dependence of the perturbed magnetic flux as a function of the distance from the resonant layer and we discuss a new effect associated with the local curvature of the equilibrium current density (i.e. the second derivative of $J_{eq}(r)$ at the resonant layer). As a result, we show that corrections to the standard dispersion relation become important for moderate values of resistivity, typically $\eta \sim 10^{-4} - 10^{-5}$ (η is the inverse of the Lundquist number). Although these values are unrealistically large if we refer, for instance, to present-day magnetic fusion experiments, they become relevant for laboratory magnetic reconnection experiments (e.g., the Magnetic Reconnection Experiment, MRX, in Princeton) and for numerical simulations.

A stability analysis leads to a boundary layer problem that corresponds to the matching of the dissipative solution in the resistive region around the rational surface, onto the ideal outer solution. The latter, in the vicinity of the rational surface, can be expressed through a series expansion containing four parameters which depend on the equilibrium configuration and on the boundary conditions. These parameters are: a large scale asymmetry index (A), a local asymmetry index (a), an index representing the curvature of the equilibrium current density (b) and the tearing mode standard linear stability parameter (Δ'). In the inner region we look for solution in the form:

$$\tilde{\psi}_{in} = \left[1 + c_1 x + (c_2/2)x^2 + (c_3/2)x\log\left(x^2+\delta^2\right) + (c_4/2)x^2\log(x^2+\delta^2)\right]\psi_0 + \psi_1(x/\delta)$$

where $\delta \propto \eta^{2/5}$ measures the scale of the resistive layer width, x represents the distance from the rational surface and the function ψ_1 is a small correction. This *ansatz*, that generalizes the standard constant-ψ scheme, grants particularly useful features to the solution. In particular, it resolves the logarithmic singularity of the outer solution, regularizing the magnetic flux inside the resistive layer and implies that the asymptotic matching to the outer solution for $x > \delta$ is automatically achieved with the correct choice of the constants $c_1..c_4$. The terms related to constants c_2, c_3, c_4 were not treated in Bertin's calculation, but produce significant extra contributions to the final dispersion relation. Using the matching condition it is possible to obtain the new dispersion relation:

$$\alpha_1 \gamma\delta/\eta = \Delta' + \alpha_1 F\delta$$

where $F=[(aA/2)+a^2\log(\delta)-1.31a^2+b]$, $\alpha_1 \approx 2.1$ and γ is the growth rate normalized to the relevant Alfven time. Note that only the first term in the expression for F appears in Bertin's work. The second and third terms originate from the logarithmic terms in the *ansatz*. Note also that the

$a^2\log(\delta)$ term prevails over the others. The last term, b, is the contribution due to the local curvature of J_{eq}. The discrepancy between the standard dispersion relation and the modified dispersion relation is significant when: $\eta^{2/5}\ln(\eta) \sim \Delta'^{4/5}/a^2$.

In conclusion, a new analytic dispersion relation is derived, which takes into account in a consistent way the effects of the asymmetries and of the local curvature of the equilibrium current density and which agrees very well with numerically computed growth rates. These features can become particularly important in cylindrical geometry, at moderate values of the electrical resistivity and of the tearing stability parameter, Δ'.

In the second work (F. Militello and F. Porcelli, *Phys. Plasmas* **11** (2004), issue 5, pp. L13-L16), a simple non-linear treatment of the reduced Magnetohydrodynamics equations it is presented, that describes the saturation of small width magnetic islands in symmetric equilibrium configurations. The tearing mode is, for its nature, a strongly nonlinear instability. Rutherford (*Phys Fluids* **16**, 1903 (1973)) gives the most convincing description of the nonlinear growth of magnetic islands. On the other hand, the saturation process is not described by the Rutherford theory. The first analysis of tearing mode saturation, based on quasilinear techniques, was developed a few years later (R. B. White, D. A. Monticello, M. N. Rosenbluth and B. V. Waddel, *Phys Fluids* **20**, 800 (1977)). The validity of the saturation criterion suggested by White *et al*. was questioned by Biskamp (D. Biskamp, *Nonlinear Magnetohydrodynamics* (Cambridge University Press, Cambridge, UK, 1993)), that showed that the theoretical predictions do not fit accurately the numerical results. On the contrary, the work that we present here gives a clear procedure to obtain an expression for the saturated island width which does not rely on the quasilinear approximation, and which agrees very well with numerical results.

Our analysis is carried out in the frame of a slab 2D geometry and the physics of the problem is described by reduced MHD equations. If the island width is small, this implies the formation of a narrow inner layer where the dissipation and the nonlinearities play an important role, and an ideal region outside this layer, where the linear solution is still valid. Also in this case, we expand the outer solution in the vicinity of the rational surface and we match it to the inner nonlinear solution. In order to obtain the inner solution, we solve the boundary layer equations neglecting inertia. From the vorticity equation we find that the current density is a function of the magnetic flux ψ only. Now, averaging Ohm's law over constant ψ surfaces, we obtain the nonlinear current density in the inner region: $J(\psi)=\langle J_{eq}\rangle_{\psi}$, where J_{eq} is the equilibrium current density (that we assume as given). In order to calculate the average, we assume that the shape of the perturbed magnetic flux function does not change considerably from the linear to the nonlinear saturated regimes. This means that even in the nonlinear phase, the fundamental harmonic is the only significant one, as far as ψ is concerned and that we have a 'nonlinear constant-ψ ' approximation. The validity of this assumption is verified *a posteriori*. From the matching condition we obtain the saturated island width relation:

$$w_s = 2.44 L^2 \Delta'$$

where L represents the length of variation of the equilibrium magnetic field, w_s is the saturated island width and Δ' is the standard linear stability parameter. It is important to note that w_s does not depend on the resistivity.

In conclusion, it is found that the symmetric tearing mode saturates at a finite amplitude, w_s, which is proportional to the linear stability parameter, Δ'. The correct value of the proportionality coefficient between w_s and Δ' is obtained in this paper, in agreement with numerical simulations. While this work was being completed, we became aware of a similar result recently obtained with a different technique by D. Escande and M. Ottaviani (*Phys. Letters A* **323,** 278 (2004)).