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Statistical model for the laser beam multiple scattering on self-induced density fluctuations

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We develop a statistical model for the laser beam multiple scattering on self-induced density fluctuations. According to the equation on the electric field correlation function, the propagation through a random media decreases the transverse correlation length of the electric field and increases the beam divergence. The correlation function of the density fluctuations driven by the randomized laser beam shows a complicated structure associated with moving and static perturbations.

In the context of Inertial Confinement Fusion it is important to control the laser beam coherence. Recent experiments[1] show that spatio-temporal smoothing could be induced by the plasma fluctuations, which is an attractive possibility for an efficient control of the laser transport and energy deposition on the target.

For laser intensities below the self-focusing threshold, the laser multiple scattering on self-induced density fluctuations has been proposed as the dominant mechanism for plasma smoothing. In this work we develop a statistical model for this process and investigate the statistical properties of plasma density fluctuations.

We consider the propagation of a laser beam through a plasma with an inhomogeneous density. The electric field envelop satisfies the paraxial equation, which is widely used to describe a regular beam propagation. However, for a random beam, it is more relevant to deal with statistical average quantities, the electric field correlation function :

$$\Gamma_{EE*}\left(\vec{R},\vec{\rho},z\right) = \left\langle E\left(\vec{R}+\frac{\vec{\rho}}{2},z\right)\,E^*\left(\vec{R}-\frac{\vec{\rho}}{2},z\right)\right\rangle ,$$

where $\langle ... \rangle$ represents a statistical average in the plane perpendicular to the propagation axis, and the density correlation function :

$$D^N(\vec{R},\vec{\rho},z) = \left\langle \delta n\left(\vec{R}+\frac{\vec{\rho}}{2},z\right)\,\delta n\left(\vec{R}-\frac{\vec{\rho}}{2},z\right)\right\rangle .$$

An equation describing the propagation of the electric field correlation function through plasma density fluctuations δn follows from the paraxial equation. It is obtained assuming the gaussian statistics and neglecting side- and back-scattering[2] :

$$\left(\frac{\partial}{\partial z} - \frac{i}{k_0}\vec{\nabla}_{\vec{R}}.\vec{\nabla}_{\vec{\rho}}\right)\Gamma_{EE*}(\vec{R},\vec{\rho},z) =$$
$$= -\sqrt{2\pi}\,L_R\,\frac{\omega_{p0}^4}{4k_0^2c^4}\left[D^N(\vec{0},z) - D^N(\vec{\rho},z)\right]\Gamma_{EE*}(\vec{R},\vec{\rho},z)\,. \qquad (1)$$

The left-hand side term describes the propagation of the correlation function in a media without fluctuations whereas the right-hand side term represents the effect of fluctuations.

An incident random gaussian beam propagating in a media without fluctuations preserves its gaussian profile with modification of the beam and hot-spots width along the propagation axis. We find the classical results for focalisation and diffraction of a spatially smoothed beam through a free media. The propagation of such a beam in a media with gaussian density perturbations decreases the transverse correlation length :

$$\xi(\Lambda) = \frac{\rho_\Lambda(z)}{\rho_c(z)} = \left(1 + \frac{\Lambda}{\Lambda_{car}}\right)^{-1/2} \text{ with : } \Lambda_{car} = \frac{a^2}{\rho_0^2}\,\frac{4k_0^2c^2}{\sqrt{2\pi}L_R\delta n_0^2\omega_{p0}^4}\,.$$

Here a is the density correlation length and the plasma length Λ is supposed to be shorter than the Rayleigh length. The effect of multiple scattering is important after the distance Λ_{car} that depends on the plasma density fluctuation level. Note that the decrease of the transverse correlation, and therefore of the Rayleigh length, induces an increase of the divergence of the transmitted light (Fig. 1).

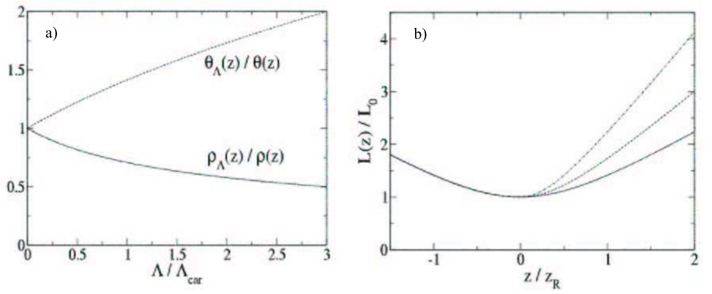

FIG. 1 – (a) Evolution of the correlation length and diffraction angle with media length Λ. (b) Evolution of the laser envelop width with z in vacuum (solid curve) for : $\Lambda = \Lambda_{car}$ (dashed curve) and $\Lambda = 2\Lambda_{car}$ (dash-dot curve).

The plasma density fluctuations responsible for the multiple diffusion of the laser beam can be induced by the laser beam itself. We consider the plasma response to the ponderomotive force of the laser beam using a simple wave equation :

$$(\partial_t^2 + c_s^2\vec{k}^2)\,\widetilde{\delta n}(\vec{k},t) = -Z/(2c\,n_c\,m_i)\,\vec{k}^2\,\tilde{I}(\vec{k},t)\,. \qquad (2)$$

It describes the propagation of a density wave with the sound velocity c_s in a transverse plane. Analytical expressions in the one- and two-dimensional cases have been obtained for regular beam. They show that the characteristic excitation time is ρ_0/c_s where ρ_0 is the transverse width of the beam. In the 1D case the density profile presents a stationary depression and two humps propagating with c_s in the opposite directions (Fig. 2a). In the 2D case the ponderomotive force is stronger which creates a hole propagating just behind the hump (Fig. 2b). Moreover, we study the dependance on the intensity raise-time t_m of the density level. It shows that the humps amplitude decreases as $1/t_m$ whereas their width increases as a linear function of t_m.

