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Kinetic drift effects in magnetic reconnection

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Introduction Magnetic reconnection is a key mechanism in transport processes both in (virtually collisionless) astrophysical plasmas and magnetic fusion experiments. The weakness of collisional dissipation challenges our understanding of high reconnection rates and release of magnetic energy. In weakly collisionless plasmas with a strong magnetic guide field, faster than resistive reconnection rates are possible due to electron inertia. Parallel electron compressibility can further accelerate this process, yielding fast reconnection rates comparable to those estimated from tokamak plasma instabilities [1]. The high reconnection rate involves the acceleration of electrons in an exceedingly thin current layer.

Reconnection in a magnetically confined plasma can bring together field lines with a temperature difference. In a collisionless plasma this leads to a temperature jump in the reconnection zone. As shown in [2], the drift-kinetic model of the electrons can resolve the temperature jump across the reconnection zone.

In the present paper, the nonlinear effect of such a temperature jump on the geometry of the reconnection zone is studied. To this end, we consider a simple equilibrium with a straight current layer that is unstable to tearing modes that can be explicitly described in drift-kinetic theory. In a quasi-linear approach, the effect of a temperature jump on the island produced by such an instability is discussed.

Drift-kinetic model We consider a strongly magnetized, low β plasma, with a strong magnetic guide field $\mathbf{B} = B_0(\mathbf{e}_z + \mathbf{e}_z \times \nabla\psi)$ and an electric field $\mathbf{E} = B_0(\mathbf{e}_z\partial_t\psi - \nabla\phi)$, where ϕ, ψ are the electrostatic and magnetic vector potential, respectively. The electrons are described by a distribution function $F(t, \mathbf{x}, v_\parallel)$ that satisfies the collisionless drift-kinetic equation

$$\partial_t F + [\phi, F] + v_\parallel \nabla_\parallel F = \Omega_e(\partial_t\psi - \nabla_\parallel\phi)\frac{\partial F}{\partial v_\parallel}.$$

The bracket is defined by $[A, B] = \mathbf{e}_z \cdot \nabla A \times \nabla B$. The ions are assumed to be cold. In order to maintain quasineutrality, they respond to parallel fluctuations of the electron density by moving perpendicular to the magnetic field with the $E \times B$-drift. This means that the electron density is related to the ion potential vorticity, $\nabla^2\phi = \Omega_i \log n_i +$ constant, with Ω_i the ion gyro-frequency, and $n_i = n_e$ the density. We obtain a closed system by taking the density and current density from the respective velocity space integrals of F. The vector potential ψ follows from Ampère's law,

$$n_i = n_e = \int dv_\parallel\, F, \qquad \nabla^2\psi = -\frac{e}{B_0}\int dv_\parallel\, v_\parallel\, F.$$

We presume z to be an ignorable coordinate so that $\nabla_\parallel = [\psi,\ ]$. The electron velocity distribution is almost homogeneous, $F(\mathbf{x}, v_\parallel, t) \approx F_0(v_\parallel)$. We use this to transform to a new function [3]

$$f(\mathbf{x}, v_\parallel, t) = F(\mathbf{x}, v_\parallel, t) - \Omega_e\psi\frac{dF_0}{dv_\parallel}.$$

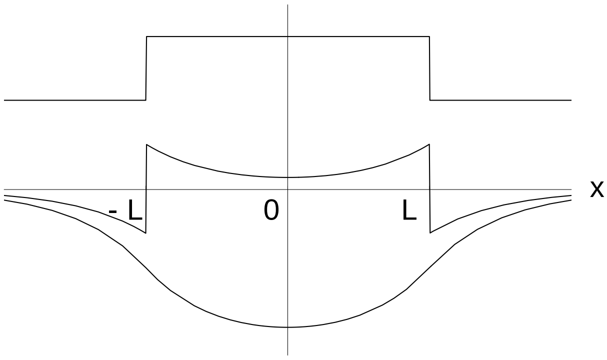

Fig. 1. The equilibrium geometry: The upper curve shows the x-dependence of the electron distribution function f. The central curve is the current density $\nabla^2\psi_0(x)$, showing a current layer at $|x| < L$ and screening at distances $> d_e$ (Here, $d_e = 0.5L$). The lower curve is ψ_0.

The purpose of this transformation is to obtain, for small ψ, a kinetic equation for the new distribution function that is to leading order a purely advective form,

$$\partial_t f + [\phi + v_\parallel \psi, f] = 0, \tag{1}$$

with $\phi + v_\parallel\psi$ a $v_\parallel$-dependent streamfunction. The system is closed by the quasi-neutrality condition and Ampère's law in the new variable,

$$\nabla^2\phi = \frac{\Omega_i}{n_0}\Big(\int dv_\parallel f - n_0\Big) \qquad \text{and} \qquad (\nabla^2 - d_e^{-2})\psi = -\frac{e}{B_0}\int dv_\parallel\, v_\parallel\, f. \tag{2}$$

Here, d_e is the electron inertial skin depth c/ω_{pe}.

