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# Small-Angle Multi-Scattering Contribution to the Doppler Reflectometry Signal

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## 1. Introduction

One of widespread methods used nowadays for plasma rotation velocity measurements is Doppler reflectometry [1]. This technique provides measuring fluctuations propagation poloidal velocity which is often shown to be dominated by plasma poloidal rotation. The method is based on plasma probing with a microwave beam which is tilted with respect to plasma density gradient. A back-scattered signal with frequency differing from the probing one is registered by a nearby standing or the same antenna. The information on plasma poloidal rotation is obtained in this technique from the frequency shift of the backscattering spectrum which is supposed to originate from the Doppler effect due to the fluctuation rotation.

Usually received signal is assumed to be produced by linear mechanism of single scattering in the cut-off vicinity. In this case the problem was investigated analytically in recent paper [2] in assumption of small enough amplitude of the fluctuations leading to the scattering allowing us to neglect multiple scattering contribution to the received signal. However, this approximation can be incorrect at high turbulence level or in large plasma devices, where probing ray trajectory is long.

In the present paper the limit of dominant small-angle multi-scattering and negligible backscattering (BS) contribution to the Doppler reflectometry signal is considered. The nonlinear theory of Doppler reflectometry is developed in assumption of long wavelength of fluctuations, which usually holds true in tokamak plasma. The turbulence is assumed to be strong enough to change significantly poloidal wavenumber of the probing wave due to multiple small-angle scattering effect and together with the reflection off the cut-off to turn the probing wave toward the receiving antenna.

## 2. Autocorrelation function of reflectometry signal

We use slab background plasma model, which is reliable for large plasma devices. We suppose that $x$-axe corresponds to the direction of the density gradient, $y$-axe determines poloidal direction, and $z$-axe is a direction of external magnetic field. We consider the same tilted gaussian antenna pattern both for probing and receiving antennas, which are supposed to be situated in the origin.

We are interested in normalized autocorrelation function of Doppler reflectometry signal

$$C(t_1 - t_2) = \frac{\langle A_s(t_1) A_s^*(t_2)\rangle - |\langle A_s\rangle|^2}{\left\langle |A_s|^2 \right\rangle - |\langle A_s\rangle|^2} \tag{1}$$

The received signal amplitude $A_s$ can be calculated using reciprocity theorem [3]

$$A_s = \frac{c}{16\pi} \int_{-\infty}^{+\infty} \mathrm{d}y E(x=0,y) E_a(y) \tag{2}$$

Here $E(y)$ is the reflected wave electric field, created by probing antenna at the plasma boundary, which is integrated over all plasma boundary with weight function proportional to electric field of the receiving antenna $E_a(y)$

$$E_a(y) = \left(\frac{8\sqrt{\pi}}{c\rho}\right)^{1/2} \mathrm{e}^{i\mathcal{K}y - y^2/2\rho^2}$$

Here $\rho$ denotes antenna beam width, $\mathcal{K}$ corresponds to the antenna tilt ($\mathcal{K} = \omega_i/c \sin\vartheta$, where $\vartheta$ is the tilt angle in respect to the density gradient).

The electric field, created by the probing antenna in the plasma volume $E(x, y)$ should be evaluated taking into account plasma turbulence. In case of long-scale turbulence, leading to probing wave forward-scattering, which could be taken into account as a probing wave phase shift, this electric field at the plasma border can be represented in the following form

$$E(l_0, y') = \exp\left\{2\mathrm{i}\int_0^{x_c} k(x')\, dx'\right\} \int_{-\infty}^{+\infty} G[l_0, y'|0, y; t] E_a^{(i)}(y)\, \mathrm{d}y$$