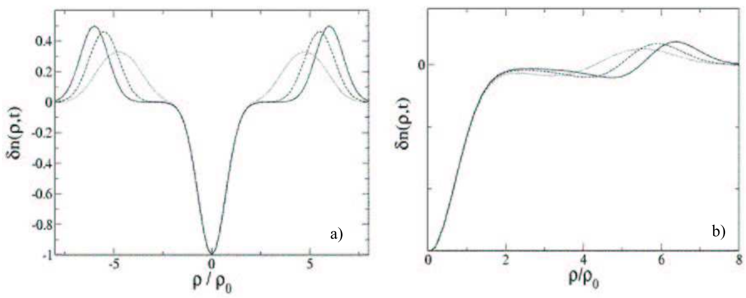

FIG. 2 – Density profile at $T \sim 6\rho_0/c_s$ in the 1D case (a) and the 2D case (b) for different raise-time : $t_m = 0$ (solid curve), $t_m \sim \rho_0/c_s$ (dashed curve), $t_m \sim 2\rho_0/c_s$ (doted curve).

For a spatially smoothed beam, one can consider that each speckle creates a stationary density depression and a propagating hump. Then by using the analytical solutions to (2) one can calculate the density correlation function (Fig. 3a) :

$$\widetilde{D_N}(\vec{k},\omega,T) = \langle \widetilde{I}(\vec{k})\,\widetilde{I}(-\vec{k})\rangle \Big[A(T,t_m,k)\,\delta(\omega) + B(T,t_m,k)\,\Big(\delta(\omega+c_sk)+ \delta(\omega-c_sk)\Big) - C(T,t_m,k)\,\Big(\delta(\omega+\frac{c_sk}{2})+\delta(\omega-\frac{c_sk}{2})\Big)\Big]. \quad (3)$$

This expression of the density fluctuations spectrum is a function of the macroscopic time T and the Fourier variables associated with the correlation time and coordinate. The first term A in brackets is responsible for diffusion without frequency shift. It is due to the correlation between stationary depressions (that does not depend on the raise-time t_m) and between humps propagating in the same direction (that decreases as $1/t_m^2$). The second term B represents correlations between humps and corresponds to the acoustic free mode of the plasma ($\omega = \pm kc_s$). Its amplitude does not depend on the time T but decreases as $1/t_m^2$. Lastly, the third term C describes the correlation between humps and stationary depressions. It decreases as $1/t_m$ so that for sufficiently long intensity raise-time, its contribution is the most important. This component is especially interesting because this resonance ($\omega = \pm k c_s/2$) is responsible for an additional component in the scattering spectrum.

These results can be obtained by deriving a general equation for the density fluctuation correlation function :

$$\left(\frac{1}{16}\partial_T^4 + \frac{1}{2}\left(\omega^2 + c_s^2 k^2\right)\partial_T^2 - \left(\omega^2 - c_s^2 k^2\right)^2\right)\widetilde{D_N} = \left(\frac{Z}{2cn_c m_i}\right)^2 \vec{k}^4\,\widetilde{\Gamma_I}\,. \tag{4}$$

Here $\widetilde{\Gamma_I}$ is the intensity correlation function. Assuming gaussian statistics it can be expressed using the electric field correlation function. Equation (4) allows us to consider spatially and/or temporally random laser beam. For beam with a coherence time smaller than ρ_0/c_s the stationary density depressions disappear as well as the resonance $\omega = \pm k c_s/2$ (Fig. 3b).

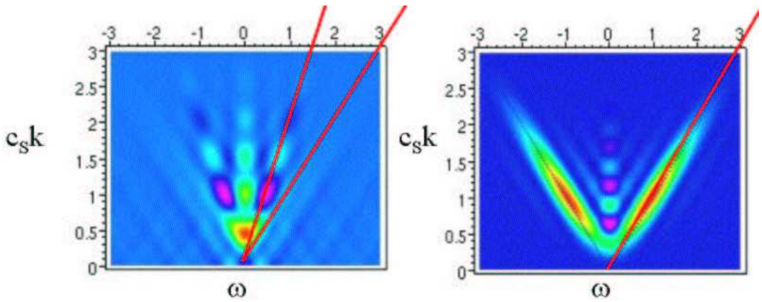

FIG. 3 – Density fluctuations spectrum derived from our model. (a) spatially smoothed laser beam (b) spatially and temporally smoothed laser beam with $\tau_c \ll \rho_0/c_s$.

As a conclusion, the laser beam multiple scattering on self-induced density fluctuations has been analysed. It is responsible for spatial and temporal coherence losses and increase of the transmited beam divergence. A spectrum for the density fluctuations has been derived and we have found a new resonance $\omega = \pm k c_s/2$. Analysis of the set of coupled equations (1) and (4) would allow to give a complete statistical description of the plasma smoothing effect.

References

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