Geometry, linear stability We consider an equilibrium in a y, z-independent slab geometry, with a current layer around $x = 0$, i.e. a layer in which $\int dv_\parallel\, v_\parallel f \neq 0$. The gradient, ∇f, is zero almost everywhere, except at the jumps at $x = \pm L$. We put (see Fig. 1)

$$f = \begin{cases} f_0 & \text{if } |x| > L, \\ f_0 + \delta f & \text{if } |x| < L. \end{cases}$$

The moments of the equilibrium distribution function are chosen to be

$$\int dv_\parallel f_0 = n_0, \quad \int dv_\parallel\, v_\parallel f_0 = 0, \quad \int dv_\parallel \delta f = 0, \quad \int dv_\parallel\, v_\parallel \delta f = -j_0\frac{B_0}{e}.$$

Ampère's law (2) gives the equilibrium expressions for ψ, shown in Fig. 1,

$$\psi_0(x) = -\frac{d_e^2}{2} j_0\, \left[\text{sign}(x+L)(1 - e^{-|x+L|/d_e}) - \text{sign}(x-L)(1 - e^{-|x-L|/d_e})\right].$$

Since f is piecewise constant and the kinetic equation (1) is purely advective, an instability can perturb f only by perturbing the boundaries at $x_\pm = \pm L$ with a $v_\parallel$-dependent displacement function $\xi(v_\parallel)$ to $x_\pm = \pm\big(L + \xi(v_\parallel)e^{i(ky-\omega t)}\big)$. The linear perturbation ξ depends on the perturbed streamfunction as $\partial_t\xi(v_\parallel) = \partial_y(\phi + v_\parallel\psi)\big|_{x=L}$. The eigenfunctions are

$$\phi = \phi_1 e^{kL}\left[e^{-k|x+L|} - e^{-k|x-L|}\right] e^{i(ky-\omega t)}, \tag{3}$$

$$\psi = \psi_1 e^{k'L}\left[e^{-k'|x+L|} + e^{-k'|x-L|}\right] e^{i(ky-\omega t)} + \psi_0(x), \tag{4}$$

with $k' = \sqrt{k^2 + d_e^{-2}}$. This yields a dispersion relation in ω and k:

$$\omega^2 = -j_0^2\frac{k}{k'}\frac{v_A^2}{4}\left(1 - e^{-2kL}\right)\left(1 + e^{-2k'L}\right), \quad \text{and} \quad \phi_1 = i\psi_1 v_A\sqrt{\frac{k'}{k}\,\frac{e^{2k'L}+1}{e^{2kL}-1}}.$$

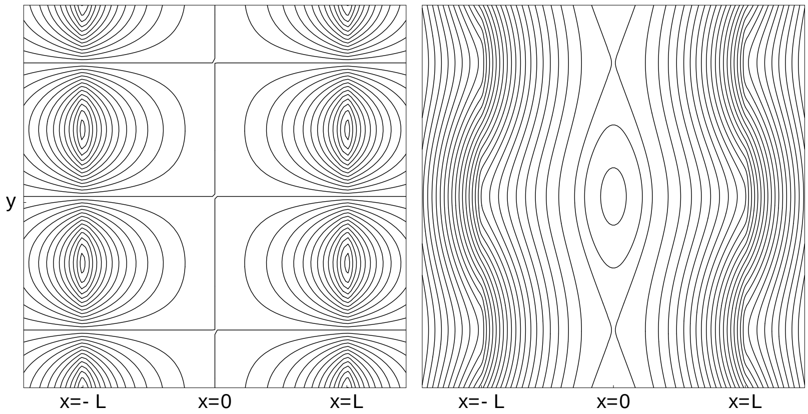

Fig. 2. The perturbed fields ϕ (left) and ψ (right). The jumps in $j(x)$ are located at $x = \pm L$

These modes are unstable for all values of k.

Temperature gradients We study the effect of a small temperature gradient dT/dx on the nonlinear reconnection process by giving the linear mode (3), (4) a finite amplitude, see Fig. 2. Here we do not consider the well-known effects of the pressure gradient and the diamagnetic velocity on the linear mode structure. Instead, we focus on the nonlinear effect of the steepening temperature gradient near the x-points as the islands grow. If the initial temperature gradient equals $dT/dx = T/L_T$, in the strongly nonlinear phase a temperature difference $\delta T = 2TL/L_T$ will build up across the x-point. There, reconnection will involve the merging of field lines that are at different temperatures. The kinetic model is necessary to describe the collisionless reconnection in the presence of this temperature jump.

Following [2], we study the nonlinear reconnection phase by assuming that the magnetic islands are well-developed and that the reconnection process is proceeding quasi-steadily. Then, the electron distribution function near the x-point is not evolving and hence is aligned with the stream-function, $[\phi + v_\parallel \psi, f] = 0$. In the plasma that streams towards the x-point from the left $(x < 0)$, this equation is trivially satisfied by a constant, hot, distribution function $f_+(v_\parallel)$ and the plasma that aproaches the reconnection zone from the right $(x > 0)$ has also a constant, but colder, distribution $f_-(v_\parallel)$. As outlined in Ref. [2], the two distribution functions meet at the separatrix of the stream function $\phi + v_\parallel \psi$, which is inside the island. However, for each value of $v_\parallel$ the separatrix is a different line through the x-point, and therefore the distribution function is position-dependent and changes smoothly from f_+ to f_- across the island.

