

Self-generate shear flows in helical turbulence.

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The effect on large-scale dynamics of small-scale kinetic helicity injection in three-dimensional resistive magnetohydrodynamic is investigated. The magnetic configuration of such α^2 prototype flows is such that, generally, the magnetic energy concentrates on one helical large-scale mode. Such a process is mainly think to result from an Alfvénization of the medium at small scales, transferring helicity from the velocity to the magnetic field, followed by an inverse cascade of the magnetic helicity. Such dynamos are thought to be important in astrophysical turbulent systems like the sun, even if the $\alpha - \omega$ dynamo is also an important player in such processes. Many mechanisms may lead to the generation of kinetic helicity as, for instance, the classical convective processes but also hydrodynamic instabilities of stratified shear flow (see [1] for a review of those works).

Generally large scale instabilities models are describe by some equations of the following form

$$\frac{\partial}{\partial t} \mathbf{b}_{LS} = \alpha \nabla \times \mathbf{b}_{LS} + (\beta + \eta) \Delta \mathbf{b}_{LS}, \quad (1)$$

where α and β are parameters controlled by turbulent and statistical properties of the velocity field, and η is the resistivity. Indeed, kinematic approaches predict that α and β are respectively functions of the fluid helicity spectrum and the kinematic spectrum [2]. In particular α effect is associated to helical flows. This linear equation, in the context of a large scale helical magnetic field, leads to an instability with a growth rate given by

$$\sigma = |\alpha|k - (\beta + \eta)k^2. \quad (2)$$

It is clear from such equation that, for a given level of hydrodynamic turbulence, instability might not occur if the typical large scale involved are not large enough, and noteworthy, when the resistivity is too high. We might also expect that unstable modes not to be located at the largest scale of the volume where fluids is present, even if the largest scale are also unstable. However, numerical simulations [3, 4], because of the nonlinear process involved in the inverse cascade of the magnetic helicity, show that, for times long enough, only the largest scale concentrate the magnetic activity, the ones where also magnetic helicity is, in the end, present.

Let us focus now to situations such that the tachocline where α^2 -dynamo and anisotropy might be important: it can be interesting to the revisit such processes by introducing anisotropic flows where the largest scales of the volume depend of the orientation. Indeed, the inverse cascade induced by the isotropic small scales might be slowed down and eventually stopped: Let us define three orthogonal directions LX , LY and LZ with $LX = LY = LZ/n$, and where $n > 1$. If the dominant modes involved in the large scale growing up magnetic field are not k_z ones', a slow down should occur and eventually the inverse cascade stopped before reaching the largest scale. What would happen then? A decreasing of the magnetic field, a multi-helical states, something else? This is what are going to briefly investigate now by means of numerical simulations.

We perform the experiment in the frame of the incompressible and resistive MHD, where dynamics is describe by the following equations

$$\frac{D}{Dt}\mathbf{u} = -\nabla p + (\mathbf{b} \cdot \nabla)\mathbf{b} + \nu \nabla^2 \mathbf{u} + \varepsilon \mathbf{f}, \quad (3)$$

$$\frac{D}{Dt}\mathbf{b} = \mathbf{b} \cdot \nabla \mathbf{u} + \eta \nabla^2 \mathbf{b}, \quad (4)$$

$$\nabla \cdot \mathbf{b} = \nabla \cdot \mathbf{u} = 0, \quad (5)$$

in the usual units and where, by definition, η is the magnetic diffusivity, ν the kinematic viscosity and the pressure term p is the sum of the hydrostatic pressure and the magnetic pressure. Here, both fluid velocity \mathbf{u} and magnetic field \mathbf{b} are expressed in Alfvèn speed units. The magnetic Prandtl number is $P_m = \eta/\nu$ and we restrict ourselves to $P_m = 1$. The parameter ε is just to emphase that the injected energy is weak.

The flow is forced by divergence free fields \mathbf{f} centered around $k = 7$ with random phases. A few modes are updated randomly at every time step in order to decorrelate the forcing in time. Moreover, in the following, we will analyze the effect of an helical forcing, *i.e* with a energy spectrum concentrated on scales between 6.5 and 7.5, and such that for a given shape of the energetic spectrum of the forcing, the helicity $\mathbf{f} \cdot \nabla \mathbf{f}$ is maximal. The sizes of the rectangular box are $LX = LY = 2\pi$ and $LZ = 8\pi$. Note that similar forcings has been used by Brandenburg [3] in a compressible flow. Grid size is 144^3 : it means that non-linearities are computed by mean of 144^3 grids to avoid aliasing and linearities by mean of 96^3 grids. Initial conditions are $\mathbf{u} = 0$ and $\mathbf{b} \sim \varepsilon^2 \mathbf{f}(t = 0)$, *i.e* we introduce a weak small scale magnetic field to avoid a null magnetic field for any time. Kinematic viscosity ν and magnetic diffusivity are both set equal to 0.008.

