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# Collisional damping of Alfvén eigenmodes on localized electrons in stellarators

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Alfvén instabilities driven by the energetic ions were observed in many experiments on stellarators. Theory predicts that they can arise also in reactor plasmas [1]. The destabilization of various Alfvén eigenmodes (AE) occur when the drive produced by the energetic ions exceeds the wave damping. Therefore, it is of importance to investigate not only the energetic particle drive (which was done in Ref.[1]) but also various damping mechanisms. To study one of them – the collisional damping on electrons – is the purpose of this work. The mentioned damping can play an important role in tokamaks, which was shown for the Toroidicity-induced Alfvén Eigenmodes (TAE). The role of the collisional damping in stellarators is not clear because it is not studied for stellarator plasmas yet. On the other hand, various AEs associated with absence of the axial symmetry of the magnetic configuration do exist in stellarators. Because of this, in many cases even rough estimates of the collisional damping cannot be done using results obtained for tokamaks. Furthermore, one can expect that a new physics of the collisional damping is involved in stellarators. The matter is that the collisional damping is determined mainly by the barely trapped particles, which are subject to the collisionless orbit transformations.

We begin with the simplest case when the equilibrium magnetic field, $B_0$, is

$$B_0 = \overline{B}[1-\varepsilon_t(r)\cos\theta - \varepsilon_h(r)\cos\eta] \ , \qquad (1)$$

where (r, θ, φ) are the flux coordinates, $\overline{B}$ is the average magnetic field at the magnetic axis, $\varepsilon_t$ is the magnitude of the toroidal modulation, $\varepsilon_h$ is the magnitude of the helical ripple, $\eta = \theta - N\varphi$ is the ripple phase, and $N$ is the number of field periods. We assume that the following two conditions are satisfied. First, we assume $\iota << |\iota - N|$, with $\iota$ the rotational transform, which means that θ remains nearly constant during the motion along the magnetic field within a ripple. Second, we take $\varepsilon_t\iota / (\varepsilon_h\,|\iota - N|) << 1$. In this case, the toroidicity of the magnetic field can be considered as a perturbation to the sinusoidal symmetry of the helical ripple well, so that only particles localized in this wells satisfy the condition $0 \le \kappa^2 \le 1$, whereas non-localized particles are locally passing and characterized by $\kappa^2 > 1$. Here $\kappa$ is the trapping parameter given by

$$\kappa^2 = \frac{\mathrm{E}/\mu_p \overline{B} - 1 + \varepsilon_t \cos\theta + \varepsilon_h}{2\varepsilon_h},$$

where E, $\mu_p$ is the particle energy and magnetic moment, respectively. In contrast to tokamaks, $\kappa$ is not a constant of the motion, allowing for the collisionless orbit transformations between the locally trapped states and locally passing states:

$$\left\langle \frac{d\kappa^2}{dt} \right\rangle_O \approx -2\left( \Omega_E - \frac{\Omega_h}{2\varepsilon_h} \right)\varepsilon_t \sin\theta \left( \frac{E}{K} - 1 + \kappa^2 \right) \quad , \tag{2}$$

where $\langle .. \rangle_O$ denote bounce averaging, $\Omega_E = -(c / r\overline{B})\partial\Phi / \partial r, \Omega_h = (c\mathrm{E}/ e\overline{B} r)\partial\varepsilon_h / \partial r$, with $\Phi$ the ambipolar potential, and *K, E* are complete elliptic integrals.

We use the linearized drift kinetic equation for the perturbed electron distribution function

$$\frac{df}{dt} + \tilde{\vec{V}}_E \bullet \nabla F_M + v_\parallel \frac{\tilde{\vec{B}}}{B_0} \bullet \nabla F_M - \frac{eF_M}{T} v_\parallel \tilde{E}_\parallel - \frac{eF_M}{T} \vec{V}_D \bullet \tilde{\vec{E}} = C(f), \tag{3}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \left( v_\parallel \frac{\vec{B}_0}{B_0} + \vec{V}_D \right) \bullet \nabla + \frac{d\kappa^2}{dt} \frac{\partial}{\partial \kappa^2} \quad ,$$

$$\tilde{\vec{V}}_E = c \frac{\tilde{\vec{E}} \times \vec{B}_0}{B_0^2}, \vec{V}_D = -\frac{v_\perp^2 / 2 + v_\parallel^2}{\omega_B} \nabla(\ln B_0) \times \frac{\vec{B}_0}{B_0} \quad ,$$

$\omega_B = eB_0 / M c$ with *M* the particle mass, $F_M$ is the equilibrium Maxwellian distribution, *C(f)* is the collisional term, and spatial derivatives are taken at constant E, $\kappa^2$.