After reconnection of a hot field line with a cold field line, the hot electrons travel along the reconnected field line more rapidly than the cold electrons, and it depends on the sign of $v_\parallel$ to which part of the island the electrons go. This inbalance produces a current perturbation inside the island,

$$j(\mathbf{x}) = -\frac{e}{B_0}\Big(\int_{-\infty}^{v_s} f_+ v_\parallel \, dv_\parallel + \int_{v_s}^{\infty} f_- v_\parallel \, dv_\parallel\Big), \tag{5}$$

where $v_s(\mathbf{x})$ is the resonant value of $v_\parallel$ at any position inside the island, $v_s = -\phi/\psi$. The island separatrices correspond to $v_s = \pm\infty$. Via Ampère's law, the current perturbation (5) perturbs the field ψ both inside and outside the island.

Sufficiently close to the x-point, the fields ϕ and ψ are both quadratic in x and y so that the separatrices of the stream function $\phi + v_\parallel \psi$ can be considered straight lines through the origin. The current perturbation is constant on these lines. Ampère's law can be integrated across the island. Thus one finds from (5) that the vector potential outside the island,

$$\psi = 2d_e^2 j_\times \cosh(x/d_e) + 2\psi_1 \cosh(k'x)\,\cos(ky), \qquad \text{with} \quad j_\times = j_0 e^{-L/d_e},$$

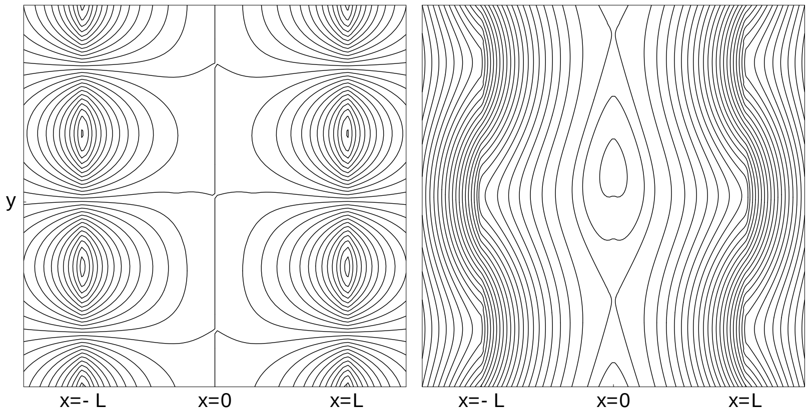

Fig. 3. The fields $\phi + \tilde{\phi}$ (left) and $\psi + \tilde{\psi}$ (right) in the presence of a temperature difference across the magnetic island. Visible are the up-down asymmetry of the island and its shift δy with respect to the outer mode structure.

is perturbed by a contribution of the form $\tilde{\psi} = \tilde{\psi}_1 \sinh(k'|x|)\sin(ky)$. This contribution perturbs the separatrix angles at the x-point, making the angles different for $y \uparrow 0$ and $y \downarrow 0$,

$$\theta = \theta_0 + \mathrm{sign}(y)\,\tilde{\psi}_1 k'k/j_\times, \qquad \text{with} \quad \theta_0 = 2k\sqrt{\psi_1/j_\times}.$$

The result is that the island becomes droplet-shaped, as shown in figure 3. The actual integration of Ampère's law across the island can be rewritten as an integral over v_s,

$$\tilde{\psi}_1 = -\frac{e\theta_0}{4kk'B_0}\int_{-\infty}^{\infty} dv_s\,|v_s|\big(f_+(v_s) - f_-(v_s)\big).$$

For distributions $f_\pm$ that have a temperature difference $\delta T = mv_{th}\delta v_{th}$, the integral yields

$$\tilde{\psi}_1 = -\frac{e\theta_0 n_0\,\delta v_{th}}{4\sqrt{\pi}k'kB_0}.$$

Hence the upper and lower separatrix angles of the island are $\theta_0\big(1 \pm \frac{1}{4}\pi^{-1/2}en_0\,\delta v_{th}/j_\times B_0\big)$. The perturbation $\tilde{\psi}_1$ also brings about a vertical shift of the x-point relative to the far fields. This shift is $\delta y = \tilde{\psi}_1/(2k\psi_1)$. The shift is in the electron diamagnetic direction and can be expressed as $\delta y = (v_{*T}/|\omega|)(v_A/v_{th})(kd_e)^{-1/2}$, where v_{*T} is the electron diamagnetic velocity calculated for the temperature gradient over the island width.

Discussion When reconnection yields a finite-size magnetic island in a plasma with a cross-field temperature gradient, a much steeper gradient builds up across the x-point. Then, reconnection gives rise to opposite island current perturbations above and below the x-point. These currents amount to a deformation of the island separatrix and shift of the x-point in the electron diamagnetic direction. This shift can be seen as the result of the fact that near the x-point, the local diamagnetic velocity is much higher than in the surrounding plasma.

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