Here $k^2(x) = \left[\omega_i^2 - \omega_{pe}^2(x)\right]/c^2$ is a square of the probing wave wavenumber, $l_0$ is a probing ray trajectory length $l_0 = \omega_i/c \int_0^{x_c} \mathrm{d}x'/k(x')$, $x_c$ is the distance between the plasma boundary and the cut-off. The function $G$ takes into account beam diffraction and turbulent phase shift

$$G\left[l_0, y'|0, y; t\right] = \sqrt{\frac{\omega_i}{2\pi c l}} \exp\left\{\frac{\mathrm{i}\omega_i}{2c}\left[\frac{(y'-y)^2}{l_0} - \int_0^{l_0} \tilde{n}\left[x(l'), y(l'), t\right]\, \mathrm{d}l'\right] - \frac{\mathrm{i}\pi}{4}\right\}$$

where $\tilde{n}(x, y, t) = \delta n(x, y, t)/n_c$ is the density fluctuation normalized to the density in the cut-off, $x(l)$, $y(l)$ denote probing ray trajectory $y(l') = y + l'(y' - y)/l_0$.

In nonlinear regime, when the following criterion [4] is fulfilled

$$\gamma = \frac{\omega_i^2}{c^2}\, \tilde{n}^2 x_c \ell_{cx} \ln\frac{x_c}{\ell_{cx}} \gtrsim 1$$

where $\ell_{cx}$ denotes radial correlation length of the turbulence, this representation allows us to obtain autocorrelation function (1) in the following form

$$C(t_1 - t_2) = \exp\left\{-\frac{(t_1 - t_2)^2}{2}\left[\hat{\mathcal{L}}\left(\Omega^2 + q^2 v^2\right) - \frac{\left(\hat{\mathcal{L}} q^2 v\right)^2}{\rho^{-2} + \hat{\mathcal{L}} q^2}\right] + \frac{2\mathrm{i}\mathcal{K}(t_1 - t_2)\hat{\mathcal{L}} q^2 v}{\rho^{-2} + \hat{\mathcal{L}} q^2}\right\}$$

Here $v = v(x)$ is a plasma poloidal rotation velocity, and an operator $\hat{\mathcal{L}}$ can be approximately represented as follows

$$\hat{\mathcal{L}}\xi \simeq \frac{\omega_i^2}{c^2}\int_0^{x_c} \mathrm{d}x\, \tilde{n}^2(x) \int_{-\infty}^{+\infty} \frac{\mathrm{d}\varkappa\, \mathrm{d}q\, \mathrm{d}\Omega}{(2\pi)^3} \left|n\left(\varkappa, q, \Omega\right)\right|^2 \xi \begin{cases} \frac{\omega_i^2 \delta(\varkappa)}{c^2 k^2(x)}, & x_c - x > \ell_{cx}/4 \\ 4x_c, & x_c - x \le \ell_{cx}/4 \end{cases}$$

The factor $\tilde{n}(x)$ denotes turbulence amplitude, which is assumed to be inhomogeneous, and $|n(\varkappa, q, \Omega)|^2$ is the turbulence spectral density, where $\varkappa$, $q$ are radial and poloidal wavenumbers, and $\Omega$ is a frequency of the turbulence.

## 3. Signal frequency spectrum

Fourier transformation of correlation function (1) gives us registered signal frequency spectrum

$$S(\omega) \propto \exp\left\{ -\frac{1}{2} \frac{\left[\omega - \omega_i + 2\mathcal{K}\hat{\mathcal{L}}q^2 v\left(\rho^{-2} + \hat{\mathcal{L}}q^2\right)^{-1}\right]^2}{\hat{\mathcal{L}}\left(\Omega^2 + q^2 v^2\right) - \left(\hat{\mathcal{L}}q^2 v\right)^2 \left(\rho^{-2} + \hat{\mathcal{L}}q^2\right)^{-1}} \right\} \tag{3}$$