In terms of the potentials $\zeta$, $\psi$ defined by $\tilde{B}^r \equiv \tilde{\vec{B}} \bullet \nabla r = \vec{B}_0 \bullet \nabla \varsigma$, $\tilde{E}_\parallel = -(\vec{B}_0 / B_0) \bullet \nabla \psi$ the perturbed distribution can be split into the adiabatic part and non-adiabatic part:

$$f = -\frac{dF_M}{dr}\varsigma + \frac{e\psi}{T} F_M + f_{na}. \tag{4}$$

Substituting Eq. (4) into Eq. (3), after some manipulations we obtain the following equation for the non-adiabatic part of the distribution function $f_{na}$:

$$\vec{v}_{\|} \bullet \nabla f_{na} = i\omega f_{na} + C(f_{na}) - \frac{d\kappa^2}{dt}\frac{\partial f_{na}}{\partial \kappa^2} - i\omega\left(\frac{\mathrm{E}}{T} - 2\right) F_M \left(\frac{d\varepsilon_h}{dr}\varsigma\cos\eta - i\frac{\varepsilon_h}{m}\frac{\partial\varsigma}{\partial r}\sin\eta\right), \quad (5)$$

where *m, n* are toroidal and poloidal mode numbers, respectively. Due to the conditions $\vec{v}_{\|} \bullet \nabla >> \omega >> \max\{d\kappa^2/dt, \nu_e/\varepsilon_h\}$ with $\nu_e$ the electron collision frequency, Eq.(5) can be solved perturbatively. With boundary condition $f_{na}=0$ at the locally trapped – locally passing boundary, a solution can be written as:

$$f_{na} = -\exp\left[\frac{im\iota\eta}{|\iota - N|}\right] F_M (1+y)\left(\frac{\mathrm{E}}{T} - 2\right)\left\langle\left(\varsigma\frac{d\varepsilon_h}{dr}\cos\eta - i\frac{\varepsilon_h}{m}\frac{\partial\varsigma}{\partial r}\sin\eta\right)\exp\left[-\frac{im\iota\eta}{|\iota - N|}\right]\right\rangle_O, \quad (6)$$

$$y(\xi) = -\exp\left[-\int_0^{\xi}\sigma(\xi')d\xi'\right],$$

$$\sigma(\xi) = \left[\left(\frac{\varepsilon_t\varepsilon_h}{\nu_e}\right)^2\left(\Omega_E - \frac{\Omega_h}{2\varepsilon_h}\right)2\sin^2\theta_0 - i\frac{\varepsilon_h\omega}{2\nu_e}\ln\frac{16}{\xi}\right]^{1/2} - \frac{\varepsilon_t\varepsilon_h}{\nu_e}\left(\Omega_E - \frac{\Omega_h}{2\varepsilon_h}\right)\sin\theta_0,$$

where $\theta_0$ is the poloidal angle of the orbit transition determined by $\kappa^2(\theta_0)=1$, and $\xi \equiv 1-\kappa^2$. Now we proceed to a more realistic magnetic configuration, relevant to the Wendelstein-line stellarators. Then $B_0$ contains more Fourier harmonics, the mirror harmonic, $\varepsilon_m$, being dominant. Assuming that $\varepsilon_m$ weakly depends on *r*,. we can easily generalize the electron response given by Eq. (6): the mirror harmonic affects only the fraction of the localized particles, therefore, we replace $\sqrt{\varepsilon_h} \to (\varepsilon_m^2 + \varepsilon_h^2)^{1/4}$; in addition, $\left\langle d\kappa^2/dt\right\rangle_O$ should be modified to take into account diamagnetic precession of the localized particles.

The damping rate can be calculated perturbatively by including the dissipative part of the transverse current caused by the wave-particle interaction to the corresponding vorticity equation. In general, this requires a numerical calculation. However, simple expressions can be obtained for the modes localized near the radius where two cylindrical Alfven branches *(m, n)* and $(m+\mu,\ n+\nu N)$ intersect, i.e., in the high mode number limit (here $\mu, \nu$ are the coupling numbers).