The typical turnover time is around one unit. However, amplitude of the coloured forcing is high enough to induce a dynamo as can be seen in figure (1) where time evolution of kinetic and magnetic energy are plotted. The long time behavior of the simulation results in a still unsaturated state. Existence of a saturated state or not in such kind of simulation will be addressed in a subsequent paper. Careful analysis shows that inverse cascade of helicity is stopped between times 100 and 200 and corresponds a concentration of helical states around $2\pi/LX = 1 \leq k \leq 2$ and is linked to the necessity of a slow change of the orientation of the helical state in order to pursue the inverse cascade.

However, once the anisotropic large scale are contaminated by magnetic helicity ($t \geq 200$), inverse cascade does not allow a pile up of most of the magnetic energy at the largest scales. Indeed, this can be seen in figure (2), where we plot the spectrum of the flow obtained when magnetic activity is practically zero at forcing scales.

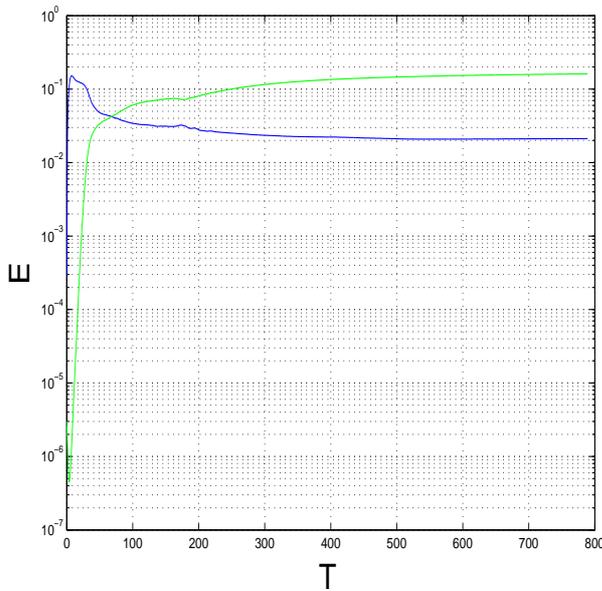


Figure 1: Time evolution of kinetic energy $E(\mathbf{u})$ (blue line), and magnetic energy $E(\mathbf{b})$ (green lines).

We observe that the magnetic mode possessing the maximal growth rate is since time $t \sim 250$ the one indicated by the arrow in the figure, $k \sim 0.350$; This also the most energetic one. In fact, all the modes where $k \leq 0.7$ have a positive growth rate, asymptotically decreasing towards zero. Moreover, we note that they are all in quasi-helical state.

An interesting consequence of such a multi-helical state is that the large scale Lorentz force is non zero. This point is to be compared with isotropic simulations [3, 4] where a single helical state is obtained and therefore $\mathbf{j} \times \mathbf{b} = 0$. The growth of a large scale shear flow is

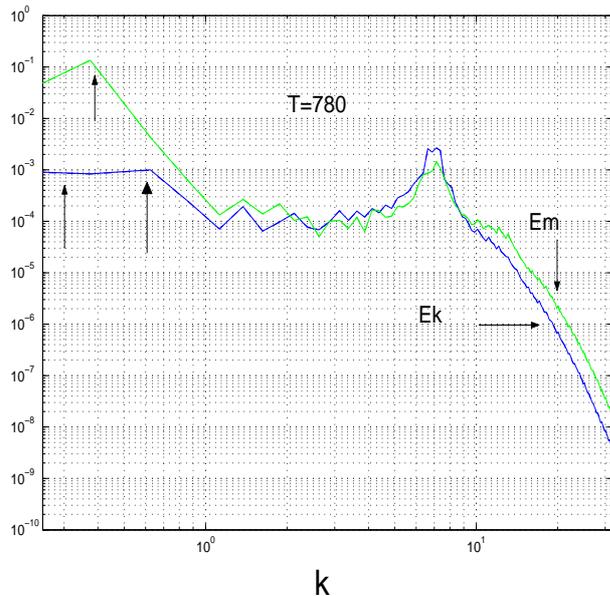


Figure 2: Spectra of kinetic (blue line) and magnetic (green line) energies at $T = 780$.

clear in the figure (2) where the energy of the large scale mode is comparable to the ones of the forcing. In fact at the end of the simulation the large scale shearflow keeps on growing (as indicated by the arrows in the figure). Those results open the question whether such kind of processes and/or mechanisms are relevant in the tachocline region where shear flow are known to be important. Can we expect, also, some kind of cycled states where shear flows instabilities generate small scales helicity which then generate shear flows and so on ...? The experiment in this paper is, of course, far from being realistic (level of the turbulence, compressibility, convection of the heat...), and it should be interesting to study it in the frame of more realistic ones.

References

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