To interpret this result we consider simple case of homogeneous plasma poloidal rotation $v(x) = v$, which gives $\hat{\mathcal{L}}q^2 v = v\hat{\mathcal{L}}q^2$. Assuming strong non-linear regime, when antenna beam divergence is completely determined by the turbulence $\left(\rho^2 \hat{\mathcal{L}}q^2 \gg 1\right)$ one can conclude from (3) that spectrum frequency shift is determined by traditional (linear) Doppler effect: $\omega_{max} = \omega_i - 2\mathcal{K}v$. On the contrary, the spectrum broadening $\Delta\omega = \sqrt{\hat{\mathcal{L}}\Omega^2}$ is strongly influenced in non-linear case by the turbulence amplitude and differs from linear one

$$\Delta\omega_{lin} = \left[\int_{-\infty}^{+\infty} \frac{dq\, d\Omega}{(2\pi)^2} \left|n(q, \Omega)\right|^2 \Omega^2\right]^{1/2}$$

In case of inhomogeneous plasma poloidal rotation the spectrum frequency shift is actually determined by the 'mean' product of the turbulence amplitude and rotation velocity

$$\omega_{max} = \omega_i - \frac{2\mathcal{K}\hat{\mathcal{L}}q^2 v}{\rho^{-2} + \hat{\mathcal{L}}q^2}$$

It means that the frequency spectrum shift can be produced by the region with high amplitude of the turbulence as well as by the region with high poloidal velocity.

The frequency spectrum broadening, which in case of homogeneous plasma poloidal rotation is caused by intrinsic frequency spectrum of fluctuations, is influenced here by additional factor associated with poloidal rotation inhomogeneity.

$$\Delta\omega = \left[\hat{\mathcal{L}}\left(\Omega^2 + q^2 v^2\right) - \frac{\left(\hat{\mathcal{L}}q^2 v\right)^2}{\rho^{-2} + \hat{\mathcal{L}}q^2}\right]^{1/2}$$

## 4. Discussion

We obtain the criterion, when the effect considered is important for diagnostics results interpretation. That is the case when small-angle scattering contribution is essential in the received signal. The condition to be met is that small-angle scattering signal amplitude should be comparable or larger than back-scattering (BS) signal, formed by linear mechanism. Small-angle scattering contribution amplitude (2) can be evaluated as

$$|A_s|^2 \sim \frac{P_i}{4}\frac{\omega_i \ell_{cy}^2}{c x_c \gamma} \exp\left\{-\frac{2\mathcal{K}^2\ell_{cy}^2}{\gamma^2}\right\}$$

The BS signal amplitude can be estimated as [2]

$$\left|A_s^{BS}\right|^2 \sim \frac{P_i}{2\sqrt{2\pi}}\gamma\frac{\omega_i \rho}{c x_c}\left|n(-2\mathcal{K})\right|^2$$

Here $|n(q)|^2$ is the fluctuation poloidal wavenumber spectrum. The ratio of the signal amplitudes

$$\alpha \sim \frac{\ell_{cy}^2}{\rho\gamma^2}\frac{\exp\left\{-2\mathcal{K}^2\ell_{cy}^2/\gamma^2\right\}}{|n(-2\mathcal{K})|^2}$$

For example, we consider turbulence spectral density $|n(q)|^2 = 4\pi\ell_{cy}\left[1+q^2\ell_{cy}^2\right]^{-3/2}$, where $\ell_{cy} \sim 2$ cm, probing frequency $f = \omega_i/2\pi = 60$ GHz, tilt angle $\theta = 30°$, $\rho = 2$ cm. Then if $\gamma = 20$ the small-angle scattering contribution larger than BS one: $\alpha \sim 1.5$.

## 5. Conclusion

The multiple forward scattering effect is taken into account in non-linear analytical theory of Doppler reflectometry, which is developed in geometrical optics approximation in slab plasma model. It is demonstrated that the frequency shift gives an information on poloidal velocity averaged over the vicinity of the cut-off, the size of which depends on the density profile and turbulence distribution. The spectrum is demonstrated to acquire additional broadening due to poloidal velocity inhomogeneity.

Thus, even in the complicated situation of multi-scattering dominance Doppler reflectometry technique is proved to be able to give more or less realistic information on plasma rotation, and the results presented allows the diagnostics results to be adequately interpreted and the spatial resolution of the method to be analyzed for real experimental conditions.

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