We revealed two regimes relevant to stellarators: (a) when the collisionless orbit transformation plays an important role $[\left\langle d\kappa^2/dt\right\rangle_O^* >> (\nu_{eff}\omega)^{1/2}$, where

$\left\langle d\kappa^2/dt\right\rangle_O^* \equiv \left\langle d\kappa^2/dt\right\rangle_O K(\kappa)\,, \nu_{eff} \equiv \nu_e/\varepsilon_h]$ (b) when the role of the orbit transformations is negligible $[\left\langle d\kappa^2/dt\right\rangle_O^* << (\nu_{eff}\omega)^{1/2}]$.

In the regime (a) we obtain for the damping rate:

$$\gamma_{\nu=1,2} \approx -\mathrm{K}_\nu \frac{19\sqrt{2}}{16}\beta_e\left(\frac{R_0}{r_0}\right)^2 \varepsilon_h^{5/2}\left[\ln\frac{\nu_e\omega^2}{|\Omega_E-\hat{\Omega}_h/2\varepsilon_h|^3\,2\varepsilon_h\varepsilon_t^3}\right]^{-2}\varepsilon_t\left|\Omega_E-\frac{93}{76}\frac{\hat{\Omega}_h}{\varepsilon_h}\right|, \tag{7}$$

$$\gamma_{\nu=0} \approx -\mathrm{K}_\mu \frac{19\sqrt{2\varepsilon_h}}{8\pi^2}\beta_e\left(R_0\frac{d\varepsilon_h}{dt}\right)^2\varepsilon_t\left|\Omega_E-\frac{93}{76}\frac{\hat{\Omega}_h}{\varepsilon_h}\right|, \tag{8}$$

where $\mathrm{K}_{\nu=1,2} = [1, (8/3\pi)^2]; \mathrm{K}_{\mu=1,2} = (1, 2/3)$, $\hat{\Omega}_h = \Omega_h(\mathrm{E}=T)$, and all the quantities dependent on the radial coordinate are taken at the point $r_0$ determined by the equation $k_\parallel(m,n,r_0) = -k_\parallel(m+\mu, n+\nu N, r_0)$, i.e., the point where two cylindrical branches of Alfven continuum intersect. Note that when collisions are very weak, the damping rates given by Eqs. (7), (8) are almost independent on the collision frequency.

In the regime (b) the damping rates take the form similar to those for the axisymmetric plasmas

$$\frac{\gamma_{\nu=1,2}}{\omega} \approx -\mathrm{K}_\nu\frac{2}{3\pi}6.58\beta_e\left(\frac{R_0\varepsilon_h}{r_0}\right)^2\sqrt{\frac{\nu_e}{\omega}}\left[\ln 8\left(\frac{2\varepsilon_h\omega}{\nu_e}\right)^{1/2}\right]^{-3/2}, \tag{9}$$

$$\frac{\gamma_{\nu=0}}{\omega} \approx -\mathrm{K}_\mu\frac{6.58}{3\pi}\left(R_0\frac{d\varepsilon_h}{dr}\right)^2\beta_e\sqrt{\frac{\nu_e}{\omega}}\left[\ln 8\left(\frac{2\varepsilon_h\omega}{\nu_e}\right)^{1/2}\right]^{1/2}. \tag{10}$$

Let us evaluate the damping rates of various AEs in the Helias reactor. In this case, only the regime (b) is relevant to the thermal electrons and, thus, the effect of the orbit transformations on the collisional damping is negligible. This implies that we have to use Eqs. (9), (10). We consider the modes localized at $r/a = 0.3\text{-}0.5$, where $T_e \sim 10\ keV$, $n_e \sim 2\times10^{14} cm^{-3}$. Calculating damping rates and comparing them with corresponding growth rates $\gamma_\alpha$ obtained in Ref.[1], we obtain:

$$\left|\frac{\gamma_e}{\gamma_\alpha}\right|_{MAE} \approx \left|\frac{\gamma_e}{\gamma_\alpha}\right|_{HAE_{11}} \approx 0.4\ ,\quad \left|\frac{\gamma_e}{\gamma_\alpha}\right|_{TAE} \approx 0.6\,,$$

and $|\gamma_e|/\gamma_\alpha > 1$ for $HAE_{22}$, $HAE_{21}$, and $EAE$.

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[1] Ya.I. Kolesnichenko *et al.,* Phys. Plasmas **9,** 517 (2002).